Binary-encounter electron production in energetic heavy-bare-ion–atom collisions

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The production of binary-encounter electrons in heavy-ion–atom collisions at high impact velocities is studied. The theoretical model used is the continuum-distorted-wave–eikonal-initial-state approximation. Deviations from classical predictions for the height and position of the associated peak in double-differential cross sections are determined. A simple adiabatic resonant-tunneling model is developed to interpret the physical meaning of the shift in the position of the binary-encounter peak.

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I. INTRODUCTION

Recent experimental studies of electrons emitted in high-velocity multiply charged *bare*-ion–atom collisions have started to look at the high energy side of the spectra. For electron emission angles θ smaller than 90°, the main feature of the spectra is the presence of the binary-encounter peak (BEP) [except at 0° where the so-called electron capture to the continuum (ECC) appears]. This peak arises from the binary collision between the impinging ion and the active electron. From the classical laws of energy and momentum conservation for the projectile-electron subsystem it is possible to obtain the position of the BEP. This is given by the well-known formula (in atomic units): $k = 2v\cos(\theta)$ [1], where k is the final electron momentum and v the projectile velocity. If the binding energy of the electron in the initial state (ε_i) is considered, the position of the BEP is shifted to lower electron energies by a factor which is proportional to ε_i (approximately $2\varepsilon_i$, see for example [2]). This results in a shift of the order of 50 eV for He. If the double-differential cross section (DDCS) is calculated using this binary collision picture, the DDCS is the Rutherford cross section averaged over the initial-state momentum distribution. This is the so-called binary-encounter approximation (BEA) [3]. In this model the DDCS is proportional to the square of the projectile charge (Z_P), which is the well-known Z_P^2-scaling law also given by the first-order Born approximation (FBA) [4].

Experimental results for proton impact are in agreement with this picture and recent measurements using multiply charged *bare* ions [5–7] show that the Z_P^2-scaling law holds for $Z_P \leq 9$. Differences appear for the position of the BEP. While Lee *et al.* [5] find that for $Z > 1$ the position of the peak is independent of Z_P, in the experiments of Pedersen *et al.* [6, 7] the shift increases, to lower electron energies, with Z_P. By applying a simple model based on ideas of Bohr and Lindhard [8], they obtained a scaling law for the shift of the BEP which is proportional to $Z_P^{1/2}$. This is in qualitative agreement with their experimental data and with calculations using the continuum-distorted-wave–eikonal-initial-state (CDW-EIS) approximation ([2] and references therein). The main idea behind the Bohr-Lindhard (BL) model is that ionization takes place in the initial channel at some internuclear distance where the attractive force of the incoming heavy ion equals the binding force of the target on the electron. In the CDW-EIS model this is taken into account by distorting the initial bound state with an eikonal phase.

From the theoretical point of view there are also different results. As was mentioned above the CDW-EIS model predicts that the shift is higher than the one predicted by the FBA or BEA in agreement with the data from Pedersen *et al.* [6, 7]. Recent calculations using a similar model, where the eikonal initial state (EIS) is replaced by the exact impulse approximation (IA) [9], give very different results. The IA predicts a shift in the BEP which is *smaller* than the one predicted by either the FBA or BEA. It was suggested by these authors that this is an *antibinding* effect because it has the opposite effect to the one introduced by the binding energy.

As the experimental results show a different behavior from the one expected from the classical analysis of the BEP, and the sophisticated quantum calculations give opposite results (*binding-antibinding*), it is then of interest to formulate some simple qualitative model to understand the process giving rise to the BEP. This appears to be a little more complicated than the classical head-on collision which has been assumed so far and which may be valid for light projectiles. These physical models can then be compared with the more complicated quantum calculations and with the experimental data.

In this work we present a simple model to explain the origin of the BEP that gives a scaling law for the position of the BEP as a function of Z_P. This is compared with experimental data and with results obtained from the CDW-EIS model which is also used to study the dependence of the DDCS at the BEP on Z_P. In this last case we find that for the systems where measurements are available [5–7] the CDW-EIS results agree with the experimental data. For higher values of Z_P, CDW-EIS deviates from the scaling law predicted by the FBA.

