Dissipative dynamics of quantum spin systems

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We analyze the quantum version of the Landau-Lifshitz equation for damped spin motion using a properly tailored Gisin approach to the dissipative quantum mechanics. Coherent- and squeezed-spin-state evolution for a single spin S and a ferromagnetically coupled spin chain are discussed in detail. The classical limit of the quantum dissipative dynamics is shown to be equivalent to the conventional description, which uses only the diagonal matrix elements for spin operators. The homogeneous and wave-vector-dependent decay of a squeezing parameter in the neighborhood of the ground state is discussed and shown to exhibit long-wavelength instability.

I. INTRODUCTION

The problem of dissipation in quantum-mechanical models attracts a lot of attention for reasons ranging from the fundamental to the practical [1–5]. In a quite ingenious work, Gisin has argued that one can suitably generalize the Schrödinger equation to obtain an (effective) nonlinear wave equation that should govern time evolution of a dissipative quantum system [6]. Gisin has applied that description to the dynamics of a harmonic oscillator and to the simple case of precessing spin $\frac{1}{2}$. The application of this equation to the analysis of two-level systems, as well as discussion of some relevant fundamental points, was given by Huang et al. in Ref. [4].

Gisin's approach possesses several features absent from other generalizations of damped quantum mechanics [7] and fits rather well into a general framework of the so-called metriplectic dynamics, an approach which tries to unite symplectic, nondissipative and metric, dissipative dynamics into one mathematical framework [8]. The aim of this work is to apply the Gisin equation to more general models of spin dynamics (arbitrary spin length and effects of an anisotropy) and to the analysis of many interacting quantum spins—a quantum-spin chain. For this purpose we shall systematically apply the coherent-spin-state representation of the wave function [9,10] to discuss spin dynamics for undamped and damped spins, following Landau and Lifshitz. Furthermore, combining the Gisin equation with a squeezed-spin-state approach [11], we derive the relaxation law for a spin squeezing.

The plan for this paper is as follows. In Sec. II we recall the Gisin equation and briefly discuss its metriplectic properties. Using an expression derivable from the Gisin equation for the time-evolution equation for the expectation value of an observable, and employing the coherent-spin-states technique, we analyze the quantum equation of motion for a spin precessing in an external magnetic field in the presence of the magnetic anisotropy. We

show that the Gisin equation in this case leads to the Landau-Lifshitz-type equation known from classical spin dynamics. Earlier we have shown that the Landau-Lifshitz type of spin dissipative dynamics represents an example of the metriplectic dynamical system [12]. Again using coherent states, we show that the same is true for the one-dimensional quantum Heisenberg chain. In Sec. III we analyze the influence of the damping on the evolution of squeezed spin states used earlier in analysis of the anisotropic chain dynamics [11]. Section IV is devoted to final comments and conclusions.

II. GISIN DAMPING FOR COHERENT SPIN STATES

Consider the quantum-mechanical system described by the Hamiltonian operator $\hat{H}$ and let $|\psi\rangle$ denote the state of that system in the Schrödinger representation. Gisin [6] has argued that one can describe the dissipative time evolution for such a system by replacing the Schrödinger equation for $|\psi\rangle$ by the nonlinear equation

$$i\hbar\frac{d|\psi\rangle}{dt}=\hat{H}\psi\rangle+i\lambda\left[\frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle}-\hat{H}\right]|\psi\rangle\ ,\qquad(2.1)$$

where λ is a dimensionless damping constant.

