Cylindrical and spherical solitons in a positive-ion–negative-ion plasma

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(Received 28 October 1991)

Experiments on the excitation of cylindrical and spherical solitons in a positive-ion–negative-ion–electron plasma are described. The results are in substantial agreement with the theoretical model proposed by Das and Singh [Aust. J. Phys. **44**, 523 (1991)]. In addition, experiments on the oblique collision of solitons in a cylindrical geometry are described.

PACS number(s): 52.35.Mw

I. INTRODUCTION

In a recent study, Das and Singh showed that it is possible to model nonlinear waves in cylindrical and spherical geometries in a plasma that consists of positive ions, negative ions, and electrons with appropriate Korteweg–de Vries (KdV) equations

$$\frac{\partial \Psi}{\partial t}+\frac{\beta}{2}\frac{\Psi}{t}+\Psi\frac{\partial \Psi}{\partial r}+\frac{1}{2}\frac{\partial^3 \Psi}{\partial r^3}=0 \tag{1}$$

where $\beta=1$ for cylindrical coordinates and $\beta=2$ for spherical coordinates. The procedure to effect such a description involved the application of the reductive perturbation technique to the relevant fluid equations in the prescribed geometry. The details of the calculation appear in Refs. [1] and [2]. Similar KdV equations have been obtained by Maxon and Viecelli in modeling a positive-ion–electron plasma [3].

A plasma that consists of positive ions and electrons has provided a convenient experimental setting in which to perform experiments designed to verify the predicted properties of KdV solitons in planar, cylindrical, and spherical geometries. A common thread found in most of the experiments was that they were performed in quiescent argon plasmas confined in large vacuum chambers, usually with a multidipole magnet confinement configuration. The majority of the experiments were performed in double-plasma (DP) machines. Reviews of these experiments and their findings have been written [4].

Nakamura and his colleagues have investigated the properties of solitons that could be launched in a DP machine in which the plasma consisted of positive ions, negative ions, and electrons [5]. They observed that both compressive and rarefactive solitons could be excited in a planar geometry. In addition, by varying the negative-ion concentration, they could excite either KdV or modified KdV solitons modeled by quadratic or cubic nonlinearities, respectively.

We have recently discovered that it is relatively easy to excite planar KdV solitons in a plasma that consisted of argon ions and electrons if an impurity gas such as sulfur hexafluoride were added [6]. The resultant plasma would consist of positive ions, negative ions, and electrons. Such a plasma permits the propagation of two low-frequency waves; one propagates faster than the ion acoustic velocity ("fast" mode) and the other slower ("slow" mode) [7]. In the experiments, we found that a nonlinear perturbation could be launched from a solid metal disk to which a small negative [$\Delta\phi \approx -2(T_e/e)$ V] voltage step was applied. This perturbation evolved into a planar KdV fast-mode soliton at the edge of a "transient" sheath. The excitation mechanism was interpreted in terms of a "klystron-bunching" mechanism. Subsequently, we confirmed the soliton interpretation of the excited signals by noting that a large amplitude soliton would have a larger velocity than a smaller amplitude one and would pass through the smaller one in an overtaking collision [8]. Solitons were also observed to be reflected at the edge of the sheath-plasma interface. Finally, we were able to create a two-dimensional soliton by reflecting a planar soliton from a concave metal surface creating a focused Fabry-Pérot configuration [9].

Motivated by the Das and Singh results and as a part of this larger study into examining the properties of KdV solitons in a three-component plasma, we performed a series of experiments where their model would be apropos. It is the purpose of this paper to describe these observations. In addition to describing solitons that are launched from just one exciter, the oblique collision of two solitons launched from two juxtaposed cylindrical exciters will be presented. This collision suggested by the Kadomtsev-Petviashvilli (KP) equation has been well documented in positive-ion–electron plasmas [10]. The experimental setup is described in Sec. II. The results are presented and interpreted in Sec. III. Section IV contains the concluding comments.