Atomic units will be used except where otherwise stated.

II. THE BINARY-ENCOUNTER PEAK POSITION IN ASYMMETRIC COLLISIONS ($Z_P > Z_T$)

We consider the collision of a bare charge Z_P with a hydrogenic atom of nuclear charge Z_T, electronic wave function $\phi_i(\mathbf{r})$, and binding energy ε_i. Eventually, Z_T, ϕ_i, and ε_i may also refer to the effective charge, wave function, and energy of the active electron in the collision, when a multielectronic atom is described by an independent-particle approximation.

The characteristic velocity of the electron in the initial state of principal quantum number n is $v_e = Z_T/n$. We will consider *fast collisions*, defined by

$$v \gg v_e \tag{1}$$

where v is the relative velocity of the $P+(T+e)$ system. As a result of the collision, the active electron is ejected with momentum $\mathbf{k}$ relative to the target. If we consider this an *impulsive process*:

$$k \gg v_e, \tag{2}$$

then the electron response time $1/v_e$ in the initial state is much longer than the collision time $1/k$ needed by the electron to leave the atom and the classical picture is valid.

The active electron is in the continuum of the projectile P in the entrance channel, and of both P and T in the ionization channel. The long-range character of these Coulomb forces should be incorporated in the asymptotic form of the electronic wave function for the initial and final states. The condition (1) of fast collision indicates that $Z_T/nv \ll 1$, so a first-order Born-type perturbation approach in the Z_T-e interaction for the transition amplitude is justified. When $Z_P \leq Z_T$ the Z_P-e interaction is also adequate for such an approximation. Since we will be interested in the case where $Z_P > Z_T$, Z_P/v will not be small in general. On the contrary, the projectile potential may be dominant and its action on the electron needed to be fully considered throughout the collision process.

Binary encounter electrons are produced in ion-atom collisions when the target potential is negligible compared with that of the projectile. Since the average T-e distance is $a_i = 3/2Z_T$ for a ground-state hydrogenic orbital, the typical P-e separations s that define the region of the binary collision are

$$s < s_b \text{ with } s_b = a_i \; Z_P/Z_T. \tag{3}$$

For light projectiles such that $Z_P < Z_T$, this region is smaller than the dimension a_i of the target orbital, so the binary collision develops inside the initial state, and both before and after the close P-e encounter the dynamics is ruled by the target interaction. The only role played by the projectile outside the region of the binary encounter will be given by the asymptotic distortion of the electron state due to the long-range P-e Coulomb interaction.

For large Z_P such that $Z_P > Z_T$, the situation is very different. The electron feels equivalent projectile and target forces at P-e distances s_b that, according to (3), are larger than the atomic radius. Therefore, the polarization of the initial state by the projectile cannot be neglected. Electron emission is no longer an impulsive process in the sense that now the time interval where the projectile action is relevant may become appreciable. For the case $Z_P > Z_T$, either ionization or charge transfer may proceed through a resonant tunneling of the electron through the potential barrier of finite width, formed by the projectile electric field superimposed on the target atomic potential. The resonance condition includes the electron translation energy, and expressed as it should in the center-of-mass system gives

$$\varepsilon_P = \varepsilon_i + \frac{1}{2}\frac{M_P - M_T}{M_P + M_T}v^2 \tag{4}$$

where ε_P is the electron energy of the final state centered on the projectile, and $M_{T,P}$ the target and projectile masses.

For small $v\,(v < Z_T)$ and large asymmetry of charges ($Z_P \gg Z_T$), there is an almost continuum density of projectile Rydberg states with energy ε_P. In our case of $v \gg Z_T$, the resonance condition is satisfied in the continuum of projectile states. Since the asymmetry also applies to the masses ($M_P \gg M_T$), the binary electron energy is obtained from (4) as

$$\varepsilon_P = \varepsilon_i + \tfrac{1}{2}v^2. \tag{5}$$

This result assumes that the electron resonant tunneling is produced at very large internuclear distances R.

