Generation of stimulated backscattered harmonic radiation from intense-laser interactions with beams and plasmas

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A comprehensive analysis is presented that describes the generation of harmonic radiation by the stimulated backscattering of intense laser fields from electron beams and from plasmas. The dynamics of the intense laser-electron interaction are analyzed using a fully nonlinear, relativistic, cold fluid model valid to all orders in the amplitude of the pump laser. In general, the backscattered radiation, from an electron beam or stationary plasma, occurs at odd harmonics of the Doppler-shifted incident laser frequency. The strength of the harmonics is strongly dependent on the incident laser intensity. The growth rate and saturation level of the backscattered harmonics are calculated, and the limitations due to thermal, space-charge, and collisional effects are discussed. Significant radiation generation at high harmonics requires sufficiently intense pump laser fields and sufficiently cold axial electron distributions. This mechanism may provide a practical method for producing coherent radiation in the xuv regime.

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I. INTRODUCTION

Recent technological advances have made possible compact terawatt laser systems having high intensities ($>10^{18}$ W/cm^2), modest energies (>1 J), and short pulses (<1 ps) [1,2]. These high-laser intensities lead to a number of laser-plasma and laser-electron beam interaction phenomena [3–11]. This paper discusses one such phenomenon, stimulated backscattered harmonic (SBH) radiation generated by the interaction of intense, linearly polarized laser fields with electron beams or plasmas [11] (see Fig. 1). The intense-laser-backscattering mechanism is essentially stimulated coherent scattering in the strong-pump regime. For sufficiently intense incident laser fields, the electron quiver velocity becomes highly relativistic. The high-laser intensity, along with the induced nonlinear relativistic electron motion, results in the generation of stimulated backscattered radiation at odd harmonics. In the interaction of an intense-laser pulse with a counterstreaming electron beam, SBH radiation is generated via a free-electron-laser (FEL) mechanism [12–22] resulting in a relativistic Doppler-frequency upshift. Hence, a laser-pumped FEL [13–17] (LPFEL) may utilize both the harmonic upshift as well as the Doppler upshift to generate short-wavelength radiation. In the interaction of an intense laser pulse with a stationary plasma, SBH radiation is generated via a nonlinear Raman backscatter mechanism [3,23] at odd multiples of the fundamental laser frequency. Previous analyses of stimulated backscattering of intense lasers from electron beams (i.e., LPFEL's) or plasmas, due to the complex dynamics of the laser-electron interaction, have been limited to studies of the fundamental backscattered mode [13–17,23].

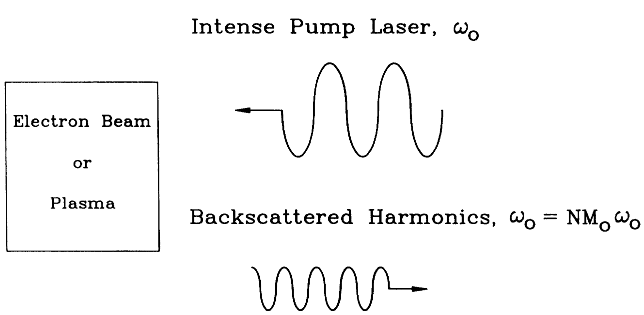

FIG. 1. Schematic of the incident pump-laser field, $\sim\exp(i\omega_0\eta/c)$, of frequency ω_0, and the backscattered harmonic radiation field, $\sim\exp(i\omega\xi/c)$, of frequency ω, as obtained via stimulated scattering from an electron beam or a plasma, where $\eta=z+ct$ and $\xi=z-ct$.

In the following, a fully nonlinear analysis of SBH generation is presented which is valid for arbitrarily high pump-laser intensities. The growth rates, saturation levels (efficiencies), and thermal limitations are obtained for the SBH radiation generated from either an electron beam or a plasma. Differences between the SBH radiation generated from an electron beam and from a plasma are discussed. Sufficiently intense pump lasers and sufficiently cold axial electron distributions are required for significant radiation generation at high harmonics. The SBH mechanism may provide a practical method for producing coherent radiation in the xuv regime.

In conventional FEL's, a linearly polarized (planar) static magnetic wiggler may be used to generate stimulated harmonic radiation at odd multiples of the relativistic Doppler upshifted fundamental FEL frequency [18–21]. In the LPFEL, the static periodic magnetic wiggler is replaced by a counterstreaming intense laser field [13–17]. For a given electron beam energy, the LPFEL can lead to substantially shorter-wavelength radiation than a conventional FEL since the pump-laser wavelength (typically ~ 1 μm) is several orders of magnitude smaller than the conventional wiggler period (typically ~ 1 cm). In the LPFEL, the free energy driving the radiation is available from both the electromagnetic energy of the pump laser and the kinetic energy of the electron beam. In conventional FEL's, free energy is available from only the kinetic energy of the electron beam and, hence, the FEL interaction vanishes in the limit of vanishing beam energy. This, however, is not the case in the LPFEL mechanism and SBH radiation may still be generated in the limit of zero electron beam energy. Hence, an intense laser field interacting with a stationary plasma may also generate SBH radiation. This process may be viewed as nonlinear Raman backscattering in the strong-pump limit (or strongly coupled regime) [23].

Significant radiation generation at high harmonics in conventional FEL's requires that the normalized amplitude of the magnetic wiggler field be somewhat greater than unity [20]. Similarly, in the LPFEL (as well as in the nonlinear Raman backscattering mechanism), it is shown that the growth rate for the higher harmonics becomes significant when the normalized pump-laser amplitude exceeds unity, $a_0 \geq 1$, where $a_0 = |e| A_0 / m_0 c^2$ is the normalized amplitude of the pump laser vector potential, A_0. The normalized laser amplitude, a_0, is related to the power, P_0 (in GW), of a linearly polarized laser by

$$P_0 = 21.5(a_0 r_0 / \lambda_0)^2 ,$$

where r_0 is the spot size of the Gaussian profile, λ_0 is the laser wavelength, and the power is in units of GW. Physically, $a_0 \geq 1$ implies that the electron quiver motion in the laser field is highly relativistic. This may be seen from conservation of canonical transverse momentum which, in the one-dimensional (1D) limit ($\lambda_0 \ll r_0$), states that $a_0 = \gamma \beta_\perp$, where γ is the relativistic mass factor and $\beta_\perp = v_\perp / c$ is the electron quiver velocity. The nonlinearities associated with the highly relativistic electron motion result in the generation of harmonic radiation. In terms of the laser intensity ($I_0 = 2P_0 / \pi r_0^2$), the quantity a_0 is given by

$$a_0 = 0.85 \times 10^{-9} \lambda_0 I_0^{1/2} ,$$

where λ_0 is in μm and I_0 is in W/cm^2. Relativistic electron motion ($a_0 \geq 1$) requires laser intensities greater than 10^{18} W/cm^2 for wavelengths of ~ 1 μm. Such intensities are now available from compact laser systems using a technique referred to as chirped pulse amplification (CPA). The CPA technique allows ultrashort pulses (≤ 1 ps) to be efficiently amplified in solid-state media such as Nd:glass, alexandrite, and Ti:sapphire [1]. Alternatively, KrF excimer laser systems may be used to generate ultrahigh-intensity, subpicosecond laser pulses [2].

