PHYSICAL REVIEW A VOLUME 45, NUMBER 7 1 APRIL 1992

Transient statistics for a good-cavity laser with swept losses

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(Received 24 April 1991; revised manuscript received 6 November 1991)

We study the statistical properties of the intensity of a good-cavity laser driven across the threshold by a linear variation in time of the cavity losses. This study is performed by using the quasideterministic theory (QDT) for two cases: In the first one, case I, the control parameter $a(t)$ changes without bound, and in the second, case II, $a(t)$ reaches a fixed value. The validity of QDT is then analyzed for both cases. We characterize the anomalous fluctuations by the time t_m at which the maximum appears and by the value of this maximum σ_m. Our results show that t_m coincides with the average of the time of maximum rate of growth of the intensity T. We show that in case I, t_m^{-1} and σ_m scale with $\sqrt{v}$ and that the relative fluctuations are given by a universal scaling function. The scaling of t_m is in agreement with experimental studies. In case II we calculate analytically T, t_m, and σ_m, and we show that the moments of the intensity are given through a scaling variable. All the results are checked with numerical simulations.

PACS number(s): 42.50.Ar, 05.40.+j

I. INTRODUCTION

The laser switch-on can be considered as a dynamical bifurcation with a control parameter that is continuously swept in time through the instability point. It has been shown that because of the critical slowing down displayed by the laser first threshold, the dynamical bifurcation is delayed with respect to the time at which the instability point is crossed [1]. Deterministic analyses of this problem in the context of laser physics include the laser with saturable absorber [2], the laser Lorenz equations [3], as well as their good-cavity limit [4, 5], class-B lasers [6], and intrinsic optical bistability [7]. Fluctuations have been considered for the laser in the good-cavity limit by numerical integration [8], analogic simulations [9], and analytic methods in the linear regime [10–12]. Experimental results for CO_2 lasers with saturable absorber [13], Ar^+ lasers [14], CO_2 lasers [15], and semiconductor lasers [16] are now available. Sweeping-rate dependence has also been discussed in connection with fluid instabilities [17] and general theories for the decay of unstable states [18].

Numerical integration of the Fokker-Planck equation, which describes the action of spontaneous-emission noise in a good-cavity laser, has shown that the delay persists when the sweeping rate is larger than the intensity of the noise [8]. The dynamical bifurcation has been characterized in terms of first-passage-times distribution [10] and using the averaged intensity [11]. However, experiments in an Ar^+ laser driven across the threshold region by a variation of the cavity losses analyze another magnitude T that is the time at which the intensity has its maximum slope. This study indicates that the sweeping rate v, and T, satisfy the relation $\sqrt{v}T \sim$ const [14]. A theoretical determination of T requires the solution of a nonlinear problem because at this time the system has already started to be attracted by the stationary solution. An analysis of the nonlinear deterministic equation has been carried out and shows the same $v^{1/2}$ dependence for T if the sweeping rate is large enough [4]. When the effect of spontaneous-emission noise is considered, the time T becomes a random variable and anomalous large transient fluctuations in the nonlinear regime appear [19, 20]. A nonlinear stochastic description is then desirable to study this problem.

In this paper the study of the anomalous fluctuations for a good-cavity laser with swept losses is made by generalizing the quasideterministic theory [21] (QDT) to this problem [22]. This generalization is performed for two cases: in the first one (case I), the control parameter increases linearly for any time with a sweeping rate v, and in the second case (case II), it increases in the same way until it reaches a fixed value a. The first case is of interest in the study of slow variations of the control parameter. The second one allows us to calculate corrections to the instantaneous-change case. In both cases, we study the validity conditions of the QDT. The QDT is shown to be valid in case I when the sweeping rate is large with respect to the intensity of spontaneous-emission noise D, and in case II when D is much smaller than the final control parameter a. We characterize the anomalous fluctuations by the time t_m at which the maximum of the variance of the intensity happens and by the value of this maximum σ_m. Numerical simulations show that if the sweeping rate is of the order of D the maximum of the fluctuations disappears and the delay in the bifurcation becomes negligible. When the sweeping rate is larger than D, t_m is found to coincide with the average of T, $\langle T\rangle$, for both cases I and II. This magnitude T is shown to have small fluctuations. These results are explained using the QDT.

In case I we find, using the QDT and numerical simulations, that t_m^{-1} and σ_m scale with $\sqrt{v}$. This is in agreement with the experimental results for $\langle T\rangle$ in an Ar^+ laser [14]. With the use of the QDT, relative fluctuations of the intensity are shown to be given by a universal scaling function. In case II we find, by using the

QDT, that the time dependence of the moments of the intensity is given through a scaling variable. This result generalizes the dynamical scaling obtained in the instantaneous-change case [19, 21]. We also obtain analytical expressions for t_m and σ_m, including corrections to the instantaneous-change case as well as for the mean value and the variance of T. We check these predictions with numerical simulations.

The outline of the paper is as follows. In Sec. II the model and QDT are presented. In Secs. III and IV the validity conditions of QDT are obtained and the analysis of anomalous fluctuations is performed for cases I and II, respectively. Finally, in Sec. V we summarize the most important results and draw conclusions.