Resonant electron capture in asymmetric collision systems has received a great deal of attention in the past [10, 11]. At the low collision energies where this process is dominant, a description in terms of adiabatic electron states is justified. Transitions to final states centered on the projectile were treated using the Landau–Zener curve-crossing model, and an effective internuclear distance R_c, where the electron transfer is produced, has been obtained as a function of Z_P [10]. Typical values obtained are $R_c(Z_P = 5) = 7$ a.u., and $R_c(Z_P = 50) = 20$ a.u. These results are consistent with accurate solutions of the Schrödinger equation to calculate the electron tunneling that, furthermore, show R_c to be proportional to $Z_P^{1/2}$ and to have a weak logarithmic dependence on v [11].

For collision energies such that $v \gg Z_T$, the assumption of electron emission by an adiabatic electron transfer may still be justified. At large R the projectile potential to leading orders in R is

$$V_P = -\frac{Z_P}{R} + Z_P \frac{\mathbf{x}\cdot\mathbf{R}}{R^3} + O(R^{-3}). \tag{6}$$

The nonadiabatic coupling associated with nonresonant transitions is produced by the time derivative of this potential, of order $v\,Z_P/R^4$. Using the values of R_c just quoted, we see that this coupling is negligible in the region $R > R_c$, where the resonant tunneling is effectively depleting the initial state, depletion that is completed at $R \simeq R_c$. In Eq. (6), $\mathbf{x}$ is the electron position vector with respect to the target nucleus.

An elementary model for the electron resonant tunneling from the initial target bound state to a final projectile continuum state will provide us with a simple expression for R_c in terms of Z_P and Z_T. To reach the continuum from the state with binding energy ε_i, the electron should tunnel through a barrier of width d. If we define R_c as the value of the internuclear distance for which the barrier width is equal to the orbital radius $d = a_i$ we obtain

$$V_P(R_c) - V_P(R_c - d) \simeq |\varepsilon_i|. \tag{7}$$

This means that at the distance R_c the binding energy of the electron has been lowered in a quantity equal to the projectile potential $-Z_P/R_c$. When the initial state is the ground hydrogenic state we have $\varepsilon_i = -Z_P^2/2$, $a_i = 3/2Z_T$, so (7) gives

$$R_c = \frac{3}{4Z_T}\left(1+\sqrt{1+\frac{48Z_P}{9Z_T}}\right). \tag{8}$$

This qualitative estimation of the effective distance of ionization is of little more than pedagogical value, but has the merits of being based on a simple picture for the electron transfer to the projectile continuum, and of being directly related to the parameters of this process. For $Z_P \gg Z_T$, it gives a $Z_P^{1/2}$ dependence as the sophisticated low energy formalism, and furthermore, the values of R_c are of the same order as those obtained with these methods; from (8) we get $R_c(Z_P = 5) = 3.1$ a.u. and $R_c(Z_P = 50) = 8.4$ a.u., corresponding to values 7 and 20 a.u. obtained by Olson and Salop [10]. This indicates that our estimated values of R_c may be smaller than the distances where the ionization is actually happening. Assuming that the effective internuclear distance R_c for electron tunneling corresponds to a barrier width larger than a_i, it is possible to get a close accord with the results of [10].

The existence of a finite distance $R = R_c$ for the target ionization will have an effect on the final binary electron energy, reducing it in the amount $-Z_P/R_c$. From (6), the correct expression for ε_P instead of Eq. (5) is

$$\varepsilon_P = \tfrac{1}{2}v^2 + \varepsilon_i - \frac{Z_P}{R_c}. \tag{9}$$

This expression is valid for heavy projectiles, when Z_P is larger than Z_T. It is defined in the projectile reference frame, so the final electron energy is $\varepsilon_P = \frac{1}{2}v_{P\text{-}e}^2$. To determine the final velocity of the binary electron in the target frame, we use the triangle relation between the vectors $\mathbf{v}_{T\text{-}e}$, $\mathbf{v}_{P\text{-}e}$, and $\mathbf{v}_{P\text{-}T}(=\mathbf{v})$:

$$v_{P\text{-}e}^2 = v_{T\text{-}e}^2 + v^2 - 2\,v\,v_{T\text{-}e}\cos(\theta_L) \tag{10}$$

where $\cos(\theta_\mathrm{L}) = \hat{\mathbf{v}}_{T\text{-}e}\cdot\hat{\mathbf{v}}$, so

$$v_{T\text{-}e} = v\cos(\theta_L) + \sqrt{v^2\cos^2(\theta_L) + v_{P\text{-}e}^2 - v^2}. \tag{11}$$

To leading orders in v and for $Z_P \gg Z_T$ it results that

$$\tfrac{1}{2}k_{\mathrm{BEP}}^2 \simeq 2v^2\cos^2(\theta_L) - 2|\varepsilon_i| - \sqrt{\frac{4Z_T^3 Z_P}{3}}. \tag{12}$$

This result is very similar to the one predicted by the BL model [8] except for the constants in the last term. The present model gives a shift of the BEP which is higher than the one given by the BL model.

III. DISTORTED-WAVE METHODS IN ELECTRON EMISSION PROCESSES

Distorted-wave methods have played an important role in the study of electron emission in ion-atom collision [2]. These methods allow one to take into account the long-range nature of the Coulomb potential in a simple and elegant way. Furthermore, analytic expressions are obtained for the transition amplitude. This simplifies the numerical computation of DDCS. A discussion of these methods and their application to study single ionization was recently given by Fainstein, Ponce, and Rivarola [2]. In the following we will use the straight-line version of the impact-parameter approximation (IPA), where $\mathbf{R} = \boldsymbol{\rho} + \mathbf{v}t$, with $\boldsymbol{\rho}$ the impact parameter and $\boldsymbol{\rho}\cdot\mathbf{v} = 0$.

A. The initial channel

There are several distorted-wave approximations of common use for the initial channel; we will consider the impulse, the continuum-distorted-wave, and the eikonal-initial-state approximations. The IA was introduced by Briggs [12] within the impact-parameter formalism. An alternative derivation was given by Jakubassa Amundsen and Amundsen [13]. It has been recently applied in its quantum version by Miraglia and Macek [9] to study electron emission. The CDW and EIS approximation are discussed in [2].

The initial target bound state, seen from the projectile reference frame, is

$$\Phi_i(t) = \phi_i(\mathbf{x})\,\exp(-i\varepsilon_i t - i\mathbf{v}\cdot\mathbf{s} - \tfrac{1}{2}v^2 t). \tag{13}$$

The IA distorted-wave function is defined as a wave packet of Coulomb waves on the energy shell, where the Fourier transform $\tilde{\phi}_i(\mathbf{q})$ of the initial wave function $\phi_i(\mathbf{x})$ only acts as the modulation factor of this packet [12, 13]:

$$\Phi_i^{\mathrm{IA}}(t) = \int d\mathbf{q}\,\psi_{\mathbf{q}-\mathbf{v}}(\mathbf{s},t)\,D^+(Z_P,\mathbf{v}-\mathbf{q}|\mathbf{s})\,\tilde{\phi}_i(\mathbf{q}) \times \exp(i\mathbf{q}\cdot\boldsymbol{\rho}) \tag{14}$$

where

$$\psi_{\mathbf{q}}(\mathbf{s},t) = (2\pi)^{-3/2}\exp(i\mathbf{q}\cdot\mathbf{s} - i\tfrac{1}{2}q^2 t) \tag{15}$$

represents a plane wave in the projectile reference frame, and

$$D^+(Z,\mathbf{q}|\mathbf{r}) = D^-(Z,\mathbf{q}|\mathbf{r})^* = N(\nu)\ {}_1F_1(i\nu;1;iqr + i\mathbf{q}\cdot\mathbf{r}) \tag{16}$$