The remainder of this paper is organized as follows. In Sec. II, the generation of coherent SBH radiation is analyzed using a relativistic, cold fluid theory. This theory includes the nonlinear effects of the pump laser to all orders in the normalized vector potential, a_0. A dispersion relation for the SBH radiation is obtained in the strong-pump regime in which the effects of the perturbed electrostatic potential are neglected. Based on particle-trapping arguments, the saturation level of the backscattered harmonics is calculated in Sec. III. Limitations of the theory due to thermal, space-charge, and collisional effects are discussed in Sec. IV. Examples of SBH generation using an electron beam and a plasma are given in Sec. V. The growth rates, saturation efficiencies, and thermal requirements for SBH generation from electron

TABLE I. Growth rates, efficiencies, and thermal requirements for stimulated backscattered harmonic generation. ω_0 is the pump-laser frequency, ω_p is the plasma frequency, γ_0 is the initial relativistic factor, $a_0 = |e| A_0 / m_0 c^2$ is the normalized pump-laser amplitude, $\gamma_{10} = (1 + a_0^2/2)^{1/2}$, $N = (2l+1)$ is the harmonic number, $F_l = b[J_l(b) - J_{l+1}(b)]^2$ is the harmonic coupling function, $b = N a_0^2 / 4\gamma_{10}^2$, and M_0 is the frequency amplification factor, $\omega = N M_0 \omega_0$.

	Laser plasma ($M_0 = 1$, $\gamma_0 = 1$)		Laser electron beam ($M_0 \gg 1$, $\gamma_0 \gg 1$)	
	Arbitrary a_0, N	$a_0 \ll 1$, $N = 1$	Arbitrary a_0, N	$a_0 \ll 1$, $N = 1$
Growth rate Γ / ω_0	$\sqrt{3} \left[\frac{\omega_p^2 F_l}{4\omega_0^2 \gamma_{10}} \right]^{1/3}$	$\sqrt{3} \left[\frac{\omega_p a_0}{4\omega_0} \right]^{2/3}$	$\frac{\sqrt{3}}{\gamma_0} \left[\frac{\omega_p^2 \gamma_{10}^2 F_l}{4\omega_0^2} \right]^{1/3}$	$\frac{\sqrt{3}}{\gamma_0} \left[\frac{\omega_p a_0}{4\omega_0} \right]^{2/3}$
Laser eff.[a] P_N / P_0	$\frac{\omega_p^2 \gamma_{10} \Gamma / N}{\sqrt{3} \omega_0^3 a_0^2 J_0^2}$	$\left[\frac{\omega_p^2}{2\omega_0^2 a_0} \right]^{4/3}$	$\frac{\omega_p^2 \gamma_0 \Gamma / N}{\sqrt{3} \omega_0^3 a_0^2 J_0^2}$	$\left[\frac{\omega_p^2}{2\omega_0^2 a_0} \right]^{4/3}$
Electron. eff. η_e			$\frac{\Gamma / N}{2\sqrt{3} \omega_0 J_0^2}$	$\frac{1}{2\gamma_0} \left[\frac{\omega_p a_0}{4\omega_0} \right]^{2/3}$
Thermal spread $\Delta E_{\text{th}} / m_0 c^2 \ll$	$\frac{1}{6} \left[\frac{\Gamma}{\omega_0 N} \right]^2$	$\frac{1}{2} \left[\frac{\omega_p a_0}{4\omega_0} \right]^{4/3}$		
Energy spread $\Delta \gamma_{\text{th}} / \gamma_0 \ll$			$\frac{\Gamma / N}{2\sqrt{3} \omega_0}$	$\frac{1}{2\gamma_0} \left[\frac{\omega_p a_0}{4\omega_0} \right]^{2/3}$

[a]Formulas valid provided $a_0 > \omega_p^2 / 2\omega_0^2$.

beams and plasmas are summarized in Table I. This paper concludes with a discussion in Sec. VI.

II. INTENSE-LASER-ELECTRON INTERACTION AND STIMULATED HARMONICS

The 1D fields associated with the pump laser, backscattered radiation, and plasma response are described by the transverse vector potential, $\mathbf{A}$, and the scalar potential, Φ. In what follows the Coulomb gauge is used, i.e., $\nabla\cdot\mathbf{A}=0$, which in 1D implies $A_z=0$, where z is along the axis of propagation of the SBH radiation. The polarization of the laser field is arbitrary. The normalized potentials satisfy the field equations

$$\left[\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{a}=k_p^2\frac{n}{n_0}\boldsymbol{\beta}_\perp=k_p^2\frac{n}{n_0\gamma}\mathbf{a}\ , \tag{1a}$$

$$\frac{\partial^2\phi}{\partial z^2}=k_p^2\left[\frac{n}{n_0}-1\right]\ , \tag{1b}$$

where $\mathbf{a}(z,t)=|e|\mathbf{A}/m_0c^2$, $\phi(z,t)=|e|\Phi/m_0c^2$, $\omega_p=ck_p=(4\pi|e|^2n_0/m_0)^{1/2}$ is the ambient plasma frequency, $n(z,t)$ is the plasma density, n_0 is the ambient plasma density, $\boldsymbol{\beta}=\mathbf{v}/c$ is the normalized plasma fluid velocity, and

$$\gamma=(1-\beta_z^2-\beta_\perp^2)^{-1/2}=(1+a^2)^{1/2}/(1-\beta_z^2)^{1/2}$$

is the relativistic factor. In obtaining the right-hand side of Eq. (1a), use is made of the fact that the transverse canonical momentum is invariant and that, prior to the laser-pulse interaction, the particle distribution is assumed to have no transverse velocity, i.e., $\boldsymbol{\beta}_\perp=\mathbf{a}/\gamma$.

The fluid quantities n, β_z, and γ are assumed to satisfy the cold, relativistic fluid equations which can be written in the form

$$\frac{\partial n}{\partial t}+c\frac{\partial}{\partial z}(n\beta_z)=0\ , \tag{2a}$$

$$\frac{d}{dt}(\gamma\beta_z)=-\frac{c}{2\gamma}\frac{\partial}{\partial z}a^2+c\frac{\partial\phi}{\partial z}\ , \tag{2b}$$

$$\frac{d\gamma}{dt}=\frac{1}{2\gamma}\frac{\partial}{\partial t}a^2+c\beta_z\frac{\partial\phi}{\partial z}\ , \tag{2c}$$

where $d/dt=\partial/\partial t+c\beta_z\partial/\partial z$. The first term on the right-hand side of Eq. (2b), proportional to a^2, represents the ponderomotive force. In the present model, thermal effects have been neglected. This is valid provided (i) the electron quiver velocity is much greater than the electron thermal velocity and (ii) the thermal energy spread is sufficiently small so that electron trapping in the plasma wave does not take place. The effect of a thermal axial velocity spread on the wave-particle resonance is discussed in Sec. IV. Also, the ions are assumed to be stationary.