Equation (2.1) has several remarkable properties. For example, it preserves the eigenspectrum of the Hamiltonian $\hat{H}$ (the eigenstates are *not* damped) and the state normalization. When the time evolution of the initial wave packet is followed, the system sets into the lowest-energy eigenstate present in the initial wave packet. The easiest way to see that is to rewrite the right-hand side of Eq. (2.1) in the form $\mathcal{D}(\psi)$, where the nonlinear operator $\mathcal{D}$ has the form

$$\mathcal{D}(\psi)\equiv(1-i\lambda\hat{Q}_\psi)\hat{H}\ ,\qquad(2.2)$$

and

$$\hat{Q}_\psi=\hat{I}-\hat{P}_\psi=\hat{I}-|\psi\rangle\langle\psi|/\langle\psi|\psi\rangle$$

is the projection operator on the subspace perpendicular to the state $|\psi\rangle$. That this is indeed the case follows from the fact that Eq. (2.1) preserves the norm of the state vector. The damped quantum evolution described by the Gisin equation goes on the hypersurface of the constant norm. This fact is quite similar to that in the classical metriplectic models used in the dynamics of either rigid [13] or classical spins [14]. This suggests that the Gisin equation can be cast in the metriplectic form. Indeed, consider the case of the wave equation describing quantum evolution of a particle in the configuration space. The Schrödinger equation can be derived from the "classical" Hamiltonian

$$H\{\psi,\psi^*\} = \int d^d\mathbf{x} \left[\frac{\hbar^2}{2m} |\nabla\psi(\mathbf{x})|^2 + V(\mathbf{x})|\psi(\mathbf{x})|^2 \right] . \quad (2.3)$$

Using the classical Poisson brackets for the field $\psi(\mathbf{x})$,

$$\{\psi(\mathbf{x}),\psi^*(\mathbf{x}')\} = \frac{1}{i\hbar}\delta(\mathbf{x}-\mathbf{x}') , \quad (2.4)$$

we obtain the Schrödinger equation from $\partial_t\psi = \{\psi,H\}$. Now, in the metriplectic dynamics one describes the damped motion of the system, for which undamped dynamics is cast in the symplectic form, by amending the Poisson bracket with a proper symmetric part [12]. In the present case this symmetric bracket reads

$$\{\{\psi(\mathbf{x}),\psi^*(\mathbf{x}')\}\} = -\frac{\lambda}{\hbar} \left[\delta(\mathbf{x}-\mathbf{x}') - \frac{\psi(\mathbf{x})\psi^*(\mathbf{x}')}{\|\psi\|^2} \right] . \quad (2.5)$$

It is now easy to check that the metriplectic equation of motion for the field ψ follows from the Hamiltonian equation (2.3).

A very similar situation is that encountered in the dynamics of a classical damped spin as described by the Landau-Lifshitz equation of motion [15–17]. The existence of the Casimir—the length of the spin, the norm of the state vector, etc.—can be incorporated into the bracket structure following rather general group-theoretical arguments leading to the damped equation of motion such as that discussed in this paper [8].

Let us return now to Eq. (2.1) and consider the expectation value $\langle\psi|\hat{A}|\psi\rangle$ of an arbitrary observable $\hat{A}$. Following Eq. (2.1), it can be shown that

$$\frac{d}{dt}\langle\psi|\hat{A}|\psi\rangle = \left\langle\psi\left|\frac{\partial\hat{A}}{\partial t}\right|\psi\right\rangle + \frac{i}{\hbar}\langle\psi|[\hat{H},\hat{A}]|\psi\rangle$$
$$- \frac{\lambda}{\hbar}(\langle\psi|[\hat{H},\hat{A}]_+|\psi\rangle$$
$$- 2\langle\psi|\hat{A}|\psi\rangle\langle\psi|\hat{H}|\psi\rangle) , \quad (2.6)$$

where [,] and [,]$_+$ denote commutator and anticommutator, respectively, and we have assumed that $\langle\psi|\psi\rangle = 1$.

The two first terms in Eq. (2.6) are the *usual* terms in the Schrödinger dynamics. The last two terms, proportional to λ, describe the damping. We note in passing that if one would like to use the Gisin equation within the realm of quantum *many-body* mechanics, and use the

Hartree-Fock-like ansatz, then

$$\langle\psi|\hat{A}\hat{H}|\psi\rangle \approx \langle\psi|\hat{A}|\psi\rangle\langle\psi|\hat{H}|\psi\rangle \approx \langle\psi|\hat{H}\hat{A}|\psi\rangle$$

and the dissipative term does vanish.