II. EXPERIMENTAL SETUP

The experiments were performed in a large multidipole plasma device that has been described elsewhere [11]. The device consists of several hollow rectangular tubes that were filled with approximately 1500 small permanent magnets. Each row had magnet pole faces pointing in the same direction and the rows alternated in magnetic pole orientation. The tubes were arranged in a cage structure (80 cm diameter $\times 110$ cm length) and the entire structure was inserted in a large vacuum chamber whose base vacuum pressure was maintained beneath 10^{-6} Torr by continuous pumping. Argon was bled into the chamber to maintain a neutral pressure of $1-2\times 10^{-4}$ Torr using a double-valve system. One valve was used to control the pressure and the second was an "open-close" valve. Ionizing electrons emitted from Joule heated filaments were accelerated by an 85-V bias supply to the cage which served as the anode. Typical plasma numbers as monitored with a Langmuir probe were electron density $n_e \approx 10^8-10^9$ cm^{-3} and electron temperature $T_e \approx 1-3$ eV. The ion temperature T_i as measured with an energy analyzer was $T_i < T_e/10$. The plasma potentials ranged from $+12$ to $+2$ V and decreased as the amount of SF_6 was increased. The plasma was quiet and the small amount of noise present decreased with the addition of the SF_6. The plasma was also relatively homogeneous in the region of the experiment.

Small amounts of sulfur hexafluoride were separately bled into the chamber through a separate double-valve system. The percentage of SF_6 was estimated in two ways. First as a crude estimate, the pressure of neutral SF_6 in the evacuated chamber was set to be a certain percentage of the known value of Ar prior to the opening of the Ar gas valve. The second technique involved monitoring the reduction of the electron saturation current of the Langmuir probe [12]. We define a parameter ϵ, the negative-ion concentration, to be

$$\epsilon = \frac{n_{SF_6^-}}{n_{Ar^+}} = \frac{n_{Ar^+} - (n_{Ar^+} - n_{SF_6^-})}{n_{Ar^+}} = 1 - \frac{I_{es}}{I_{es}^{(0)}} \qquad (2)$$

where I_{es} and $I_{es}^{(0)}$ are the Langmuir probe saturation currents with and without the SF_6. Other negative ions such as SF_5^-, F^-, etc. may have also been present in the device. Their presence will not alter the conclusions that are reached in this work so no effort was made to quantify their amounts.

Waves were excited by applying a voltage signal to one of the three aluminum exciters depicted in Fig. 1. The amplitude $\Delta\phi$ of the applied voltage signal was $\Delta\phi \approx -50$ V for the hemisphere and $\Delta\phi \approx -150$ V for the rods. The repetition frequency of the voltage step was $\approx 500-1500$ Hz. The first exciter, a hollow aluminum hemisphere 20 cm in diameter [Fig. 1(a)], was used to launch hemispherically symmetric density perturbations that evolved into solitons. Since the probe was scanned only on the convex side, we approximate the observed results as spherical solitons. The second exciter [Fig. 1(b)] was a single aluminum rod 8 mm in diameter that launched cylindrically symmetric density perturbations which evolved into cylindrical solitons. The third exciter [Fig. 1(c)] consisted of two aluminum rods of diameter 1.9 cm that were separated by 17.6 cm.

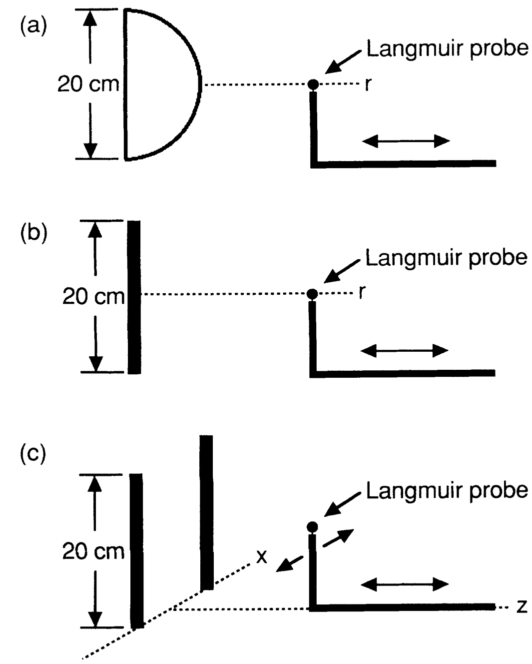

FIG. 1. Launchers of solitons. (a) Hemispherical aluminum surface. (b) Cylindrical aluminum rod of diameter 8 mm. (c) Two cylindrical rods of diameter 1.9 cm. The rods were separated by 17.6 cm.