It proves convenient to replace the independent variables (z,t) with the independent variables (η,ξ), where $\eta=z+ct$ and $\xi=z-ct$. To transform Eqs. (1) and (2) from z,t to η,ξ variables, note that $\partial/\partial z=\partial/\partial\xi+\partial/\partial\eta$ and $\partial/\partial t=-c(\partial/\partial\xi-\partial/\partial\eta)$. In the new variables, Eqs. (1) and (2) become

$$\left[\frac{\partial}{\partial\eta}\frac{\partial}{\partial\xi}-\frac{1}{4}k_p^2\frac{n}{\gamma n_0}\right]\mathbf{a}=0\ , \tag{3a}$$

$$\left[\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\xi}\right]^2\phi=k_p^2\left[\frac{n}{n_0}-1\right]\ , \tag{3b}$$

$$\frac{\partial}{\partial\eta}[(1+\beta_z)n]-\frac{\partial}{\partial\xi}[(1-\beta_z)n]=0\ , \tag{3c}$$

$$\gamma\left[(1+\beta_z)\frac{\partial}{\partial\eta}-(1-\beta_z)\frac{\partial}{\partial\xi}\right](\gamma\beta_z)=-\left[\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\xi}\right]\frac{a^2}{2}+\gamma\left[\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\xi}\right]\phi\ , \tag{3d}$$

$$\gamma\left[(1+\beta_z)\frac{\partial}{\partial\eta}-(1-\beta_z)\frac{\partial}{\partial\xi}\right]\gamma=\left[\frac{\partial}{\partial\eta}-\frac{\partial}{\partial\xi}\right]\frac{a^2}{2}+\gamma\beta_z\left[\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\xi}\right]\phi\ . \tag{3e}$$

Introducing the new fluid quantities $g=\gamma(1-\beta_z)$, $h=\gamma(1+\beta_z)$, and $\rho=(\gamma_0 n)/(\gamma n_0)$, where γ_0 is a constant equal to the initial relativistic factor of the electrons prior to the laser interaction, Eqs. (3) become

$$\left[\frac{\partial}{\partial\eta}\frac{\partial}{\partial\xi}-4\gamma_0^{-1}k_p^2\rho\right]\mathbf{a}=0\ , \tag{4a}$$

$$\left[\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right]^2\phi=k_p^2\left[\frac{\gamma\rho}{\gamma_0}-1\right]\ , \tag{4b}$$

$$\frac{\partial}{\partial\eta}(\rho h)-\frac{\partial}{\partial\xi}(\rho g)=0\ , \tag{4c}$$

$$h\frac{\partial h}{\partial\eta}-g\frac{\partial h}{\partial\xi}=-\frac{\partial}{\partial\xi}a^2+h\left[\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\xi}\right]\phi\ , \tag{4d}$$

$$g\frac{\partial g}{\partial\xi}-h\frac{\partial g}{\partial\eta}=-\frac{\partial}{\partial\eta}a^2+g\left[\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right]\phi\ , \tag{4e}$$

where $gh=1+a^2$, $\gamma=(h+g)/2$, and $\beta_z=(h-g)/(h+g)$.

To proceed with the analysis, the radiation is assumed to consist of a large-amplitude incident (pump) field traveling towards the left and a small-amplitude backscattered field traveling towards the right. The total radiation field is denoted by

$$\mathbf{a}=\mathbf{a}^{(0)}(\eta)+\mathbf{a}^{(1)}(\xi,\eta)\ ,$$

where $\mathbf{a}^{(0)}$ is the incident pump (equilibrium) field, $\mathbf{a}^{(1)}$ is the backscattered (perturbed) field, and $|\mathbf{a}^{(0)}|\gg|\mathbf{a}^{(1)}|$. The pump field, $\mathbf{a}^{(0)}$, is taken to be a function only of η (group and phase velocity equal to c, assuming $\omega_p\ll\omega_0$), which implies that the envelope of $\mathbf{a}^{(0)}$ is nonevolving, i.e., pump depletion effects are neglected. Since the backscattered field may be temporally growing, it is a function of both ξ and η. The fluid quantities all have the general form

$$\mathbf{Q}(\eta,\xi)=\mathbf{Q}^{(0)}(\eta)+\mathbf{Q}^{(1)}(\xi,\eta)\ ,$$

where $\mathbf{Q}^{(0)}$ represents the fluid quantity in the presence of only the incident field, $\mathbf{Q}^{(1)}$ is the perturbed contribution due to the backscattered field, and $|\mathbf{Q}^{(1)}| \ll \mathbf{Q}^{(0)}|$.

A. Equilibrium

Since the equilibrium quantities are functions of only the variable η, Eqs. (4) become

$$\frac{\partial^2}{\partial\eta^2}\phi^{(0)}=k_p^2(\gamma^{(0)}\rho^{(0)}/\gamma_0-1) , \tag{5a}$$

$$\frac{\partial}{\partial\eta}(\rho^{(0)}h^{(0)})=0 , \tag{5b}$$

$$\frac{\partial}{\partial\eta}(h^{(0)}-\phi^{(0)})=0 , \tag{5c}$$

$$h^{(0)}\frac{\partial}{\partial\eta}g^{(0)}=\frac{\partial}{\partial\eta}(a^{(0)})^2-g^{(0)}\frac{\partial}{\partial\eta}\phi^{(0)} , \tag{5d}$$

where the superscript (0) refers to quantities in the presence of only the incident field, $\mathbf{a}^{(0)}(\eta)$, and $g^{(0)}=[1+(a^{(0)})^2]/h^{(0)}$.

The equilibrium state of the electron fluid in the presence of the pump field can be obtained from Eqs. (5) which, in terms of $\phi^{(0)}$, give

$$h^{(0)}=\gamma_0(1+\beta_0)+\phi^{(0)}(\eta) , \tag{6a}$$

$$\rho^{(0)}=\gamma_0(1+\beta_0)/h^{(0)}(\eta) , \tag{6b}$$

$$g^{(0)}=\{1+[a^{(0)}(\eta)]^2\}/h^{(0)}(\eta) , \tag{6c}$$

where the pump field, $\mathbf{a}^{(0)}(\eta)$, is assumed to be known, $\gamma_0=(1-\beta_0^2)^{-1/2}$, and β_0 is the axial velocity of the fluid (electron beam or plasma) in the absence of the pump field. The self-consistent scalar potential is given by

$$\frac{\partial^2}{\partial\eta^2}\psi^{(0)}=\frac{\tilde{k}_p^2}{2}\left[\frac{[1+(a^{(0)})^2]}{(1+\psi^{(0)})^2}-1\right] , \tag{7a}$$

where $\psi^{(0)}=\phi^{(0)}/\gamma_0(1+\beta_0)$ and $\tilde{k}_p=k_p/\gamma_0^{3/2}(1+\beta_0)$. In the limit that $|\psi^{(0)}|\ll 1$, Eq. (7a) reduces to

$$\left[\frac{\partial^2}{\partial\eta^2}+\tilde{k}_p^2\right]\psi^{(0)}=\tilde{k}_p^2(a^{(0)})^2/2 . \tag{7b}$$

The equilibrium between the pump laser and the electrons is given by the expressions in Eqs. (6) together with the solution of Eq. (7a) for the potential $\psi^{(0)}$. The solution of Eq. (7a) is, in general, highly nonlinear. However, simple solutions can be obtained under conditions which are relevant to a wide range of applications. The pump-laser–electron fluid equilibrium will be considered in two limits. In these limits the characteristic temporal variation of the pump-laser envelope, τ_L (typically the pump-laser rise time), is compared to an effective plasma period $(c\tilde{k}_p)^{-1}$.

In the short-pulse limit, $\tau_L \ll (c\tilde{k}_p)^{-1}$, the magnitude of $\psi^{(0)}$ is much less than unity, $|\psi^{(0)}|\ll 1$, provided [6] $|a_0|<2/(c\tau_L\tilde{k}_p)$, where a_0 is the normalized amplitude of the pump-laser envelope, e.g., $a^{(0)}=a_0\cos k_0\eta$. In this limit, $\psi^{(0)}$ can be neglected in Eqs. (6).

In the long-pulse limit, $\tau_L \gg (c\tilde{k}_p)^{-1}$, the left-hand side of Eq. (7a) can be neglected. Furthermore, it can be shown that the fast oscillatory part of $\psi^{(0)}$ (on the laser frequency time scale) can be neglected [6] and, hence, $\psi^{(0)}\simeq(1+a_0^2/2)^{1/2}-1$.