with $\nu = Z/q$, ${}_1F_1$ the confluent hypergeometric function and $N(\nu)$ the normalization (Coulomb) factor. Thus, the IA replaces the free-particle components of the initial bound state by projectile-centered Coulomb waves. It is important to remark that the IA uses on-shell Coulomb waves [the time phase of the $\psi_{\mathbf{q}-\mathbf{v}}(\mathbf{s},t)$ term in (14) is $(\mathbf{q}-\mathbf{v})^2t/2$, which corresponds to the physical energy of a Coulomb wave function of momentum $\mathbf{q}-\mathbf{v}$].

The continuum-distorted-wave approximation to the initial state is

$$\Phi_i^{\rm CDW}(t) = \phi_i(\mathbf{x})\ \psi_{-\mathbf{v}}(\mathbf{s},t)\ \exp(-i\varepsilon_i t)\ D^+(Z_P,\mathbf{v}|\mathbf{s}). \tag{17}$$

In the CDW approximation the target bound state conserves its form but multiplied by a function which modifies $\phi_i(\mathbf{x})$ both in phase and amplitude with the corresponding loss of normalization. Even though one is tempted to identify the CDW and IA initial wave functions, a fundamental difference between them must be noted. The CDW approximation describes the electron under the simultaneous action of the projectile and target centers at *all* collision times. The IA allows the electron-target interaction up to a certain time t when the electron starts to evolve in the projectile field [13] (considering the electron-target interaction only to first order). For this reason it is not possible to obtain the CDW approximation from the IA.

The eikonal-initial-state approximation further reduces the action of Z_P on $\phi_i(\mathbf{x})$ by replacing the Coulomb wave of CDW by its asymptotic form:

$$\lim_{s\longrightarrow\infty} D^+(Z_P,\mathbf{v}|\mathbf{s}) = \exp[i(Z_P/v)\ln(vs + \mathbf{v}\cdot\mathbf{s})] \tag{18}$$

so

$$\Phi_i^{\rm EIS}(t) = \phi_i(\mathbf{x})\ \psi_{-\mathbf{v}}(\mathbf{s},t)\ \exp(-i\varepsilon_i t) \times \exp[i(Z_P/v)\ln(vs + \mathbf{v}\cdot\mathbf{s})]. \tag{19}$$

This is the minimal distortion to $\phi_i(\mathbf{x})$ that satisfies the correct boundary conditions for the electronic motion. The initial state conserves both its form and norm, and only its phase its modified.

B. The final channel

For the final states of the ionization channel, target and projectile play equivalent roles, and it is not possible to select one of them as the dominant interaction. The CDW approximation incorporates both potentials on the modeling of the wave function and describes properly the electron ejection close to either target or projectile:

$$\Phi_f^{\rm CDW}(t) = \psi_{\mathbf{k}}(\mathbf{x},t)\ D^-(Z_T,\mathbf{k}|\mathbf{x})\ D^-(Z_P,\mathbf{p}|\mathbf{s}) \tag{20}$$

where $\mathbf{p} = \mathbf{k} - \mathbf{v}$ is the final electron momentum with respect to the projectile reference frame.

Extensive comparison with experimental data [2] has shown that this CDW approximation to the final state describes the main features of the electron emission process, which has been termed *two-center electron emission* [14].

IV. CALCULATIONS OF THE BINARY PEAK POSITION

In this section we will study the different aspects of the BEP by comparing results from the CDW-EIS model with experimental data and with the model introduced in Sec. II that we will call for simplicity the tunneling model (TM).