All signals were detected with a 3-mm-diameter spherical Langmuir probe that was biased positive with respect to the plasma potential in order to detect perturbations in the electron saturation current. The probe could be scanned in the entire region in front of the launching structures. The perturbations in current were passed through a resistor to ground and the resulting voltage perturbations were displayed on an oscilloscope that was triggered from the signal generator.

III. EXPERIMENTAL RESULTS AND INTERPRETATION

A series of experiments that has direct bearing on the theoretical investigation of Das and Singh [1,2] was performed. A second series of experiments that we describe does not yet have a theoretical foundation for a three-component plasma.

In Fig. 2, a sequence of pictures taken at 2-cm incre-

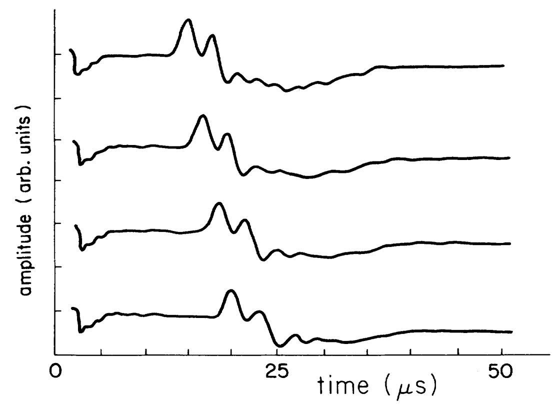

FIG. 2. Hemispherically expanding solitons. The pictures were taken at 2-cm increments starting 20 cm in front of the hemisphere.

ments starting 20 cm in front of the hemisphere is shown. Two signals emerge ahead of a trailing rarefactive depression that have characteristics of spherically expanding solitons. The trailing depression would be caused by a depletion of ions that were compressed to form the solitons.

From the experimental data, part of which are shown in Fig. 2, we can plot the time-of-flight trajectory and the amplitude variation as a function of position. This is shown in Fig. 3. From the trajectory shown in Fig. 3(a), we note that signal initially travels faster and then slows to its final velocity. This behavior is also characteristic of ion acoustic waves in a positive-ion–electron plasma [13]. The $t=0$ intercept indicates that the transient sheath in this three-component plasma is approximately 2.1 cm [6,14]. The wave velocity of 1.3×10^6 cm/s identifies it as the fast mode in a positive-ion–negative-ion–electron plasma. Previous studies in planar geometry have identified the fast-mode wave as a soliton [6,8].

The amplitude decay depicted in Fig. 3(b) can be understood in terms of KdV soliton dynamics also. It was examined in the work of Das and Singh [1,2]. Here, we will invoke a simpler model. A spherical (or cylindrical) soliton has the KdV property that

$$\Psi_0 W^2 \approx \text{const} \tag{3}$$

where Ψ_0 is the amplitude of the soliton and W is its width. The energy in a spherical (or cylindrical) shell at a radius r is defined as

$$\Psi_0^2 W r^{\beta} \approx \text{const} \tag{4}$$

where $\beta=2$ in spherical coordinates and $\beta=1$ in cylindrical coordinates. Eliminate W between (2) and (3) and write the amplitude geometrical decay as

$$\Psi_0 \approx \begin{cases} \dfrac{1}{r^{4/3}}, & \text{spherical} \quad (5a) \\[2ex] \dfrac{1}{r^{2/3}}, & \text{cylindrical .} \quad (5b) \end{cases}$$

The solid line in Fig. 3(b) is from this model for spherical geometry.

If the hemisphere is replaced with the long slender rod depicted in Fig. 1(b), cylindrical solitons are launched. The amplitude dependence as a function of radius is depicted in Fig. 3(c) with the solid line from this model for cylindrical geometry.

These experimental results are in substantial agreement with the Das and Singh theoretical studies [1,2]. A second series of experiments was performed to ascertain if a positive-ion–negative-ion–electron plasma admitted a resonance interaction during an oblique collision.