For applications which utilize intense pump lasers with pulse lengths on the order of a ps, the short-pulse limit is relevant to interactions with electron beams with densities $n_0/\gamma_0^3 \ll 10^{-16}$ cm^{-3}, whereas the long-pulse limit is relevant to interactions with stationary ($\gamma_0=1$) plasmas with densities $n_0 \gg 10^{16}$ cm^{-3}. It proves convenient to define the equilibrium parameters h_0 and ρ_0, such that $h^{(0)}=h_0$ and $\rho^{(0)}=\rho_0$. The values of the parameters h_0 and ρ_0 depend on which of the above two equilibriums is being examined, i.e.,

$$h_0=\begin{cases}\gamma_0(1+\beta_0), & e \text{ beam (short pulse)},\\ \gamma_{\perp 0}, & \text{plasma (long pulse)},\end{cases} \tag{8a}$$

$$\rho_0=\begin{cases}1, & e \text{ beam (short pulse)},\\ 1/\gamma_{\perp 0}, & \text{plasma (long pulse)}\end{cases} , \tag{8b}$$

where $\gamma_{\perp 0}=(1+a_0^2/2)^{1/2}$. Hence, the equilibrium fluid quantities $\beta_z^{(0)}$, $\gamma^{(0)}$, and $n^{(0)}$ are given by

$$\beta_z^{(0)}=[h_0^2-1-(a^{(0)})^2]/[h_0^2+1+(a^{(0)})^2] , \tag{9a}$$

$$\gamma^{(0)}=[h_0^2+1+(a^{(0)})^2]/2h_0 , \tag{9b}$$

$$n^{(0)}=(n_0\rho_0/2h_0\gamma_0)[h_0^2+1+(a^{(0)})^2] . \tag{9c}$$

Physically, the difference between the plasma (high-density) equilibrium and the electron beam (low-density) equilibrium is due to the fact that, in the high-density regime, the ponderomotive force associated with the laser envelope variation is balanced by the space-charge force, whereas in the low-density regime the ponderomotive force dominates.

B. Backscattered radiation

To analyze the SBH radiation, Eqs. (4) are expanded about the equilibrium state given in Eqs. (6). In the following, the perturbed electrostatic field, $\phi^{(1)}$, is neglected. This is valid provided [12,20,23] the temporal growth rate of the backscattered harmonic radiation is much greater than the relativistically corrected plasma frequency, $\omega_p/\gamma_0^{3/2}$. In this strong-pump regime, the SBH radiation is completely described by the fluid equations

$$\left[h^2\frac{\partial}{\partial\eta}-(1+a^2)\frac{\partial}{\partial\xi}\right]h=-h\frac{\partial}{\partial\xi}a^2 , \tag{10a}$$

$$\left[h^2\frac{\partial}{\partial\eta}-(1+a^2)\frac{\partial}{\partial\xi}\right]\rho=-\rho\frac{\partial}{\partial\eta}h^2 , \tag{10b}$$

along with the wave equation, Eq. (4a), and the equilibrium given by Eq. (8).

The self-consistent closed set of perturbed quantities $h^{(1)}$, $\rho^{(1)}$, and $\mathbf{a}^{(1)}$ satisfy

$$\left[\frac{\partial}{\partial\eta}-\mu(\eta)\frac{\partial}{\partial\xi}\right]h^{(1)}=-\frac{2}{h_0}\frac{\partial}{\partial\xi}(\mathbf{a}^{(0)}\cdot\mathbf{a}^{(1)}) , \tag{11a}$$

$$\left[\frac{\partial}{\partial\eta}-\mu(\eta)\frac{\partial}{\partial\xi}\right]\rho^{(1)}=-\frac{2}{h_0}\rho_0\frac{\partial}{\partial\eta}h^{(1)}\ , \qquad (11b)$$

$$\left[\frac{\partial^2}{\partial\eta\partial\xi}-\frac{k_p^2\rho_0}{4\gamma_0}\right]\mathbf{a}^{(1)}=\frac{k_p^2}{4\gamma_0}\mathbf{a}^{(0)}\rho^{(1)}\ , \qquad (11c)$$

where $\mu(\eta)=[1+(a^{(0)})^2]/h_0^2$. To proceed, the perturbed quantities are represented by the form $\mathbf{Q}^{(1)}=\mathbf{Q}_1(\eta)e^{ik\xi}/2+\text{c.c.}$, where k is complex and the amplitude is a function of η. Using this representation, the perturbed amplitudes are given by

$$\left[\frac{\partial}{\partial\eta}-ik\mu(\eta)\right]h_1(\eta)=-\frac{2ik}{h_0}\mathbf{a}^{(0)}(\eta)\cdot\mathbf{a}_1(\eta)\ , \qquad (12a)$$

$$\left[\frac{\partial}{\partial\eta}-ik\mu(\eta)\right]\rho_1(\eta)=-\frac{2}{h_0}\rho_0\frac{\partial}{\partial\eta}h_1(\eta)\ , \qquad (12b)$$

$$\left[ik\frac{\partial}{\partial\eta}-\frac{k_p^2\rho_0}{4\gamma_0}\right]\mathbf{a}_1(\eta)=\frac{k_p^2}{4\gamma_0}\mathbf{a}^{(0)}(\eta)\rho_1(\eta)\ . \qquad (12c)$$

Equations (12a) and (12b) have the solutions

$$h_1(\eta)=e^{i\theta(\eta)}\int^{\eta}e^{-i\theta(\eta')}\left[-\frac{2ik}{h_0}\mathbf{a}^{(0)}(\eta')\cdot\mathbf{a}_1(\eta')\right]d\eta'\ , \qquad (13a)$$

$$\frac{\rho_1(\eta)}{\rho_0}=-\frac{2}{h_0}h_1(\eta)+e^{i\theta(\eta)}\int^{\eta}e^{-i\theta(\eta')}\left[-\frac{2ik}{h_0}\mu(\eta')h_1(\eta')\right]d\eta'\ , \qquad (13b)$$

where $\theta(\eta)=k\int^{\eta}\mu(\eta'')d\eta''$.

Taking the incident field to be $\mathbf{a}^{(0)}=\mathbf{a}_0\cos k_0\eta$ and the complex amplitude of the backscattered harmonic field to be $\mathbf{a}_1=\hat{\mathbf{a}}_1\exp(i\Delta k\eta)$, where Δk is complex, the solutions to Eqs. (13a) and (13b) become

$$h_1(\eta)=\frac{k}{h_0}\mathbf{a}_0\cdot\hat{\mathbf{a}}_1\sum_{l,n=-\infty}^{\infty}\frac{(-1)^lJ_n(b)[J_l(b)-J_{l+1}(b)]}{[\bar{k}-k_0(1+2l)-\Delta k]}\exp(i\{k_0[1+2(l+n)]+\Delta k\}\eta)\ , \qquad (14a)$$

$$\frac{\rho_1(\eta)}{\rho_0}=\frac{2k\bar{k}}{h_0^2}\mathbf{a}_0\cdot\hat{\mathbf{a}}_1\sum_{l,n=-\infty}^{\infty}(-1)^lJ_n(b)\frac{[J_l(b)-J_{l+1}(b)]}{[\bar{k}-k_0(1+2l)-\Delta k]^2}\exp(i\{k_0[1+2(l+n)]+\Delta k\}\eta)\ , \qquad (14b)$$

where $\bar{k}=k\gamma_{10}^2/h_0^2$ and $b=ka_0^2/(4k_0h_0^2)$. In obtaining the expressions for $h_1(\eta)$ and $\rho_1(\eta)$, the Bessel identity,

$$\exp(ib\sin x)=\sum_{-\infty}^{\infty}J_n(b)\exp(inx)\ ,$$

was used. Also, near resonance the first term on the right of Eq. (13b) has been neglected and the initial-value terms in Eqs. (13a) and (13b) have been neglected compared to the exponentially growing modes.