II. MODEL AND QUASIDETERMINISTIC THEORY (QDT)

The single-mode on-resonance laser can be described near threshold and in the good-cavity limit by the following equation:

$$\dot{E} = aE - B \mid E \mid^2 E + \xi(t), \tag{2.1}$$

where $E = E_1 + iE_2$ is the electric-field complex amplitude, $a = \Gamma - \kappa$ (Γ and κ are the gain and loss parameters, respectively), and $B = \beta\Gamma$ (β is a positive parameter, which involves the coupling constant and the polarization and population-inversion decay rates). The complex random term $\xi = \xi_1 + i\xi_2$ models the spontaneous-emission noise. It is taken as a Gaussian white noise of zero mean and correlation

$$\langle \xi^*(t)\xi(t')\rangle = 2D\delta(t-t'). \tag{2.2}$$

We will study a laser that is continuously driven from below to above threshold at a sweeping rate v, by changing the loss parameter in the following way: $\kappa(t) = \kappa_0 - vt$, with a fixed gain parameter. This corresponds to the experimental setup of Ref. [14]. Therefore the control parameter is $a(t) = a_0 + vt$ ($a_0 < 0$, $v > 0$) where $a_0 = \Gamma - \kappa_0$. The time at which $a(t)$ changes sign is denoted by $\bar{t}$, ($\bar{t} = -a_0/v$). The static bifurcation is then reached at $\bar{t}$. We will distinguish two cases. In the first one (case I) the control parameter is changed without bound,

$$a(t) = \begin{cases} a_0 & \text{if } t < 0 \\ a_0 + vt & \text{if } t > 0. \end{cases} \tag{2.3}$$

In the second case (case II) $a(t)$ reaches a fixed value a at $t_1 > \bar{t}$,

$$a(t) = \begin{cases} a_0 & \text{if } t < 0 \\ a_0 + vt & \text{if } 0 < t < t_1 \\ a = a_0 + vt_1 & \text{if } t > t_1. \end{cases} \tag{2.4}$$

The first case is of interest when the stationary-state value is obtained as a function of the control parameter. The second case is of interest in the study of fast variations of the control parameter when switching on the laser.

From a deterministic analysis, a time t^* at which the system becomes dynamically unstable can be defined as the time at which the solution of the linearized deterministic equation starts to grow exponentially [1],

$$\int_0^{t^*} a(s)ds = 0. \tag{2.5}$$

It can be shown that in cases I and II, if $t_1 > 2\bar{t}$, $t^* = 2\bar{t}$, and in case II, if $t_1 < 2\bar{t}$, $t^* = (a - a_0)^2/2av$. In this deterministic framework $t^* - \bar{t}$ is the delay in the bifurcation. In a stochastic description the dynamical bifurcation point has been characterized in terms of first-passage times [10] and using the time dependence of the mean intensity [11]. Numerical integration of the Fokker-Planck equation has shown that for a linear variation in time of the gain parameter, the presence of the spontaneous-emission white noise decreases the delay with respect to the deterministic value [8]. In fact, this delay disappears when $v \sim D$. We have checked with numerical simulations [23] of Eq. (2.1) that this result also holds in the case of swept losses [22].

The stochastic analysis of the dynamical bifurcation point only involves the linear regime. In order to analyze the anomalous fluctuations, nonlinear terms must be taken into account. To perform this analysis we generalize the QDT [21] to the case of a linear variation of the cavity losses [22].

Let us assume that the initial electric field $E_i(0)$ is distributed according to the Gaussian distribution below threshold. Therefore the initial intensity has an exponential distribution with mean value, $\langle I(0)\rangle = D/\mid a_0 \mid$. This is valid when nonlinear terms are negligeable, i.e., $\langle I(0)\rangle \ll \mid a_0 \mid /B$. The linearized version of (2.1) can be solved with this initial condition. The solution for the intensity is

$$I(t) = \mid h(t) \mid^2 e^{2\int_0^t a(s)\,ds}, \tag{2.6}$$

where $h(t)$ is a complex Gaussian process with variance [10]:

$$\langle \mid h(t) \mid^2 \rangle = \frac{D}{\mid a_0 \mid} + 2D \int_0^t e^{-2\int_0^s a(s')\,ds'} ds. \tag{2.7}$$

In the linear solution $\mid h(t) \mid^2$ plays the role of an effective random initial condition for the deterministic evolution. The QDT consists in replacing the actual process (2.1) by a process obtained from the nonlinear deterministic solution of (2.1), changing the initial condition by $\mid h(t) \mid^2$,

$$I(t) = \frac{\mid h(t) \mid^2 e^{2\int_0^t a(s)\,ds}}{1 + 2B \mid h(t) \mid^2 \int_0^t e^{2\int_0^s a(s')\,ds'} ds}. \tag{2.8}$$

This approximation is valid whenever two different stages of evolution can be distinguished: an initial linear fluctuating regime and a nonlinear regime where the evolution is essentially deterministic. This corresponds to the existence of a time t_0 in the linear regime, such that for times larger than t_0 the process $\mid h(t) \mid^2$ becomes a random independent variable $\mid h(\infty) \mid^2$. For these times the evolution will be deterministic. In the following sections we use this criterion to obtain the regions of validity of the QDT.

