Intrinsic optical bistability with squeezed vacuum

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Intrinsic optical bistability due to dipole-dipole interactions among interacting two-level atoms shows strong phase dependence in the presence of a squeezed vacuum. We show that by varying the relative phase between the squeezed vacuum and the incident field, it is possible to excite the system of two-level atoms from the lower to upper branch of the bistable curve. On this basis, an optical switch that utilizes this phase dependence for intrinsic optical bistability is proposed.

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I. INTRODUCTION

The squeezed vacuum (SV) has been shown to have remarkable effects on several phenomena involving two-level atoms. Gardiner [1] has shown that the two quadratures of the atomic dipole decay at unequal rates in the presence of the SV. Carmichael, Lane, and Walls [2] studied the fluorescence spectrum from a system of two-level atoms damped by a SV. Their results show remarkable differences from the spectrum for ordinary resonance fluorescence. They also demonstrated that for strong coherent fields, the Mollow triplet depends on the relative phase between the driving field and the SV. The mean atomic polarization was also shown to be phase sensitive. Recently, Shevy [3] has applied the concept of SV to the phenomenon of laser cooling of atoms. The effects of SV on cavity optical bistability has also been studied recently by Galatola *et al.* [4]. They investigated bistability with a cavity as described by the model of Lugiato and coworkers [5,6]. These and other works involving SV have exploited the distinctive feature of squeezed states in their phase sensitivity. In general, most nonlinear phenomena should exhibit remarkable differences in the presence of SV. The question that naturally arises is, what is the effect of SV on the intrinsic optical bistability, i.e., bistability without a cavity, or external feedback? In this work we report our results describing the intrinsic optical bistability (IOB) due to near-dipole-dipole (NDD) interactions in the presence of a squeezed vacuum. In Sec. II we extend the existing theory of Ben-Aryeh, Bowden, and Englund [7] to include the effects of squeezed-vacuum field. The equations of motion for the atomic polarizations are found to depend upon the phase of the squeezed vacuum. In Sec. III the steady-state equation for atomic excitation, the equation of state, which describes optical bistable effects, is derived. The critical points and the threshold condition for optical bistability are determined. The phase sensitivity of the IOB is explicitly demonstrated and discussed in detail. In Sec. IV we discuss the occurrence of IOB as a function of various system and input field parameters and verify the stability conditions. A type of effect in IOB is demonstrated which occurs entirely due to the variation of the relative phase between the input field and the SV, for a fixed input intensity. Finally, Sec. V serves as the summary of the results and conclusion.

II. FORMULATION

We consider a system consisting of a dense collection of two-level atoms (having many atoms within a cubic resonance wavelength) embedded in a squeezed vacuum. Such a scheme may be realized, for instance, by sandwiching a thin film of the atomic medium of thickness d between a parametric oscillator material, the dimensions of which are much larger compared to the former. Since optical bistability in the system is intrinsic, d can be much smaller than a resonance wavelength, $d \ll \lambda$. As is well known, if the input field is ordinary vacuum, the quantized field inside the parametric material is squeezed [8]. The system is also driven by an externally applied classical coherent field. This field is assumed to propagate in the z direction and is linearly polarized in the x direction. The Hamiltonian of the total system in the rotating-wave approximation is

$$\begin{aligned} H &= H_0 + H' , \\ H_0 &= \frac{\hbar}{2}\omega_a \sum_{i=1}^{N} \sigma_z^{(i)} + \hbar \int d\omega\, \omega a^\dagger(\omega) a(\omega) , \\ H' &= -i\hbar \sum_{i=1}^{N} \int d\omega\, g^{(i)}(\omega) a(\omega) \sigma_+^{(i)} e^{i\mathbf{k}\cdot\mathbf{r}_i} \\ &\quad - \frac{\hbar}{2}\Omega \sum_{i}^{N} \sigma_+^{(i)} e^{i(\mathbf{k}_L\cdot\mathbf{r}_i - \omega_L t)} + \text{H.c.} , \end{aligned} \tag{2.1}$$