C. Dispersion relation

The dispersion relation for the SBH modes may be obtained from the perturbed wave equation. Substituting Eq. (14b) into Eq. (12c) and requiring that both sides of the equation have the same η dependence, yields

$$4k\Delta k+k_p^2\frac{\rho_0}{\gamma_0}=-\frac{\rho_0k_p^2\bar{k}k}{\gamma_0h_0^2}a_0^2\sum_{l=-\infty}^{\infty}\frac{(J_l-J_{l+1})^2}{[\bar{k}-(2l+1)k_0-\Delta k]^2}\ , \qquad (15)$$

where $\mathbf{a}_0\cdot\hat{\mathbf{a}}_1=a_0\hat{a}_1$ has been assumed. The frequency, ω, and temporal growth rate, Γ, may be determined from the above expression by setting $k=\omega/c-\Delta k$. The backscattered radiation has the phase dependence $a^{(1)}\sim\exp[i(\omega/c)\xi+i2ct\Delta k]$, where the temporal growth rate is given by $\Gamma=-2c\,\text{Im}(\Delta k)$. The dispersion relation indicates that the resonant frequency of the lth mode is

$$\omega=NM_0\omega_0\ , \qquad (16)$$

where $N=2l+1$ is the harmonic number and $M_0=(h_0/\gamma_{10})^2$ is the frequency multiplication factor which is dependent on the particular equilibrium being examined, i.e.,

$$M_0=\begin{cases}\gamma_0^2(1+\beta_0)^2/(1+a_0^2/2), & e\text{ beam},\\ 1, & \text{plasma}\ .\end{cases} \qquad (17)$$

Assuming $|\omega/c|\gg|\Delta k|\gg k_p/\gamma_0^{3/2}$, the dispersion relation reduces to

$$\Delta k^3=-\frac{\rho_0k_p^2k_0M_0F_l(b_l)}{\gamma_0(1+M_0)^2}\ , \qquad (18)$$

where $F_l=b_l[J_l(b_l)-J_{l+1}(b_l)]^2$ is the harmonic coupling function [20] and $b_l=(2l+1)a_0^2/4\gamma_{10}^2$. The temporal growth rate is

$$\Gamma=\sqrt{3}c\left|\frac{\rho_0k_p^2k_0M_0F_l}{\gamma_0(1+M_0)^2}\right|^{1/3}\ . \qquad (19a)$$

A plot of the function $F_l^{1/3}$ vs $b_0=a_0^2/4\gamma_{10}^2$ for $N=1+2l=1,3,5.\ .\ .\ ,19$ is shown in Fig. 2. For a rela-

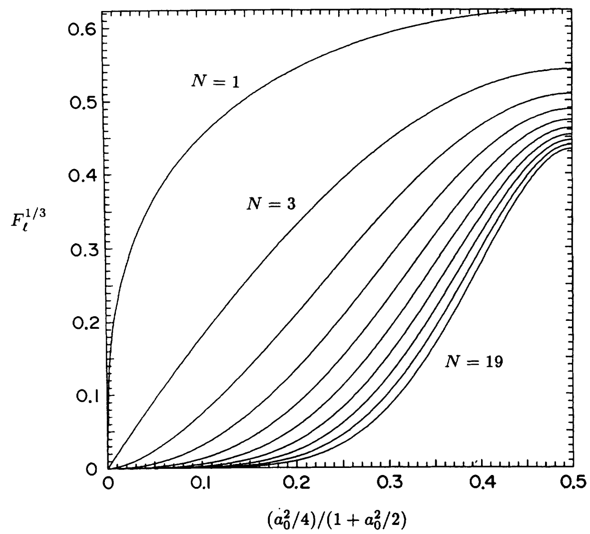

FIG. 2. The function $F_l^{1/3}$ (proportional to the growth rate Γ) vs the parameter $b_0=(a_0^2/4)(1+a_0^2/2)$ for the harmonics $N=1,3,5\ldots,19$.

tivistic electron beam in which $h_0 \simeq 2\gamma_0 \gg \gamma_{\perp 0}$, the growth rate is given by

$$\Gamma \simeq \sqrt{3}c(k_p^2 k_0 \gamma_{\perp 0}^2 F_l/4\gamma_0^3)^{1/3} . \qquad (19b)$$

For a plasma, the growth rate is given by

$$\Gamma \simeq \sqrt{3}c(k_p^2 k_0 F_l/4\gamma_{\perp 0})^{1/3} . \qquad (19c)$$

The asymptotic value of the harmonic coupling function $F_l(b_l)$ for $a_0^2 \gg 1$ and $l \gg 1$ can easily be found. In this asymptotic limit, $b_l \simeq l$ and $J_l(l)-J_{l+1}(l) \simeq J_l'(l) \simeq 0.411 l^{-2/3}$. Hence,

$$F_l \simeq 0.169/l^{1/3} . \qquad (20a)$$

Notice that asymptotically $(l \gg 1, a_0^2 \gg 1)$, $\Gamma \sim l^{-1/9}$, i.e., the growth rate is a weakly decreasing function of l.

For small arguments, $b_l < 1$, the harmonic coupling function is given by

$$F_l \simeq \frac{2(b_l/2)^{2l+1}}{(l!)^2}\left[1-\frac{b_l}{2(l+1)}\right]^2 , \qquad (20b)$$

which, for $a_0^2/2 \ll 1$, gives

$$F_l \simeq 2[(2l+1)a_0^2/8]^{2l+1}(l!)^{-2} . \qquad (20c)$$

Hence, in the limit $(2l+1)a_0^2/2 \ll 1$, the growth rate is a rapidly decreasing function of l. Notice that, for $a_0^2/2 \ll 1$ and $l=0$, Eqs. (19c) and (20c) indicate the growth rate of the fundamental backscattered radiation from a plasma is $\Gamma=\sqrt{3}(\omega_p^2\omega_0 a_0^2/16)^{1/3}$, which is the standard result for the Raman backscatter instability in the strong-pump or strongly coupled regime for a linearly polarized, low-power $(a_0^2 \ll 1)$ laser [23].

III. SATURATION

The present analysis indicates that the SBH radiation grows exponentially, with a growth rate given by Eq. (19). This growth continues until nonlinear effects (i.e., particle trapping) limit the amplitude. The saturated value of the backscattered radiation may be determined from arguments based on electron trapping in the ponderomotive wave. The growth of the radiation will cease when the generalized ponderomotive wave traps the electrons. For small-amplitude pump fields, $|a_0| \ll 1$, the ponderomotive wave is proportional to the product of the stimulated backscattered radiation and the pump field, $\mathbf{a}^{(1)}\cdot\mathbf{a}^{(0)}$. In the present analysis a generalized ponderomotive wave is obtained for arbitrarily large values of $|a_0|$. The nonlinear effects of the pump field, in the generalized ponderomotive wave, are included to all orders.