III. ANOMALOUS FLUCTUATIONS: CASE I

In this section we consider case I [see (2.3)]. We begin our analysis of the validity conditions of QDT for this case by calculating the average of the process $|h(t)|^2$. If we substitute (2.3) in (2.7) we obtain

$$\langle |h(t)|^2\rangle = \frac{D}{|a_0|}\left(1+\frac{2|a_0|}{\sqrt{v}}e^{a_0^2/v}\int_{-|a_0|/\sqrt{v}}^{\sqrt{v}(t-\bar{t})} e^{-s^2}\,ds\right). \tag{3.1}$$

This stochastic process becomes a time-independent random variable $|h(\infty)|^2$ when $\sqrt{v}(t-\bar{t})>2$. In this time regime the QDT expression for the intensity (2.8) can be written as

$$I(t)=\frac{|h(\infty)|^2\,e^{v(t-\bar{t})^2-v\bar{t}^2}}{1+2B\,|h(\infty)|^2\int_0^t e^{v(s-\bar{t})^2-v\bar{t}^2}\,ds}. \tag{3.2}$$

We can also deduce (3.2) if there is a matching time t_0 ($t_0>\bar{t}$), such that the nonlinear terms are negligible for times smaller than t_0 and $\sqrt{v}(t_0-\bar{t})>2$. If this last relation holds, $|h(t_0)|^2=|h(\infty)|^2$ and the evolution for $t>t_0$ is then deterministic. The evolution before t_0 is well approximated by the linearized equation (2.6) if the following condition is satisfied:

$$\langle I(t_0)\rangle \ll (a_0+vt_0)/B, \tag{3.3}$$

where $I(t_0)$ is given by

$$I(t_0)=|h(t_0)|^2\,e^{v(t_0-\bar{t})^2-v\bar{t}^2}, \tag{3.4}$$

and the second term is the stationary intensity at t_0. The intensity for times greater than t_0 is the solution of the nonlinear deterministic equation with the random initial condition $I(t_0)$:

$$I(t)=\frac{I(t_0)e^{2\int_{t_0}^{t}(a_0+vs)ds}}{1+2BI(t_0)\int_{t_0}^{t}e^{2\int_{t_0}^{s}(a_0+vs')ds'}ds}. \tag{3.5}$$

If we substitute $I(t_0)$ given by (3.4) in (3.5) we obtain the following expression for the intensity:

$$I(t)=\frac{|h(t_0)|^2\,e^{v(t-\bar{t})^2-v\bar{t}^2}}{1+2B\,|h(t_0)|^2\left(\int_0^t e^{v(s-\bar{t})^2-v\bar{t}^2}ds-\int_0^{t_0}e^{v(s-\bar{t})^2-v\bar{t}^2}ds\right)}. \tag{3.6}$$

We can consider two cases $\sqrt{v}>|a_0|$ (fast sweeping) and $\sqrt{v}\ll|a_0|$ (slow sweeping). If $\sqrt{v}>|a_0|$ the condition (3.3) is equivalent to

$$\frac{BD}{|a_0|\sqrt{v}}\ll x_0\,e^{-x_0^2}, \tag{3.7}$$

where $x_0=\sqrt{v}(t_0-\bar{t})$. If this last condition holds, it can be shown that

$$2B\langle |h(t_0)|^2\rangle\int_0^{t_0}\exp[v(s-\bar{t})^2-v\bar{t}^2]ds\ll 1. \tag{3.8}$$

Therefore if (3.7) holds with $x_0>2$, we obtain the QDT expression (3.2) from (3.6). The validity condition of QDT in the case of fast sweeping is then given by

$$\frac{BD}{|a_0|\sqrt{v}}\ll 10^{-2}\quad\text{if } \sqrt{v}>|a_0|\,. \tag{3.9}$$

We now consider the case of slow sweeping ($\sqrt{v}\ll|a_0|$). For the linear description to be valid up to a time $t_0>\bar{t}$, the average of the second term in the denominator of (2.8) has to be very small,

$$2B\langle |h(\bar{t})|^2\rangle\int_0^{\bar{t}}e^{2\int_0^s(a_0+vs')ds'}ds\ll 1. \tag{3.10}$$

After some calculations we then obtain a necessary condition for the validity of the QDT in the case of slow sweeping:

$$\frac{BD}{|a_0|\sqrt{v}}\ll e^{-a_0^2/v}\quad\text{if } \sqrt{v}\ll|a_0|\,. \tag{3.11}$$

These conditions agree when $\sqrt{v}>|a_0|$ with the ones [18] found for the validity of the Suzuki matching procedure [19]. However, when $\sqrt{v}\ll|a_0|$ we get a more stringent condition. The difference between both cases is that in the first case $\langle I(t)\rangle$ is for times $t\approx\bar{t}$ of the same order as $\langle I(0)\rangle$, whereas in the second case $\langle I(\bar{t})\rangle\gg\langle I(0)\rangle$. Therefore, for a linear description to be valid up to the matching time $t_0>\bar{t}$, the sweeping across the bifurcation must be faster in the second case.

We have checked with numerical simulations that if the condition (3.9) is satisfied in the case $\sqrt{v}>|a_0|$, the QDT describes correctly the anomalous fluctuations, as can be seen in Fig. 1. We also give an example in Fig. 2 in which the condition (3.9) is not verified. In this last figure we see how the QDT description of the average and variance fails to describe the results of the numerical simulations of Eq. (2.1).