In Fig. 1 we plot the experimental data from Lee *et al.* [5] for the 1.5 MeV/amu-H^+, F^{9+} + He system. In the measurements of [5] the emission angle is chosen as 0°. Together with the data we plot the binary-encounter and plane-wave Born (PWBA) approximations used in [5] and the present CDW-EIS calculations. In the BEA the ionization process is considered as the binary collision of the projectile and the active electron. The resulting (Rutherford) cross section is averaged over the initial momentum distribution of the target electron. The PWBA is a simplification of the FBA, where the final target continuum state is replaced by a plane wave. The differ-

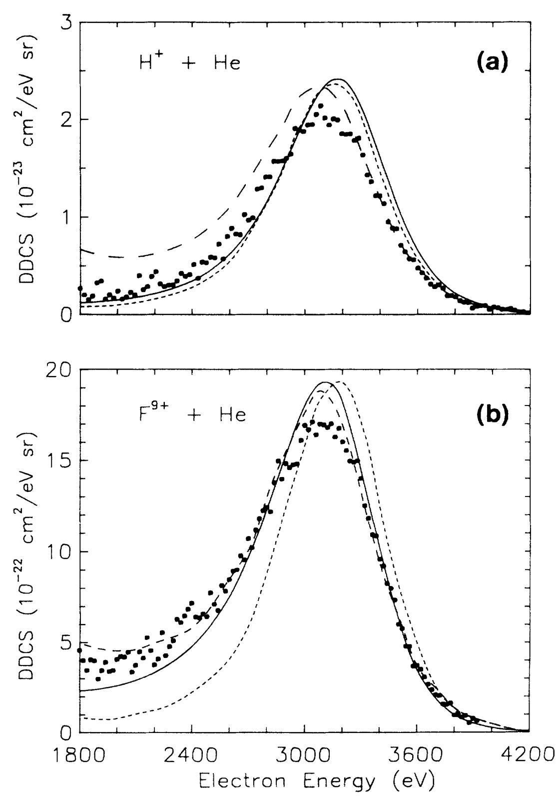

FIG. 1. DDCS for single ionization of He by 1.5 MeV/amu (a) H^+, (b) F^{9+} impact as a function of the electron emission energy in the laboratory reference frame, and for $\theta = 0°$. Solid line: CDW-EIS present results; short-dashed line: PWBA [5]; long-dashed line: BEA [5]; experiment: • from [5].

ences between the PWBA and FBA appear in the region of low-velocity electrons where the approximation of the Coulomb continuum by a plane wave is not valid. For high-energy electrons both models give similar results. From the comparison, it is not possible to say which theory gives the best result, certainly not the PWBA. It must be noted that the experimental results are normalized to the BEA, so it is not surprising that it gives good results. CDW-EIS also gives a good representation of the data for both projectiles, but the most important feature is that it gives the shift in the position of the BEP in the same direction as the experimental data. This aspect is not present in the BEA and PWBA because in these models the position of the peak is independent of the projectile charge. The PWBA gives better results for H^+ while the BEA does for F^{9+} impact. The only improvement of these models with respect to the classical result is to account for the initial state binding energy. Calculations with the IA [9] show a different behavior. The BEP is shifted from the classical result but the shift is lower than the one given by the PWBA. Calculations with the CDW-EIS model but for H_2 targets [9] are also in good agreement with experiments [5].

To study the behavior of the DDCS at the BEP as a function of Z_P, we plot in Fig. 2 the DDCS in the projectile reference frame for different projectiles impinging on He with the same velocity (corresponding to an impact energy of 1 MeV/amu). The solid lines give the results obtained with the CDW-EIS model. Again we are considering the case of 0° emission angle. The stars indicate the position of the maximum, which is obtained from a quadratic interpolation at the top of the BEP, and the dashed line is used to guide the eye. The arrow shows the position of the peak corresponding to the classical result. This plot shows the shift in the peak,