We shall now specify our physical system to be a spin **S** precessing in the magnetic field **B** in the presence of magnetic anisotropy C. The system is then described by the following quantum Hamiltonian:

$$\hat{H} = -B\hat{S}^z - C(\hat{S}^z)^2 . \quad (2.7)$$

We assume now that the wave function $|\psi\rangle$ is represented by the coherent spin state [9]

$$|\psi\rangle \equiv |\mu\rangle = |\theta,\phi\rangle = (1+|\mu|^2)^{-S}\exp(\mu\hat{S}^-|0\rangle . \quad (2.8)$$

Here $\mu = \tan(\theta/2)\exp(i\phi)$, $\hat{S}^\pm = \hat{S}^x \pm i\hat{S}^y$, and $|0\rangle$ is the ground state of the system, i.e., $\hat{S}^z|0\rangle = S|0\rangle$. Choosing now as the observable $\hat{A}$ the spin-raising operator $\hat{S}^+$, and after some algebra, we obtain from Eqs. (2.7) and (2.8) a single differential equation for the complex amplitude $S\sin\theta\exp(i\phi)$. Decomposing it into real and imaginary parts we obtain

$$\frac{d\theta}{dt} = -\frac{\lambda}{\hbar}[B\sin\theta + C(S-\tfrac{1}{2})\sin(2\theta)] ,$$
$$\frac{d\phi}{dt} = -\frac{B}{\hbar} - \frac{2C}{\hbar}(S-\tfrac{1}{2})\cos\theta . \quad (2.9)$$

On the other hand, the Landau-Lifshitz equation [15–17] that describes the time evolution of the classical damped spin can be written as

$$\hbar\frac{d\mathbf{S}}{dt} = \mathbf{S}\times\mathbf{B}_{\text{eff}} - \frac{\lambda}{|\mathbf{S}|}\mathbf{S}\times(\mathbf{S}\times\mathbf{B}_{\text{eff}}) . \quad (2.10)$$

In Eq. (2.10), $\mathbf{B}_{\text{eff}} = -\partial H/\partial\mathbf{S}$ denotes an effective magnetic field acting on the spin and λ is the Gilbert damping coefficient. In our previous papers [12,14,17] we have shown how the Landau-Lifshitz equation can be fitted into the general metriplectic formalism. Indeed, the first term on the right-hand side of Eq. (2.10) is just the (symplectic) Poisson bracket of the spin vector with the Hamiltonian, while the second one is the metric bracket evaluated with the use of the fundamental metric bracket

$$\{\{S^\alpha,S^\beta\}\} = -\lambda|\mathbf{S}| \left[\delta^{\alpha\beta} - \frac{S^\alpha S^\beta}{\mathbf{S}^2} \right] . \quad (2.11)$$

Defining as usual the polar decomposition of the spin as

$$\mathbf{S} = S(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta) ,$$

it is easy to check that the two equations of motion resulting from Eq. (2.10) for the polar angles are equivalent to the coherent-state version of the Gisin equation provided that all the factors CS are replaced by $C(S-\tfrac{1}{2})$.

Now let us consider a problem of a quantum chain of N spins described by the following Hamiltonian:

$$\hat{H} = -\sum_n [J\hat{\mathbf{S}}_n\cdot\hat{\mathbf{S}}_{n+1} + B\hat{S}_n^z + C(\hat{S}_n^z)^2] , \quad (2.12)$$

where J (>0) is a ferromagnetic exchange constant and

summation runs over all the lattice sites. Following Ref. [10], we introduce states $|\Psi\rangle$ which are the direct products of coherent spin states for each spin in the chain

$$|\Psi\rangle = \mathop{\otimes}\limits_{n=1}^{N} |\mu_n\rangle , \tag{2.13}$$

where the coherent states for each individual spin $|\mu_n\rangle$ are defined in Eq. (2.8).