We now study the transient anomalous fluctuations of the intensity using the QDT. It is well known that in the decay of an unstable state (instantaneous change of the pump parameter), fluctuations, $\sigma^2=\langle I^2\rangle-\langle I\rangle^2$, have an anomalously large maximum [18–20]. However, when $v\approx D$ the maximum disappears, as can be seen in Fig. 3. In this situation the intensity follows adiabatically the steady-state value associated with the instantaneous control parameter [8]. We will characterize anomalous fluctuations by the time at which the maximum takes

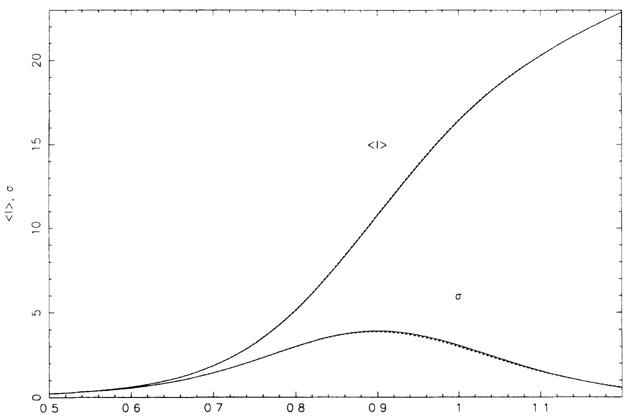

FIG. 1. Average intensity and fluctuations vs time for $B = 1/2$, $D = 10^{-3}$, $a_0 = -0.05$, and v=10 calculated from simulation (solid line) and from QDT (dotted line).

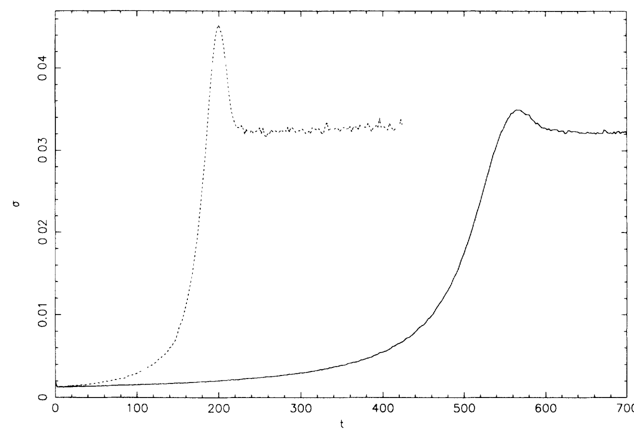

FIG. 3. Fluctuations vs time for $B = 1/2$, $D = 10^{-3}$, $a_0 = -0.5$, and $v = 0.003$ (dash-dotted line) and 0.001 (solid line). The anomalous fluctuations peak disappears if $v \to D$.

place, t_m, and by the value of this maximum σ_m. We also consider the time T at which the intensity reaches its maximum rate of growth. This time has been measured for an Ar^+ laser with swept losses [14].

We have performed simulations of (2.1) for $B = \frac{1}{2}$, D=0.001, $a_0 = -0.5$, and $a_0 = -0.05$, and for several values of the sweeping rate. In Fig. 4, $\log_{10}\langle T\rangle$ and $\log_{10} t_m$ are represented versus the logarithm of the sweeping rate for $a_0 = -0.5$. In the figure we see that t_m coincides with the average of T for a large range of values of v. This last fact can be explained in the framework of the QDT by considering that the largest amplification of the initial fluctuations occurs at the time of maximum rate of growth of the intensity, T. This time T is well defined (relative fluctuations smaller than 0.06) when $v > 1$ for $a_0 = -0.5$. This corresponds to the region of validity of QDT. We also observe in Fig. 4 that for these values of v, ($| a_0 | < \sqrt{v}$), the scaling $\sqrt{v}\langle T\rangle \approx$ const is verified. This scaling is in agreement with experimental data [14] and with deterministic analysis for large enough v [4, 5]. An explanation in the framework of QDT for both the scaling $\sqrt{v}\,\langle T\rangle \approx$ const and for the small relative fluctuations $\sigma_T/\langle T\rangle$ (where $\sigma_T^2 = \langle T^2\rangle - \langle T\rangle^2$) can be given in the following way. An equation for T can be obtained from (3.2) by setting $d^2I/dt^2|_{t=T} = 0$:

$$\frac{e^{X^2}}{A + \int_0^X e^{u^2} du} = \frac{X}{2}(3 - \sqrt{1 - 4/X^2}), \qquad (3.12)$$

where

$$A = \frac{\sqrt{v} e^{a_0^2/v}}{2B \mid h(\infty) \mid^2} + \int_0^{|a_0|/\sqrt{v}} e^{u^2} du, \qquad (3.13)$$

and $X = \sqrt{v}(T - \bar{t})$. This condition is similar to the one found in the deterministic analysis of Ref. [4]. The difference lies in the random character of the initial condition $| h(\infty) |^2$. The condition (3.12) is an implicit equation whose solution only depends on the random variable A.