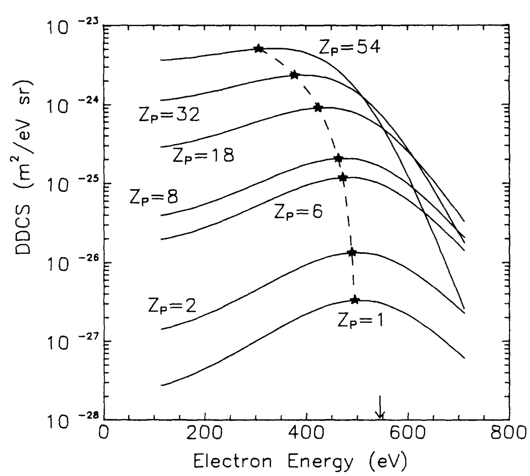

FIG. 2. DDCS for single ionization of He by impact of 1 MeV/amu bare ions with nuclear charge Z_P as a function of the electron emission energy in the projectile reference frame, and for $\theta = 0°$. Solid lines: present CDW-EIS results for different projectiles. The ⋆ gives the position of the BEP calculated as indicated in the text. The vertical arrow indicates the position of the BEP as predicted by the classical result [1].

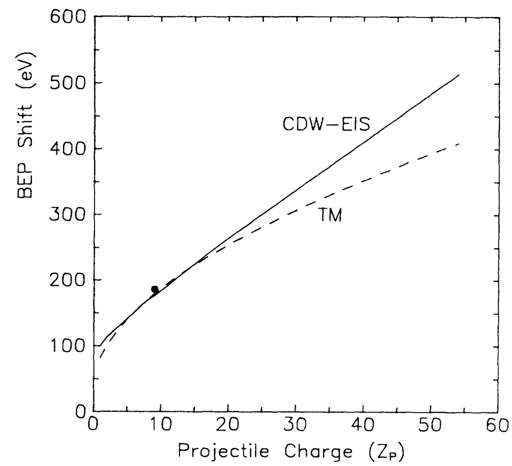

FIG. 3. BEP shift from the classical prediction, for impact of 1 MeV/amu bare projectiles on He, as a function of Z_P. The electron emission angle is $\theta = 0°$. Solid line: present CDW-EIS calculations; dashed line: present calculations with the TM; •, experimental data from [5].

a feature not present in the BEA and PWBA. Similar results to the present CDW-EIS ones were obtained by Olson, Reinhold, and Schultz [15] using the classical trajectory Monte Carlo (CTMC) method but for Ar targets. Like the CTMC results, the present CDW-EIS show that as Z_P increases the BEP disappears in the background of soft electrons. At higher energies than the BEP position, the DDCS are lower for the higher projectile charges. This is a behavior associated with the asymptotic form of the two-center potential and is discussed in detail in [2].

The TM gives only the peak position, so we study in Fig. 3 the shift ΔE of the BEP with respect to the classical value as a function of Z_P. The calculations are performed with the values of R_c obtained from Eq. (8) that are replaced in Eq. (9). We consider the case of 1 MeV/amu impact energy and a He target. For the TM we use the binding energy of He as given in Table I of [16] and the effective target charge is calculated according to the hydrogenic relation $Z_T^{\rm eff} = (-2\varepsilon_i)^{1/2}$. The TM and CDW-EIS are in good agreement with each other

TABLE I. BEP shift (eV) with respect to the classical value as a function of the projectile charge, for an impact energy equal to 1 MeV/amu.

Z_P	$\Delta E_{\rm TM}$	$\Delta E_{\rm CDW\text{-}EIS}$	$\Delta E_{\rm expt}$ [5]
1	81	100	
2	100	114	
6	151	151	
8	170	170	
9	179	176	186±8
18	243	250	
32	317	353	
54	409	514	

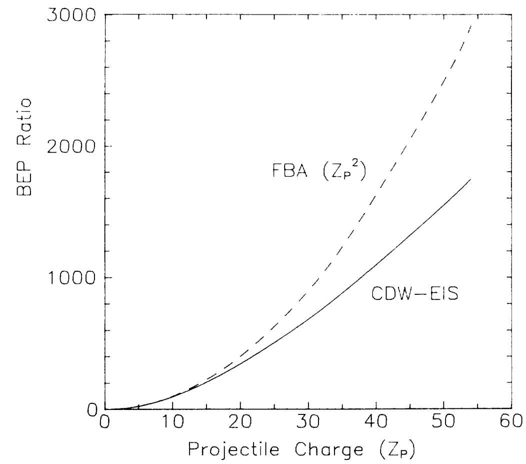

FIG. 4. Ratio between the DDCS for ionization of He by a 1 MeV/amu bare ion of nuclear charge Z_P and the corresponding one for H^+ impact with the same velocity, as a function of Z_P. The electron emission angle is $\theta = 0°$. Solid line: present CDW-EIS results; dashed line: FBA (Z_P^2-scaling law).