The combined action of the pump laser and the stimulated backscattered radiation results in a generalized ponderomotive wave which produces a fluid wave, $\rho^{(1)}$, with the same phase,

$$\rho^{(1)} \sim \sum_{l,n} C_{l,n}\exp\{i[(2l+1+2n)k_0+\Delta k]\eta+ik\xi\} , \qquad (21)$$

as indicated by Eq. (14b), where $\rho^{(1)} = \rho_1(\eta)\exp(ik\xi)/2+\text{c.c.}$ and $C_{l,n}$ are constant coefficients. The generalized ponderomotive wave consists of a sum of individual modes characterized by the mode numbers (l,n). Since $\xi=z-ct$, $\eta=z+ct$, and $k=\omega/c-\Delta k$, the normalized phase velocity for a particular generalized ponderomotive mode (l,n) in Eq. (21) is given by

$$\beta_p(l,n)=\frac{\omega/c-(2l+2n+1)k_0-2\Delta k}{\omega/c+(2l+2n+1)k_0} . \qquad (22a)$$

At a particular resonant frequency $\omega=(2l+1)M_0\omega_0$ of the lth mode, the phase velocity in Eq. (22a) becomes

$$\beta_p(l,n)=\frac{(M_0-1)(2l+1)k_0-2nk_0-2\Delta k}{(M_0+1)(2l+1)k_0+2nk_0} . \qquad (22b)$$

Particle trapping for a particular n mode occurs when the longitudinal fluid velocity of that mode is equal to the phase velocity in Eq. (22b). Trapping occurs first for the $n=0$ mode since it is the mode with phase velocity closest to the equilibrium velocity of the electrons.

The longitudinal fluid velocity of the electrons is given by $\beta_z=(h-g)/(h+g)$, which can be written as

$$\beta_z=\frac{h^2-(1+a^2)}{h^2+(1+a^2)} . \qquad (23)$$

In terms of equilibrium and perturbed quantities, $\beta_z=\beta_z^{(0)}+\beta_z^{(1)}$, where the average value of $\beta_z^{(0)}$ is $\langle\beta_z^{(0)}\rangle \simeq (M_0-1)/(M_0+1)$. Expanding Eq. (23), the leading-order contribution to the perturbed velocity at resonance is given by

$$\beta_z^{(1)} \simeq \frac{4M_0}{(1+M_0)^2}\frac{h^{(1)}}{h_0} , \qquad (24)$$

where $h^{(1)}$ is composed of a collection of n modes, the phase velocity of the nth mode given in Eq. (22b). Using Eq. (14a), the perturbed fluid velocity of the nth mode is given by

$$|\beta_z^{(1)}| \simeq \left| \frac{16\hat{a}_1 M_0^2 k_0}{a_0(1+M_0)^3 \Delta k} (bF_l)^{1/2} J_n \right| . \quad (25)$$

Particle trapping occurs first for the $n=0$ mode, which is the mode with phase velocity closest to the equilibrium fluid velocity, $\langle \beta_z^{(0)} \rangle$,

$$\beta_p(l, n=0) = \langle \beta_z^{(0)} \rangle + \delta\beta_p , \quad (26a)$$

where

$$\delta\beta_p = -2\Delta k/(2l+1)(1+M_0)k_0 . \quad (26b)$$

The saturated level of the radiation field may be obtained from the condition for deeply trapped electrons, $|\beta_z^{(1)}| \simeq 2|\delta\beta_p|$, which gives for the $n=0$ mode,

$$\frac{|\Delta k|^2}{k_0^2} = \left| \frac{4\hat{a}_1(2l+1)M_0^2}{a_0(1+M_0)^2} (bF_l)^{1/2} J_0 \right| . \quad (27)$$

A numerical factor of order unity has been neglected on the left-hand side of Eq. (27). Using the dispersion relation, the ratio of the radiation power in the Nth harmonic at saturation to that in the pump laser, $P_N/P_0 = M_0^2 N^2 |\hat{a}_1|^2 / a_0^2$, is given by

$$\frac{P_N}{P_0} = \left[\frac{\rho_0 k_p^2 (1+M_0)}{\gamma_0 k_0^2 M_0^{1/2}} \right]^{4/3} \frac{F_l^{1/3}}{16 b_l J_0^2} . \quad (28)$$

A plot of the function $F_l^{1/3}/b_l$ vs $b_0 = a_0^2/4\gamma_{10}^2$ for $N = 1+2l = 1, 3, 5 \ldots, 19$ is shown in Fig. 3.

Note that asymptotically ($l \gg 1, a_0^2 \gg 1$), $P_N/P_0 \sim l^{-10/9}/J_0^2$. Furthermore, the efficiency may be substantially increased by operating in a regime in which $b_l = (1+2l)a_0^2/4\gamma_{10}^2$ is near a zero of J_0, i.e., $J_0(b_l) \simeq 0$. Physically, the regime for which $J_0(b_l) \simeq 0$ for a particular l mode corresponds to minimizing the perturbed fluid velocity $\beta_z^{(1)}(l, n=0)$ of that mode. This implies that a larger amplitude of backscattered radiation will be reached before saturation occurs. At $J_0(b_l) = 0$, Eq. (28) no longer applies, and saturation due to particle trapping must be determined by consideration of the higher n modes. In general, Eq. (28) is valid provided $P_N/P_0 < 1$. If this is violated, then saturation will be caused by some other mechanism besides particle trapping, i.e., plasma thermalization or pump depletion. In particular, notice that, for a plasma with $l=0$ and $a_0^2 \ll 1$, Eq. (28) indicates $P_N/P_0 \simeq (k_p^2/2a_0 k_0^2)^{4/3}$ and, hence, $P_N/P_0 < 1$ implies $a_0 > k_p^2/2k_0^2$.

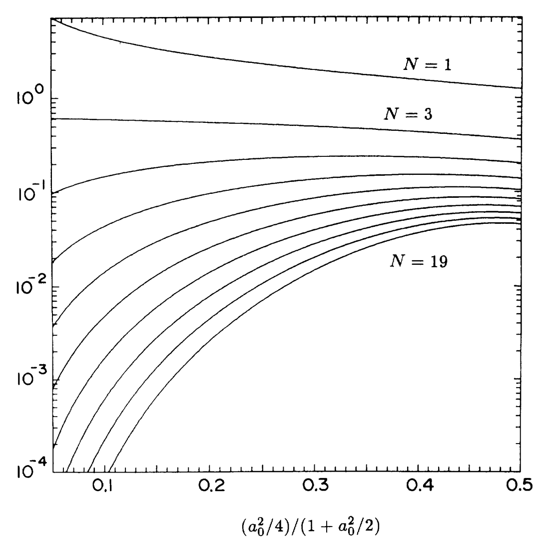

FIG. 3. The function $F_l^{1/3}/b_l$ (proportional to the power at saturation P_N/P_0) vs the parameter $b_0 = (a_0^2/4)/(1+a_0^2/2)$ for the harmonics $N = 1, 3, 5 \ldots, 19$.

It is also convenient to define an electronic efficiency, η_e, which is the ratio of the backscattered radiation power to the electron beam power,

$$\eta_e \equiv \frac{M_0^2 N^2 k_0^2 |\hat{a}_1|^2}{2k_p^2 \beta_0(\gamma_0 - 1)} . \quad (29a)$$

Using Eq. (28), the electronic efficiency may be written as

$$\eta_e = \frac{\rho_0(1+M_0)^2 \gamma_{10}^2/\gamma_0^2}{8J_0^2 M_0 \beta_0(1-1/\gamma_0)} \frac{|\Delta k|}{N k_0} , \quad (29b)$$

where $|\Delta k| = \Gamma/\sqrt{3}c$ is given by Eq. (19). Notice that, for the fundamental ($l=0$) in the limit $\gamma_0 \gg 1$ and $a_0^2 \ll 1$, $\eta_e = \Gamma/2\sqrt{3}\omega_0$, as is the case for conventional strong-pump FEL's [12], where $\Gamma = \sqrt{3}(\omega_p^2 \omega_0 a_0^2/16\gamma_0^3)^{1/3}$.