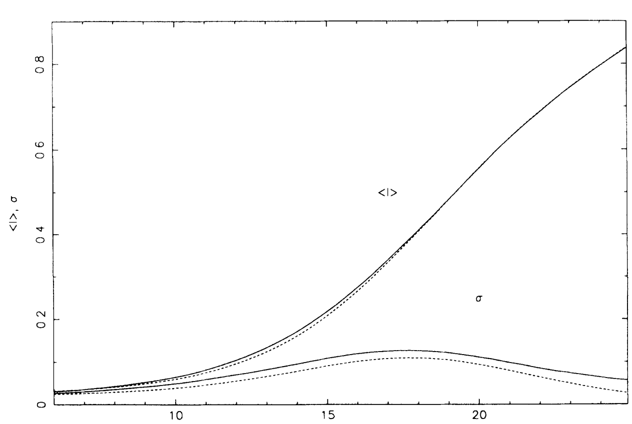

FIG. 2. Average intensity and fluctuations vs time for $B = 1/2$, $D = 10^{-3}$, $a_0 = -0.05$, and v=0.02 calculated from simulation (solid line) and from QDT (dotted line).

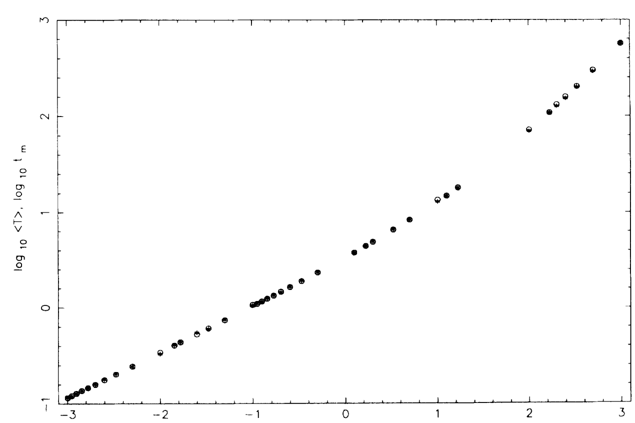

FIG. 4. $\log_{10}\langle T\rangle$ (circles), and $\log_{10} t_m$ (stars) vs $\log_{10} v$. If $\sqrt{v} >| a_0 |$ the points describe a straight line with a slope 0.499.

It can be shown by deriving (3.12) with respect to A that the variation of the root X is $\Delta X \approx e^{-X^2}\Delta A$. When $\sqrt{v} >| a_0 |$, it is easy to see from (3.1) that large variations of v change slightly the distribution of $| h(\infty) |^2$. Then the changes in A are essentially due to the $\sqrt{v}$ factor. Due to the e^{-X^2} factor there is a large range of sweeping rates such that $\Delta X \ll 1$ and then $X \approx$ const (note that in the nonlinear regime $X > 2$). Then, in this case ($\sqrt{v} >| a_0 |$) we get $\sqrt{v}\langle T\rangle \approx$ const with small fluctuations for T.

Therefore, we have shown that the $T = O(v^{-1/2})$ law is correct, provided that $\sqrt{v} >| a_0 |$. This result agrees with the deterministic analysis [5]. If $\sqrt{v} \ll| a_0 |$ the deterministic result [4, 5] is different: $T = O(v^{-1})$. The same conclusion can be obtained, including fluctuations, when the QDT is valid, that is, for small enough values of the intensity of the noise with respect to the sweeping rate [see (3.11)]. In the limit $v \to 0$ we get from (3.12) and (3.13) the following expression for T:

$$T \approx 2\frac{| a_0 |}{v}\left[1 + \frac{1}{4}\frac{v}{a_0^2}\ln\left(1 + \frac{| a_0 |}{B\,| h(\infty) |^2}\right)\right]. \tag{3.14}$$

This result is in agreement with the deterministic analysis [4]: the domain where $T \approx t^* = 2 | a_0 | /v$ increases when $| a_0 |$ increases or when the initial condition increases.

The expression for σ_m can be found from (3.2) by averaging over the initial condition $| h(\infty) |^2$

$$\frac{\sigma_m}{\sqrt{v}} = \frac{\beta_m\left(\frac{1}{\beta_m} - e^{\beta_m}E_1(\beta_m) - e^{2\beta_m}E_1(\beta_m)^2\right)^{1/2}}{2Be^{-x_m^2}\int_{-|a_0|/\sqrt{v}}^{x_m} e^{u^2}du}, \tag{3.15}$$

where β_m is the value of the following function:

$$\beta = \frac{\sqrt{v}e^{a_0^2/v}}{2B\langle| h(\infty) |^2\rangle\int_{-|a_0|/\sqrt{v}}^{x} e^{u^2}du} \tag{3.16}$$

at t_m, $x_m = \sqrt{v}(t_m - \bar{t})$, and $E_1(\beta)$ is the exponential-integral function [25]. As can be seen from Fig. 5, where $\log_{10}\sigma_m$ is plotted vs $\log_{10}v$, $\sigma_m/\sqrt{v} \approx$ const when $\sqrt{v} >| a_0 |$. We can understand qualitatively this scaling in the following way. First, the denominator in (3.15) does not change with v, due to the scaling $x_m \approx \langle X\rangle \approx$ const. Second, the numerator is a function that changes slowly for the values of β_m found in the simulations. For instance, β_m varies between 0.17 and 0.22 when v varies between v=6 and 600 for the values of the parameters $a_0 = -0.5$, $D = 10^{-3}$, and B=0.5. This variation of β_m produces a change in the numerator of (3.15) smaller than 3%. Note that if we take into account that X has small fluctuations and $\langle X\rangle \approx x_m$, we can write (3.12) in the form

$$\frac{1}{(1+\beta_m)e^{-X^2}\int_{-|a_0|/\sqrt{v}}^{X} e^{u^2}du} = \frac{X}{2}(3 - \sqrt{1 - 4/X^2}). \tag{3.17}$$

Therefore, for the scaling $X \approx$ const to be correct $(1+\beta_m)$ should vary slowly with v.