By applying the same excitation voltage signal to the

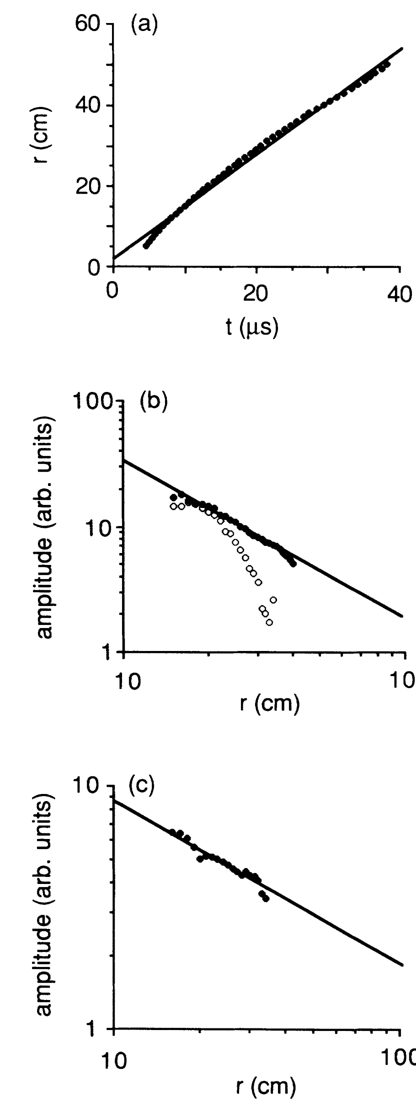

FIG. 3. Summary of the experimental results. (a) Time of flight of the leading hemispherical soliton. The slope of the line yields a velocity of propagation of 1.3×10^6 cm/s and the $t=0$ intercept indicates a transient sheath of 2.1 cm. (b) Amplitude decay as a function of radius for the leading hemispherical soliton (solid circle) and the second soliton (open circle). The line has a slope of $r^{-(4/3)}$. (c) Amplitude decay as a function of radius for the cylindrical soliton. The line has a slope of $r^{-(2/3)}$.

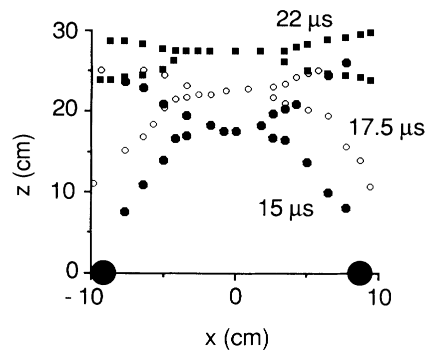

FIG. 4. Equiphase contours of the solitons launched from the two cylindrical rods.

two parallel rods shown in Fig. 1(c), we simultaneously excited two cylindrical solitons of equal amplitude. This experiment is similar to the type that has been performed in a positive-ion–electron plasma [10]. We noted an enhancement in the amplitude of the new soliton in the present experiment at a critical angle with $\Psi_3 > \Psi_1 + \Psi_2$. Measured equiphase contours, shown in Fig. 4, clearly demonstrate that the expected resonant interaction during the oblique collision of solitons has occurred in this three-component plasma. They also show that a new soliton propagates in a direction determined by satisfying the following conservation laws:

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 \ , \tag{6}$$

$$\omega_1 + \omega_2 = \omega_3 \tag{7}$$

where $\mathbf{k}_j$ and ω_j are the wave vector and the dispersion relation of the jth soliton.

IV. CONCLUSION

Experiments have been performed to test the results of a theoretical investigation of Das and Singh, who investigated cylindrical and spherical soliton propagation in a positive-ion–negative-ion–electron plasma [1,2]. In addition, the oblique collision of solitons was experimentally investigated in the same plasma. The results of this experiment invite a theoretical description.

ACKNOWLEDGMENTS

The authors wish to acknowledge Dr. G. C. Das and Dr. K. I. Singh for communicating their theoretical results prior to publication and Al Scheller for constructing most of the experimental apparatus. This work was supported in part by the National Science Foundation Grant No. ESC 90-06921.

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