It has been shown in Ref. [10] that the evolution of expectation values of the single-site operators calculated in this basis, with the help of a standard Schrödinger equation, is equivalent to a classical (undamped) equation of motion with some renormalization of anisotropy constants. Here we would extend that result by applying Eq. (2.6) to $\hat{S}_n^+$ operators and then analyze resulting discrete chain equations in the *continuum limit*. That is, we expand the resulting c-number equation into the formal power series with respect to *small* parameter $l\partial_R$, where l is the lattice spacing and R is the distance along the chain. Following that procedure, we obtain a system of two partial differential equations for fields $\theta(R,t)$ and $\phi(R,t)$:

$$\frac{\partial\theta(R,t)}{\partial t} = -\frac{J}{\hbar}Sl^2\left[\sin\theta\frac{\partial^2\phi}{\partial R^2}+2\cos\theta\left[\frac{\partial\theta}{\partial R}\right]\left[\frac{\partial\phi}{\partial R}\right]\right]$$
$$+\frac{\lambda}{\hbar}\left\{JSl^2\left[\frac{\partial^2\theta}{\partial R^2}-\tfrac{1}{2}\sin(2\theta)\left[\frac{\partial\phi}{\partial R}\right]^2\right]\right.$$
$$\left. -B\sin\theta-C(S-\tfrac{1}{2})\sin(2\theta)\right\} , \tag{2.14}$$

$$\sin\theta\frac{\partial\phi}{\partial t} = \frac{J}{\hbar}Sl^2\left[\frac{\partial^2\theta}{\partial R^2}-\tfrac{1}{2}\sin(2\theta)\left[\frac{\partial\phi}{\partial R}\right]^2\right]$$
$$-\frac{B}{\hbar}\sin\theta-\frac{C}{\hbar}(S-\tfrac{1}{2})\sin(2\theta)$$
$$+\frac{\lambda}{\hbar}JSl^2\left[\sin\theta\frac{\partial^2\phi}{\partial R^2}+2\cos\theta\left[\frac{\partial\theta}{\partial R}\right]\left[\frac{\partial\phi}{\partial R}\right]\right] . \tag{2.15}$$

Similarly as for Eqs. (2.9), these equations are equivalent to their classical counterparts following from the Landau-Lifshitz equations, provided that we replace S in all terms proportional to the anisotropy constant C by $(S-\tfrac{1}{2})$. Equations (2.14) and (2.15) were used in various contexts in the nonlinear spin dynamics. It is relatively easy to write down the generalization of those equations for the case when the magnetic degrees of freedom are coupled to the elastic displacements of the spins from their equilibrium (rigid lattice) positions. If the elastic degrees of freedom are then quantized, and combined-system dynamics is analyzed by means of spin *and phonon* coherent states, one arrives at the set of equations used in Ref. [10(c)] to discuss the elastomagnetic properties of the quantum Heisenberg chain.

We conclude therefore that the Gisin generalization of the Schrödinger equation for a damped quantum system as applied to the quantum spin dynamics is equivalent to the damped spin dynamics proposed by Landau and Lifshitz. In Sec. III we shall investigate the Gisin equation as applied to the squeezed-spin-state evolution.