IV. THERMAL, SPACE-CHARGE AND COLLISIONAL EFFECTS

A. Thermal effects

The above results were obtained using cold fluid theory, i.e., electron thermal effects were neglected. For a sufficiently thermal electron distribution, however, it is possible for the longitudinal energy spread to become large enough so as to degrade the resonant interaction between the backscattered wave and the electron distribution. This thermal electron interaction regime corresponds to a weak resonant instability in which the growth rate of the backscattered radiation is greatly reduced.

It is possible to estimate how large a thermal electron velocity spread can be tolerated before the resonant wave-particle interaction is degraded. The growth rate of the cold electron instability is determined largely by the resonant denominator, D, in the dispersion relation, Eq. (15),

$$D \equiv [\bar{k} - (2l+1)k_0 - \Delta k]^2 , \quad (30)$$

where $\bar{k} = k/M_0$ and M_0 is given in Eq. (17). The effects of a longitudinal thermal velocity spread β_{th} on the resonance may be estimated by letting $\beta_0 \to \beta_0 + \beta_{th}$ in the expression for D, where $|\beta_{th}/(1+\beta_0)| \ll 1$. At resonance, $\omega/c = M_0(2l+1)k_0$, one finds

$$D \simeq [2\gamma_0^2(1+2l)k_0\beta_{th} + (1+1/M_0)\Delta k]^2 . \quad (31)$$

Hence, in order to neglect for the effects of the thermal velocity spread β_{th}, it is necessary that

$$\beta_{\rm th} \ll \frac{(1+M_0)|\Delta k|/k_0}{2\gamma_0^2 M_0(2l+1)} . \tag{32}$$

Using the dispersion relation, Eq. (32) yields the condition

$$\beta_{\rm th} \ll \frac{1}{2\gamma_0^2 N}\left|\frac{\rho_0 k_p^2(1+M_0)}{k_0^2\gamma_0 M_0^2}F_l\right|^{1/3} . \tag{33}$$

A plot of the function $F_l^{1/3}/N$ vs $b_0=a_0^2/4\gamma_{10}^2$ for $N=1+2l=1,3,5\ldots,19$ is shown in Fig. 4. Asymptotically ($l \gg 1$ and $a_0^2 \gg 1$), the right-hand side of Eq. (33) scales as $l^{-10/9}$. For a stationary plasma, the thermal energy of the plasma is $\Delta E_{\rm th}=m_0c^2\beta_{\rm th}^2/2$. For an electron beam with initial velocity $\beta_0 \gg \beta_{\rm th}$ and $\gamma_0 \gg 1$, the normalized energy spread is given by $\Delta\gamma_{\rm th}/\gamma_0=\gamma_0^2\beta_{\rm th}$. Note that, for the fundamental ($l=0$) in the limit $\gamma_0 \gg 1$ and $a_0^2 \ll 1$, the thermal requirement is $\Delta\gamma_{\rm th}/\gamma_0 \ll \eta_e$, as is the case for conventional strong-pump FEL's operating at the fundamental [12]. The usual requirement regarding FEL's operating at the fundamental, $\Delta\gamma_{\rm th}/\gamma_0 \ll \eta_e$, however, does not apply in general to harmonic generation.

B. Space-charge effects

The expressions for the growth rate of the SBH radiation, the amplitude of the radiation at saturation, and the allowable thermal energy spread are all increasing functions of the electron density. Hence, optimal generation of the SBH radiation implies operating in a regime of high electron density. However, the fluid theory of Sec. II used to describe the backscattered radiation assumed the strong-pump limit, i.e., assumed that the electron density was sufficiently low so that the effects of the perturbed electrostatic potential may be neglected. This is valid provided the relativistic plasma frequency is small

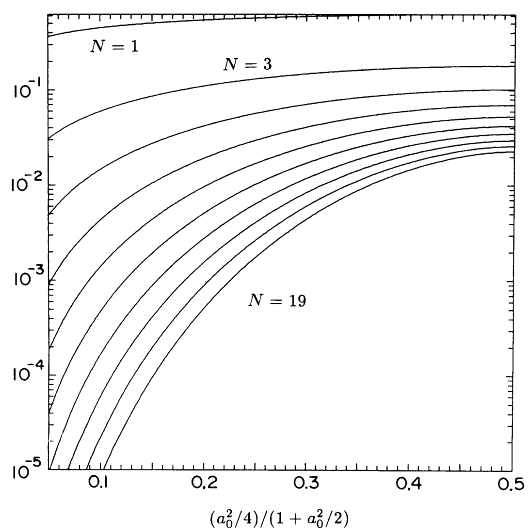

FIG. 4. The function $F_l^{1/3}/N$ (proportional to the upper limit of the thermal velocity spread, $\beta_{\rm th}$) vs the parameter $b_0=(a_0^2/4)/(1+a_0^2/2)$ for the harmonics $N=1,3,5\ldots,19$.

compared to the growth rate of the radiation. More specifically, it can be shown that the strong-pump limit is valid provided

$$\frac{\rho_0 k_p^2 M_0}{\gamma_0(1+M_0)^2} \ll |\Delta k|^2 . \tag{34}$$

This inequality gives an upper limit to the electron density for which the strong-pump limit remains valid. Using the dispersion relation, Eq. (18), gives

$$k_p^2/k_0^2 \ll \gamma_0(1+M_0)^2F_l^2/\rho_0M_0 . \tag{35}$$

Note that, asymptotically ($l \gg 1$ and $a_0^2 \gg 1$), the right-hand side of Eq. (35) scales as $l^{-2/3}$. Strictly speaking, the results of the strong-pump theory obtained in the previous sections are valid provided Eq. (35) is satisfied.

C. Collisional effects

The effects of collisions on the backscattered harmonics may be neglected provided the damping rate of the radiation field due to collisions [3], $\nu \simeq (\omega_p^2/\omega^2)\nu_{ei}$, is small compared to the growth rate of the radiation field, Γ, where ν_{ei} is the electron-ion collision frequency, i.e., $\Gamma \gg (\omega_p^2/\omega^2)\nu_{ei}$. The relativistic collision frequency is given by [24]

$$\nu_{ei}[{\rm s}^{-1}] \simeq 4.3\times10^{-14}n_0(\ln\Lambda)(\beta^{(0)})^{-3}(\gamma^{(0)})^{-2} , \tag{36}$$

where $\ln\Lambda$ is the Coulomb logarithm and n_0 is in cm^{-3}. The collision frequency ν_{ei} will be largest for electrons in a dense plasma. For plasma electrons undergoing relativistic quiver motion in the intense pump-laser field,

$$(\beta^{(0)})^{-3}(\gamma^{(0)})^{-2} = \frac{\gamma_{\perp0}[1+(a_0^2/4\gamma_{10}^2)\cos 2k_0\eta]}{a_0^3[\cos^2k_0\eta+(a_0^2/16\gamma_{10}^2)\cos^2 2k_0\eta]^{3/2}} . \tag{37}$$

An upper limit on Eq. (37) is given by

$$(\beta^{(0)})^{-3}(\gamma^{(0)})^{-2} < \frac{\gamma_{\perp0}(1+a_0^2/4\gamma_{10}^2)}{a_0^3(a_0^2/16\gamma_{10}^2)^{3/2}} . \tag{38}$$

In the limit $a_0^2/2 \gg 1$, the right-hand side of Eq. (38) reduces to $24/a_0^2$. Hence, for intense laser fields where $a_0^2 \gtrsim 1$, the condition that $\Gamma \gg \nu$ is easily satisfied.