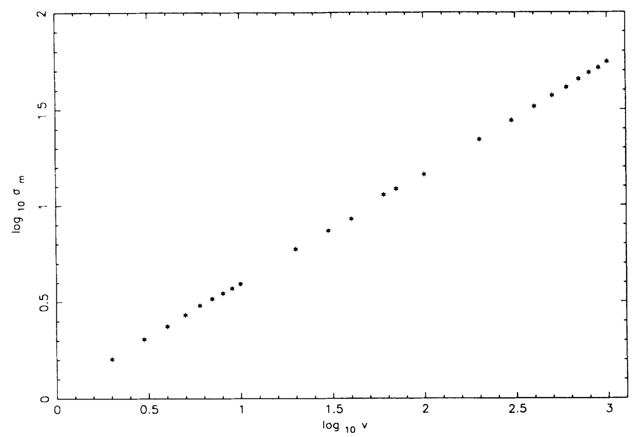

FIG. 5. Maximum fluctuations vs $\log_{10} v$ for $B = 1/2$, $D = 10^{-3}$, and $a_0 = -0.5$. When $\sqrt{v} >| a_0 |$ the points describe a straight line with a slope 0.567.

The relative fluctuations for all times are given from (3.2) by the following universal scaling function:

$$\frac{\sigma}{\langle I\rangle} = \frac{\beta\left(\frac{1}{\beta} - e^{\beta}E_1(\beta) - e^{2\beta}E_1^2(\beta)\right)^{1/2}}{1 - \beta e^{\beta}E_1(\beta)}. \tag{3.18}$$

In Fig. 6 we show that by increasing v, this scaling function is approached. This corresponds to the fact that the accuracy of the QDT approximation increases with v [see (3.9)]. This scaling has a different nature than the one found in the instantaneous-change case [18, 19] for the time dependence of $\langle I^n(t)\rangle$. Note that due to the fact that $a(t)$ is always changing with time, (2.3), the moments of the intensity [see, for instance, (3.15)] do not scale directly with $\beta(t)$. In case II we find a generalization of the dynamical scaling for $\langle I^n(t)\rangle$.

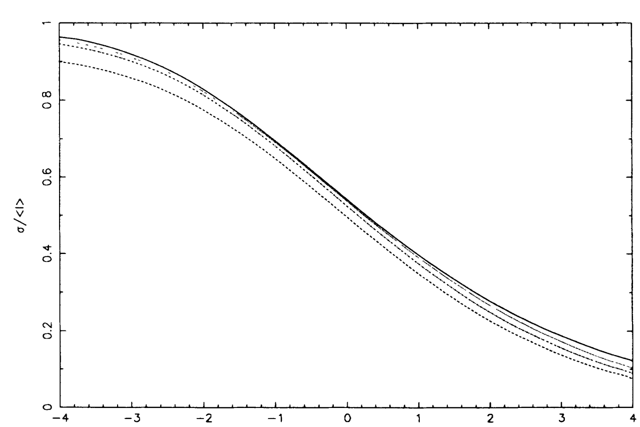

FIG. 6. Relative fluctuations vs $\ln\beta^{-1}$ for the scaling function (solid line) and $B = 1/2$, $D = 10^{-3}$, $a_0 = -0.5$, and $v =$ 5 (dashed line), 50 (dash-dotted line), and 500 (dotted line).

IV. ANOMALOUS FLUCTUATIONS: CASE II

In this case the control parameter reaches a fixed value a in t_1 ($t_1 > \bar{t}$) [see (2.4)]. This is an interesting case because it permits us to calculate corrections, owing to the finite sweeping rate, to the instantaneous change assumption made in classical studies of transient statistics [18, 19].

We can distinguish two cases: In the first one, $a \gg \sqrt{v}$, the anomalous fluctuations peak occurs before t_1. This corresponds to case I (slow sweeping). In the second case, $a < \sqrt{v}$, the times of interest are larger than t_1. This is an interesting case because the instantaneous change case is included here. As in the preceding section we begin our analysis of the validity conditions of QDT in this case by calculating the average of the process $| h(t) |^2$ from (2.4) and (2.7)

$$\langle | h(t) |^2 \rangle = \left(\frac{D}{| a_0 |} + \frac{2D}{\sqrt{v}} e^{v\bar{t}^2} \int_{-\sqrt{v}\,\bar{t}}^{\sqrt{v}(t_1-\bar{t})} e^{-u^2} du \right) + \frac{D}{a} e^{v t_1^2} (e^{-2at_1} - e^{-2at}), \tag{4.1}$$

valid for times $t > t_1$. This stochastic process becomes a time-independent random variable $|h(\infty)|^2$ when $2at \gg 1$. In this time regime the QDT expression for the intensity (2.4) can be written as

$$I(t) = \frac{| h(\infty) |^2 \exp(-vt_1^2 + 2at)}{1 + 2B \, | h(\infty) |^2 \left(\frac{e^{-v\bar{t}^2}}{\sqrt{v}} \int_{-\sqrt{v}\,\bar{t}}^{\sqrt{v}(t_1-\bar{t})} e^{u^2} du + \frac{e^{-vt_1^2}}{2a} (e^{2at} - e^{2at_1}) \right)}. \tag{4.2}$$