III. SQUEEZED SPIN STATES AND THEIR DAMPING

In this section we shall investigate the influence of the Gisin damping on the behavior of squeezed spin states (SSS) as defined in Ref. [11]:

$$|\theta,\phi,a\rangle = \exp(-i\phi\hat{S}^z)\exp(-i\theta\hat{S}^y)\hat{U}(a)|0\rangle , \tag{3.1}$$

where $\hat{U}(a)$ is the squeezing operator

$$\hat{U}(a) = \frac{1}{\sqrt{\mathcal{N}(a)}}\exp\left[\frac{a}{4S}(\hat{S}^-)^2\right] . \tag{3.2}$$

In the above, $a\in\mathbb{C}$ is the (complex) squeezing parameter and the normalization constant $\mathcal{N}(a)$ equals

$$\mathcal{N}(a) = \sum_{n=0}^{S}\frac{(2n)!}{(n!)^2}\frac{(2S)!}{(2S-2n)!}\left[\frac{|a|}{4S}\right]^{2n} . \tag{3.3}$$

The semiclassical approximation $(S\to\infty)$ of Eq. (3.3) gives, for the case of moderate squeezing [11], i.e., $|a|<1$,

$$\mathcal{N}_\infty(a) \approx \sum_{n=0}^{S}\frac{(2n)!}{(n!)^2}\left[\frac{|a|}{2}\right]^{2n} = \frac{1}{\sqrt{1-|a|^2}} . \tag{3.4}$$

For $a=0$, the SSS coincides with the coherent state given by Eq. (2.8) up to some phase factor [9,10].

That $\hat{U}(a)$ is indeed the squeezing operator can be shown by evaluating the expectation values for the x and y components of the spin operator:

$$\langle a|\hat{S}^{x(y)}|a\rangle = \tfrac{1}{2}[S+(2s-1)E_1-E_2]\mp\frac{1}{|a|^2}S\,\mathrm{Re}(a)E_1 , \tag{3.5}$$

where the state $|a\rangle = |\theta=0,\phi=0,a\rangle$ can be understood as a "squeezed ground state" and

$$E_n = \frac{1}{\mathcal{N}}|a|^n\frac{\partial^n\mathcal{N}}{\partial|a|^n} . \tag{3.6}$$

We see that squeezing causes variances $\langle a|(\hat{S}^{x(y)})^2|a\rangle$ of the spin components perpendicular to the quantization axis to be different from the standard coherent-state value $S/2$. The difference of both these variances is an obvious measure of an asymmetry of the wave packet.

We shall now discuss the time evolution of the squeezing parameter a in the presence of the damping. Let us start from the simplest case, i.e., from Hamiltonian equation (2.7) with the anisotropy coefficient $C=0$. In the absence of damping, one can show [11] that $dE_1/dt=0$ and one sees only the precession of the squeezing parameter a, phase α. To see the effects of damping, we use Eq. (2.6) with $\hat{A}=\hat{S}^z$. We get immediately

$$\frac{d\langle\hat{S}^z\rangle}{dt} = \frac{2\lambda B}{\hbar}[\langle(\hat{S}^z)^2\rangle-\langle\hat{S}^z\rangle^2] , \tag{3.7}$$

where

$$\langle\ \rangle = \langle\theta,\phi,a|\cdots|\theta,\phi,a\rangle .$$

The matrix elements occurring in Eq. (3.7) have been explicitly obtained in Ref. [11]. Using those expressions, we have

$$\frac{d}{dt}[\cos\theta(S-E_1)]$$

$$=\frac{2\lambda B}{\hbar}\left[(E_1+E_2-E_1^2)\cos^2\theta\right.$$

$$+\frac{1}{2}\left[S+(2S-1)E_1-E_2\right.$$

$$\left.\left.+\frac{2}{|a|^2}S\,\mathrm{Re}(a)E_1\right]\sin^2\right]\,. \tag{3.8}$$