for $6 < Z_P < 15$ and both agree with the experimental data of [5] for $Z_P = 9$. Even when outside this interval the two models give different results, they are in qualitative agreement for all Z_P in the sense that ΔE increases as a function of Z_P. For high values of Z_P, such that $Z_P > Z_T$, the TM gives a $Z_P^{1/2}$ dependence as the BL model introduced by Pedersen *et al.* [7]. In Table I we give the values plotted in Fig. 3.

From the measurements of [5] we can study the velocity dependence of ΔE. In Table II the experimental data for He targets are compared at a fixed value of the projectile charge ($Z_P = 9$) with calculations using the TM and CDW-EIS models. The impact energy varies from 1 to 2 MeV/amu. The TM gives a very weak energy dependence, while the experiments and CDW-EIS calculations show a more important variation. The overall change in the shift in this 1 MeV/amu interval is 2 eV in the TM, 31 eV in the CDW-EIS, and 17 eV in the experimental data (33 eV if we consider the maximum difference introduced by the experimental uncertainties). In this case CDW-EIS gives a better description. To be able to represent this behavior with the TM it is necessary to introduce the time dependence of the interaction.

Finally, in Fig. 4 we look at the Z_P dependence of the DDCS at the BEP. We can see that the Z_P^2-scaling law predicted by the FBA (and also by the BEA and PWBA) is verified by CDW-EIS for $Z_P < 15$, but for higher values the increase of the DDCS is slower than the one predicted by the FBA. A similar result was obtained by Miraglia and Macek [9], but these authors show the dependence of the DDCS choosing for the different values of Z_P the same value of the electron velocity. This velocity corresponds to the BEP position for $Z_P = 1$. Instead we plot the yield for different electron velocities; each one is the BEP position for the corresponding value of the projectile charge.

TABLE II. BEP shift (eV) with respect to the classical value as a function of the projectile velocity, for $Z_P = 9$.

E (MeV/amu)	$\Delta E_{\rm TM}$	$\Delta E_{\rm CDW\text{-}EIS}$	$\Delta E_{\rm expt}$ [5]
1.00	179	176	186±8
1.25	178	164	178±8
1.50	178	156	174±8
1.75	177	150	171±8
2.00	177	145	169±8

V. CONCLUSIONS

We have studied the main features of the BEP in heavy-bare-ion–atom collisions at high impact velocities. The yields at the BEP calculated with the CDW-EIS approximation show important deviations with respect to the Z_P^2-scaling law predicted by theories like the FBA, PWBA, or BEA. The behavior of the DDCS calculated with CDW-EIS, in the region of the BEP, is the same as the one obtained previously with the CTMC method. It is found that the BEP is washed out in the background of soft electrons. More detailed information on the production of BEP electrons is obtained by looking at the shift ΔE of the peak with respect to the classical result. First-order approximations like the FBA, PWBA, and BEA give values of ΔE that are independent of the projectile charge. On the contrary, more complete theories like the IA and CDW-EIS show that ΔE is a function of Z_P but the results from the two theories are different, predicting that ΔE is smaller than or larger than the value obtained from FBA, respectively. These differences may come from the fact that the CDW-EIS is a two-center approximation, and on the contrary the IA is a one-center one as discussed above. A simple model, based on the idea of adiabatic resonant tunneling, shows the same behavior as existing experimental results and CDW-EIS. In this model it is assumed that the electron is transferred to a continuum state in the initial channel at a finite projectile-target distance, where the projectile electric field brings down the target potential barrier allowing for resonant tunneling.

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