In a way similar to case I, the QDT will be valid when there exists a time t_0 in the linear regime such that $at_0 \gg 1$ and $\langle I(t_0) \rangle \ll a/B$. When $a \gg | a_0 |$ this is equivalent to the following condition:

$$\frac{BD}{| a_0 | \, a} \ll 10^{-2}. \tag{4.3}$$

Numerical simulations show that when (4.3) is fulfilled the approximation given by (4.2) is accurate. Recently another approximation based on a step function for $I(t)$ has been considered [12]. This kind of approximation gives very good results in the relaxation from a marginal state [24]. However, Eq. (4.2) shows that the step function approximation cannot be correct in the relaxation from an unstable state.

If we consider a range of parameters such that (4.3) is fulfilled, an analysis of the anomalous fluctuations can be performed by using the QDT. The advantage of this case with respect to case I is that we are able to get analytical expressions of t_m and $\langle T \rangle$ in which the corrections to the instantaneous change case due to the finite sweeping rate are included. The moments of the intensity can be calculated by averaging (4.2) over the distribution of $| h(\infty) |^2$. They show a temporal dependence given by a dynamical scaling parameter θ:

$$\theta = \frac{1}{2B \langle | h(\infty) |^2 \rangle \left(\alpha + \frac{\gamma}{2} (e^{2a(t-t_1)} - 1) \right)}, \tag{4.4}$$

where

$$\gamma = \frac{e^{(a^2 - a_0^2)/v}}{a}, \tag{4.5}$$

$$\alpha = \frac{e^{-a_0^2/v}}{\sqrt{v}} \int_{-|a_0|/\sqrt{v}}^{a/\sqrt{v}} e^{u^2} du. \tag{4.6}$$

This generalizes the dynamical scaling found in the instantaneous-change case [18, 19]. Unlike case I, this scaling holds directly for $\langle I^n(t) \rangle$. This is due to the fact that for the times of interest ($t > t_1$) the pump parameter is constant. We can now write the averaged intensity and the variance as functions of θ:

$$\langle I(\theta) \rangle = \frac{a}{B} (1 + C\theta) [1 - \theta e^{\theta} E_1(\theta)], \tag{4.7}$$

$$\sigma(\theta) = \frac{a}{B} (1 + C\theta) \theta [1/\theta - e^{\theta} E_1(\theta) - e^{2\theta} E_1^2(\theta)]^{1/2}, \tag{4.8}$$

where

$$C = B \langle | h(\infty) |^2 \rangle (\gamma - 2\alpha). \tag{4.9}$$

The moments depend on θ and C in the same way as in the instantaneous-change case, but changing C and the scaling parameter θ. The corresponding parameters θ_∞ and C_∞ of the instantaneous-change case are

$$\theta_\infty = \frac{a}{BD \left(\frac{1}{a} + \frac{1}{| a_0 |} \right) (e^{2at} - 1)}, \tag{4.10}$$

$$C_\infty = \frac{BD}{a} \left(\frac{1}{a} + \frac{1}{| a_0 |} \right). \tag{4.11}$$

When $v \gg a^2$ we obtain that θ and C coincide with the corresponding ones (θ_∞ and C_∞) of the Q-switching.

Now we will characterize the anomalous fluctuations by t_m and σ_m, as in the previous case. The value of θ at t_m can be calculated from the condition $d\sigma/d\theta|_{\theta=\theta(t_m)} = 0$. This condition is an implicit equation whose root depends very slightly on C when (4.3) holds, because $C \approx BD/a \, | a_0 |$ is very small. The root of the equation is $\theta(t_m)$=0.4188. Now t_m can be found from (4.4)

$$t_m = \frac{1}{2a} \ln \left(\frac{1}{B\theta(t_m)\gamma \langle | h(\infty) |^2 \rangle} - \frac{2\alpha}{\gamma} + 1 \right) + t_1. \tag{4.12}$$

We now calculate T by setting $d^2I/dt^2|_{t=T} = 0$, where $I(t)$ is given by (4.2)

$$T = \frac{1}{2a}\left[\ln\frac{1}{B\gamma\langle | h(\infty) |^2\rangle} + \ln\left(\frac{\langle | h(\infty) |^2\rangle}{| h(\infty) |^2} - C\right)\right] + t_1. \tag{4.13}$$

The average and variance of T are easily calculated by averaging over the exponential distribution of $| h(\infty) |^2$, and they read

$$\langle T\rangle = \frac{1}{2a}\left(\ln\frac{1}{B\gamma\langle | h(\infty) |^2\rangle} + \gamma_{\rm Euler}\right) + t_1, \tag{4.14}$$

$$\sigma_T = \frac{0.6413}{a}, \tag{4.15}$$

where $\gamma_{\rm Euler}$=0.5772. In the case $a \gg | a_0 |$, simple expressions of t_m and $\langle T\rangle$ with corrections of the order (a/v) to the instantaneous change case can be calculated:

$$t_m \approx \frac{1}{2a}\left(\ln\frac{a\,| a_0 |}{BD} + 0.8704\right) + \frac{a}{2v}, \tag{4.16}$$

$$\langle T\rangle \approx \frac{1}{2a}\left(\ln\frac{a\,| a_0 |}{BD} + 0.5772\right) + \frac{a}{2v}. \tag{4.17}$$