To gain some insight into the time evolution of the squeezed state without undue numerical calculation, we simplify the algebra by assuming that $\theta\approx0$ and $B>0$, i.e., we consider relaxation of the spin squeezing in the neighborhood of the ground state. Moreover, we use the identity $E_{2(sc)}=E_{1(sc)}+3E_{1(sc)}^2$, valid in the semiclassical limit. Having done that, we can solve Eq. (3.8) for $E_1(t)$ (t_0 is a constant depending on the initial conditions):

$$E_1(t)=\frac{1}{\exp[4\lambda B(t-t_0)/\hbar]-1}\,. \tag{3.9}$$

It follows from Eqs. (3.4) and (3.6) that

$$E_{1(sc)}=|a|^2/(1-|a|^2)\,;$$

thus, Eq. (3.9) gives

$$|a(t)|=\exp[-2\lambda B(t-t_0)/\hbar]\,, \tag{3.10}$$

which implies that in the limit $t\to\infty$ the wave function relaxes to the nonsqueezed state with $a=0$. This fact seems to be connected with the universal property of coherent and squeezed states that the value $E_1=0$ is a stationary value in any quantum evolution. In fact, one can show [11] that

$$-(S-E_1)\frac{\partial E_1}{\partial t}$$

$$=\frac{1}{2}\left[\frac{\partial\langle\hat{S}^z\rangle^2}{\partial t}+2\,\mathrm{Re}\left[\langle\hat{S}^-\rangle\frac{\partial}{\partial t}\langle\hat{S}^+\rangle\right]\right]\,. \tag{3.11}$$

Evaluating all the matrix elements on the right-hand side of Eq. (3.11) in the coherent states basis, one can see that it vanishes.

The solution (3.10) contains no information on the phase a of the complex parameter a. To get the evolution of its phase $\alpha(t)$, we consider expectation values of higher-order operators. Again using Eq. (2.6) for $\hat{A}=(\hat{S}^+)^2$, we find that in the semiclassical limit, Eq. (3.4), and in the neighborhood of the ground state $(\theta\approx0)$, there is no influence of damping on the evolution of α; thus,

$$a(t)=a_0\exp(-2iB\tau/\hbar)\,, \tag{3.12}$$

where $\tau=t(1-i\lambda)$ is the "rescaled" evolution parameter identical to that occurring in the work of Lakshmanan

and Nakamura on damped classical spin systems [16].

Using Eqs. (3.9) and (2.12), we can write

$$\Delta\langle(\hat{S}^{\mathrm{vert}})^2\rangle=\langle a|(\hat{S}^x)^2|a\rangle-\langle a|\hat{S}^y)^2|a\rangle$$

$$=S\frac{\cos[2B(t-t_0)/\hbar]}{\sinh[2\lambda B(t-t_1)/\hbar]}\,, \tag{3.13}$$

where t_0 and t_1 are some constants. One sees from the above that damping causes a decrease of the asymmetry of quantum fluctuations.

We shall now discuss the relaxation of the squeezing for the anisotropic spin chain described by the Hamiltonian (2.12). In order to have the ground state defined such that $S^z=S$, we assume $C>0$. Using the squeezed states for the chain in the form $|\Psi\rangle=\otimes_{n-1}^N|\theta_n,\phi_n,a_n\rangle$ and writing an equation analogous to (3.8) we get (in the neighborhood of the ground state $\theta=0$ and in the semiclassical limit)

$$\frac{\partial E_1}{\partial t}=-4\frac{\lambda}{\hbar}E_1(1+E_1)\left[2J(S-E_1)+B\right.$$

$$-2C(1-S+3E_1)$$

$$\left.-Jl^2\frac{\partial^2E_1}{\partial R^2}\right]\,. \tag{3.14}$$

This is a highly nontrivial nonlinear partial-differential equation that might be cast into a form of the nonlinear diffusion-reaction equation with "concentration"-dependent diffusion coefficient $D=4Jl^2\lambda E_1(1+E_1)/\hbar$ and the Ginzburg-Landau type of "free energy." We were unable to solve this equation in its general form. Below we shall present its linear-stability analysis.

Consider first the space homogeneous solutions of Eq. (3.14). There are three real solutions of the equation $\partial E_1/\partial t=0$,

$$E_1^{\mathrm{hom}}=0,\quad E_1^{\mathrm{hom}}=\frac{2JS=B+2C(S-1)}{2(J+3C)},\quad E_1^{\mathrm{hom}}=-1\,. \tag{3.15}$$

Out of those solutions, the third one, $E_1^{\mathrm{hom}}=-1$, is unphysical, as can be seen from Eq. (3.6).