V. EXAMPLES

The SBH instability described in the previous sections may be used as a mechanism to generate coherent radiation in the xuv regime. Conceptionally, a device may be designed to amplify a small xuv input signal injected into a plasma or a copropagating electron beam. The input xuv signal may be obtained by the output of an incoherent source, such as a flash lamp, or by using the incoherent single-particle (spontaneous) radiation generated by the interaction of the pump laser with the electron distribution. The xuv radiation will be amplified as it propagates, via the interaction with the intense, counterpropagating pump-laser field. Two examples will be discussed; one utilizing a stationary plasma and the other utilizing a relativistic electron beam. In both examples, the pump laser is taken to be an intense Nd:glass laser ($\lambda_0=1$ μm) with an intensity of $I_0=9.7\times10^{18}$ W/cm^2 ($a_0=2.6$) and a spot size of $r_0=10$ μm, which gives a pump laser power of $P_0=15$ TW.

A. Backscattering from a plasma

Consider the amplification of coherent radiation using a plasma of density $n_0=10^{19}$ cm^{-3}, which corresponds to fully ionized H_2 at about 0.1 atm. Neglecting the effects of relativistic optical guiding [5–8], the interaction length is approximately either two vacuum Rayleigh lengths, $2Z_R=2\pi r_0^2/\lambda_0=630$ μm, or one-half the pump-laser pulse length (which is 150 μm for a 1-ps laser pulse), whichever is shorter. As an example, consider the amplification of the third, $l=1$, harmonic (and the fifth, $l=2$, harmonic) at a wavelength of $\lambda=3300$ Å (2000 Å). The e-folding length is $c/\Gamma=1.8$ μm (2.0 μm). At saturation, the ratio of the harmonic power to the pump-laser power is $P_N/P_0=1.0\times10^{-4}$ (3.8×10^{-4}), which implies a saturation power of $P_N=1.5$ GW (5.8 GW). (The larger saturated power for the fifth harmonic is due to the factor $1/J_0^2$.) The pulse length of the SBH radiation is approximately the transit length of the pump-laser pulse through the plasma. Optical guiding [5–8] may substantially increase this length beyond the vacuum diffraction limit of $2Z_R=630$ μm. The thermal requirement on the longitudinal energy spread is $E_{\rm th}<77$ eV (22 eV). Plasmas with sufficiently cold longitudinal temperatures may be produced by laser-induced ionization [25].

B. Backscattering from an electron beam

Consider amplification of coherent radiation using an intense electron beam with a current of 15 A, a beam radius of 10 μm (a current density of 4.8 MA/cm^2), and an energy of 250 keV ($\gamma_0=1.5$). The interaction length is approximately either two vacuum Rayleigh lengths, $2Z_R=630$ μm, or one-half the laser pulse length, whichever is shorter. As an example, consider the amplification of the third, $l=1$, harmonic at a wavelength of $\lambda=\lambda_0/3M=2200$ Å. The e-folding length is $c/\Gamma=31$ μm. The thermal requirement on the longitudinal energy spread is $\Delta\gamma/(\gamma_0-1)<0.18\%$. The pulse length of the SBH radiation is approximately the transit length of the pump-laser pulse through the electron beam, i.e., $2Z_R=630$ μm. At saturation, the ratio of the third harmonic power to the pump-laser power is $P_N/P_0=1.1\times10^{-9}$, which implies a saturation power of $P_N=17$ kW. The electronic efficiency at saturation is $\eta_e=0.87\%$.

VI. DISCUSSION

The generation of coherent, stimulated backscattered harmonic radiation by the interaction of an intense, linearly polarized laser pulse with an electron beam or a plasma has been analyzed using relativistic, cold fluid equations. This theory includes the nonlinear effects of the pump laser to all orders in the normalized vector potential, a_0. A dispersion relation for the SBH radiation was obtained in the strong-pump regime. In this regime, the effects of the perturbed electrostatic potential are neglected. The resonant frequency of the lth mode, corresponding to the harmonic number $N=2l+1$, is given by $\omega/c=NM_0k_0$, where M_0 is the frequency multiplication factor arising from the relativistic Doppler upshift. The temporal growth rate is given by Eq. (19). Significant high harmonic generation, $N\gg1$, requires intense pump lasers, $a_0>1$. The SBH radiation from an electron beam may be viewed as a nonlinear laser-pumped FEL mechanism [12–22], whereas the SBH radiation from a plasma may be viewed as a nonlinear Raman instability mechanism [23], both in the strong-pump regime.

The saturation amplitude of the SBH radiation was calculated based on particle-trapping arguments. Particle trapping (wavebreaking) occurs when the amplitude of the perturbed longitudinal fluid velocity, $\beta_z^{(1)}$, becomes equal to the phase velocity, β_p. The perturbed fluid velocity, $\beta_z^{(1)}$, consists of a sum of individual waves characterized by the mode numbers (l,n). For a particular l resonance, particle trapping occurs first for the $n=0$ mode, which is the component of $\beta_z^{(1)}$ with phase velocity closest to the equilibrium longitudinal electron velocity, $\langle\beta_z^{(0)}\rangle$. The resulting saturation amplitude of the backscattered radiation is given by Eq. (28). Furthermore, the saturation amplitude of the backscattered radiation may be enhanced by operating in a regime in which $b_l=(2l+1)a_0^2/4\gamma_{\perp0}^2$ is close to a zero of the Bessel function $J_0(b_l)$. Physically, $J_0=0$ corresponds to minimizing the amplitude of the $n=0$ component of the perturbed fluid velocity $\beta_z^{(1)}$. When this occurs, saturation may be determined from consideration of the next higher n mode. It should also be pointed out that particle trapping only leads to saturation for sufficiently intense pump laser amplitudes for which $P_N/P_0<1$. When this inequality is not satisfied, saturation occurs by other nonlinear effects, such as electron thermalization or pump depletion.

The most stringent constraint on the production of SBH radiation is the restriction on the longitudinal thermal velocity spread. If the longitudinal electron temperature is sufficiently high, the wave-particle resonance is degraded and the growth rate of the backscattered radiation is greatly reduced. The expression for the allowable thermal velocity spread is given by Eq. (33). In general, this implies that very small thermal spreads are required in order to generate high-order harmonic radiation. The expressions for the growth rate, saturation amplitude, and allowable thermal velocity spread indicate that high-electron densities are required. This appears to favor the use of stationary plasmas over that of relativistic electron beams. However, the use of an electron beam has an advantage in that the frequency of the harmonic radiation is relativistically upshifted (in addition to the harmonic upshift), which implies the radiation frequency may be tuned by adjusting the energy of the electron beam or the amplitude of the pump laser, as indicated by Eq. (16). The growth rates, saturation efficiencies, and thermal spread requirements for the SBH radiation are summarized in Table I. As the capability for producing dense plasmas and electron beams with small thermal spreads improves, along with future advances in ultrahigh-power laser technology, SBH generation may provide a practical method for producing coherent radiation in the xuv regime.

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