The main contribution to t_m and $\langle T\rangle$ comes from the logarithmic term, because if QDT is valid then this term dominates, as can be seen from (4.3). Due to the same main dependence of t_m and $\langle T\rangle$ we obtain, as in case I, that the time at which the maximum appears is nearly the average of T. In fact, t_m is always slightly greater than $\langle T\rangle$. The fact that t_m and $\langle T\rangle$ are nearly equal can be inferred from the analysis made in the instantaneous-change case [19–21]. In this paper we show this equivalence in an analytic way. In Fig. 7 the theoretical expressions show a good agreement with the results of the simulations, when the condition $a < \sqrt{v}$ is fulfilled. We also see in this figure that the relation $t_m > \langle T\rangle$ is verified according to Eqs. (4.16) and (4.17). Another interesting fact is that the variance of T only depends on the final value of the control parameter. The comparison between the results of the simulations for σ_T and those from Eq. (4.15) can be made showing a relative error always lower than 2% for the parameters of Fig. 5.

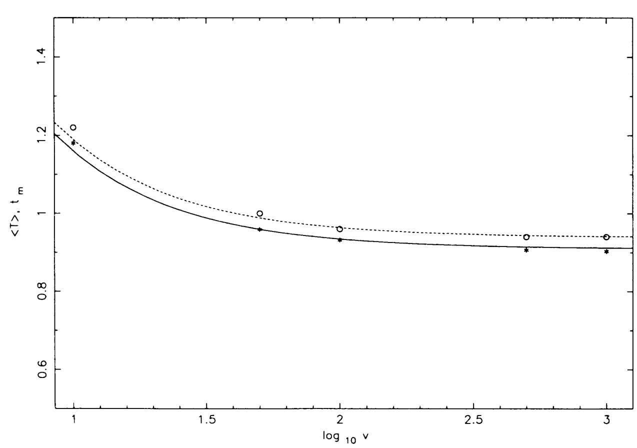

FIG. 7. $\langle T\rangle$ and t_m vs $\log_{10} v$ for $B = 1/2$, $D = 10^{-3}$, $a = 5$, and $a_0 = -0.5$. Circles ($\langle T\rangle$) and stars (t_m) are the results of the simulations and the lines are calculated from (4.16) and (4.17).

Finally, the value of σ_m can be easily calculated from (4.8) for $\beta = 0.4188$; therefore σ_m is proportional to the stationary value of the intensity:

$$\sigma_m = 0.2381\frac{a}{B}. \tag{4.18}$$

This expression has been also checked with numerical simulations.

V. CONCLUSIONS

In this paper the quasideterministic theory is generalized to explain the statistical properties of the intensity of a good-cavity laser when the losses are varied linearly in time. This generalization has been performed in two cases, I and II. In the first case the control parameter changes without bound and in the second one it reaches a fixed value. We have shown that QDT is valid when the spontaneous-emission noise is much smaller than the sweeping rate, in case I, and much smaller than the final control parameter, in case II. We have checked the validity conditions of QDT using numerical simulations.

The QDT has also been used [26, 27] to study the statistical properties of the transient response of a gain-switched semiconductor laser. When saturation effects are negligible, this problem is equivalent to a type-A laser with swept losses. The intensity fluctuations σ also show, for semiconductor lasers, large transient anomalous fluctuations associated with the decay of an unstable state as in lasers of type A. However, due to the relaxation oscillations that characterize the nonlinear regime of lasers of type B, these fluctuations follow the relaxation oscillations. Another difference is the existence of a local minimum in σ at the time that the mean intensity reaches a maximum in the oscillation.

The QDT is used to analyze the anomalous fluctuations of the intensity appearing in the nonlinear regime of type-A lasers with swept losses. To characterize these anomalous fluctuations, we have studied the time t_m at which the maximum fluctuations appear and the value of this maximum σ_m. t_m is found to coincide with the average of the time T at which the intensity has its maximum slope.

In case I, when the sweeping rate is large enough with respect to the initial value of the parameter, we have found that t_m and σ_m scale with the sweeping rate in this way: $t_m^{-1} \approx \sqrt{v}$ and $\sigma_m \approx \sqrt{v}$. We have also shown that the dependence on time of the relative fluctuations is given through a scaling variable In case II, when the sweeping rate is large enough with respect to the final value of the pump parameter, we have seen that the moments of the intensity depend on time through a scaling variable that generalizes the dynamical scaling found in the instantaneous-change case. Due to the fact that the control parameter is constant at the times of interest we have been able to calculate t_m and $\langle T\rangle$, showing in an analytic way that $t_m \approx \langle T\rangle$. To finish the characterization of the anomalous fluctuations we have also calculated σ_m, which is proportional to the stationary intensity. All these facts, predicted by the quasideterministic theory, have been checked with numerical simulations.

ACKNOWLEDGMENT

Financial support from DGICYT (Spain) Project No. PB 87-0014 is acknowledged.

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