Following standard procedure, we linearize Eq. (3.14) around the stationary solution E_1^{hom}, assuming

$$E_1(R,t)=E^{\mathrm{hom}}+\delta E_1(R,t)$$

$$=E^{\mathrm{hom}}+\delta E_1^0\exp(i\omega t-iqR)\,,$$

and obtain the dispersion relation $\omega=\omega(q,E_1^{\mathrm{hom}})$. We found that both physically admissible stationary points given by (3.15) have different linear-stability properties as described by that dispersion relation. The stationary point $E_1^{\mathrm{hom}}=0$ is linearly stable, as expected, with

$$i\omega=-(4\lambda/\hbar)[B+2JS+2C(S-1)]\,.$$

More interesting is the second stationary solution, for which we obtain

$$i\omega = \frac{4\lambda Jl^2}{\hbar} E_1^{hom}(1 + E_1^{hom}) \left[\frac{2(J + 3C)}{Jl^2} - q^2 \right] . \quad (3.16)$$

This implies the long-wavelength instability of the squeezing amplitude. The squeezing amplitude is stable with respect to fluctuations with the wave vector q larger than the critical one, $q_c = \sqrt{2(1 + 3C/J)}/l$, and unstable for shorter wave-vector undulations—a typical hydrodynamic type of instability. Note that in the isotropic chain case, the only characteristic length for the chain is the lattice spacing; thus in that case the critical wave vector becomes $\sqrt{2}/l$.

Equation (3.14) also contains spatially periodic stationary solutions given by

$$E = \frac{2JS + B + 2C(S - 1)}{2(J + 3C)} + E_0 \cos[k(x - x_0)] , \quad (3.17)$$

with the wave vectors given by

$$k = \frac{1}{l} \left[\frac{2(J + 3C)}{J} \right]^{1/2} . \quad (3.18)$$

Note that these are the same wave vectors for which the corresponding homogeneous solution becomes unstable. That implies the emergence of the spatial patterns of the squeezing amplitude and its instability akin to the instability of the finite-amplitude spin waves discussed in our previous work [17].

IV. CONCLUSIONS

In Sec. II we have combined the Gisin version of a damped Schrödinger equation with the coherent-spin state approach. We have analyzed the time evolution of diagonal matrix elements of spin operators for a single spin S in the presence of magnetic field and anisotropy and for the chain of ferromagnetically coupled spins. We have shown that the resulting equations are nearly equivalent to the classical Landau-Lifshitz equation of a damped magnetic moment. The only difference is that if one starts from the quantum model, then the influence of the magnetic anisotropy is proportional to the factor $S - \frac{1}{2}$ instead of the factor S that one obtains starting from the classical Hamiltonian. This correspondence with the Landau-Lifshitz equation coincides with the result of Gisin for a spin $S = \frac{1}{2}$. We would like to point out, however, that although any wave function of the model $S = \frac{1}{2}$ is a coherent spin state, this is not true for higher spin values.

In Sec. III we used the Gisin equation together with the squeezed-spin state approach (for a single spin and for a chain of spins). As a result, we obtained the equation describing the relaxation of spin squeezing. In the neighborhood of the ground state, this equation possesses a simple solution, i.e., there is an exponential decay of the squeezing parameter while the difference between variances of spin components perpendicular to the quantization axis decays inversely proportional to $\sinh(t)$. Coherent states are always *fixed points* of the squeezing dynamics; however, we also found some nontrivial incoherent solutions to the fixed point squeezing time evolution. It would be interesting to observe experimentally such behavior of the squeezing parameter using, for example, magnetic-resonance techniques. Finally, the squeezing amplitude for the anisotropic magnetic chain exhibits interesting long-wavelength instability, which shows similarity to the finite-amplitude spin-wave instabilities.

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