where H_0 is the Hamiltonian for the free atoms and the free quantized radiation field. H' describes the interaction between the atoms, and the quantized and the externally applied radiation fields. H.c. denotes Hermetian conjugate, $a(\omega)$ is the annihilation operator for the field in mode k, with a frequency $\omega = ck$, $\sigma_z^{(i)}$ is the atomic population inversion operator, and $\sigma_\pm^{(i)}$ are the raising and lowering operators for an atom at a position $\mathbf{r}_i$ with a resonance frequency ω_a. $g^{(i)}(\omega)$ is the coupling between the atoms and the quantized radiation field. Ω is the

Rabi frequency associated with the external field that has frequency ω_L and a wave vector in the z direction, k_L. The quantized field is assumed to be in a broadband squeezed-vacuum state with a central frequency around the externally applied field frequency ω_L, with the following properties [1,2]

$$\langle a(\omega)\rangle=\langle a^\dagger(\omega)\rangle=0\ ,$$
$$\langle a^\dagger(\omega)a(\omega')\rangle=N(\omega)\delta(\omega-\omega')\ , \quad (2.2)$$
$$\langle a(\omega)a(\omega')\rangle=M(\omega)\delta(\omega+\omega'-2\omega_L)\ .$$

The Heisenberg equations of motion for the atomic variables are obtained in the bad cavity limit, by adiabatically eliminating the variables associated with quantized field modes [9]. Assuming the coupling $g^{(i)}(\omega)$ to be weak, an iterative solution in the powers of $g^{(i)}$ [10], together with Eq. (2.2) for the case of a thin film [7], yields the following equations of motion for the expectation values of the slowly varying atomic variables:

$$\langle\dot\sigma_-\rangle=-i(\Delta+\epsilon\langle\sigma_z\rangle)\langle\sigma_-\rangle - \left[\gamma_2(2N+1)-\frac{\langle\sigma_z\rangle}{\tau_R}\right]\langle\sigma_-\rangle -\gamma_1 M\langle\sigma_+\rangle-\frac{i}{2}\Omega\langle\sigma_z\rangle\ , \quad (2.3)$$

$$\langle\dot\sigma_z\rangle=-\gamma_1[(2N+1)\langle\sigma_z\rangle+1] +i\Omega\langle\sigma_+\rangle-i\Omega^*\langle\sigma_-\rangle-\frac{4}{\tau_R}|\langle\sigma_+\rangle|^2\ , \quad (2.4)$$

where [11]

$$\frac{1}{\tau_R}=\frac{2\pi u^2 n}{\hbar}kd \quad (2.5)$$

and

$$\epsilon=\frac{4\pi u^2 n}{3\hbar}\ . \quad (2.6)$$

Here n is the atomic density, γ_1 (γ_2) is the relaxation (dephasing) rate due to the unsqueezed vacuum, Δ ($=\omega_a-\omega_L$) is the detuning between the atomic resonance and the driving field, u is the absolute value of the transition dipole moment, N is proportional to the number of photons in the SV, and the complex parameter $M=|M|e^{i\phi_M}$ characterizes the degree of squeezing; due to the Heisenberg uncertainty one must have $|M|^2\le N(N+1)$ where the equality holds for minimum uncertainty squeezing [1,2]. The squeezing parameter M couples $\langle\sigma_+\rangle$ and $\langle\sigma_+\rangle^*=\langle\sigma_-\rangle$, and thus introduces the phase-dependent characteristics of the SV. For no squeezing, $N=M=0$, Eqs. (2.3) and (2.4) reduce to the usual optical Bloch equations, in the presence of a normal vacuum modified according to the NDD interactions in a thin film medium [7,11,12], characterized by the parameters [11] (2.5) and (2.6). Equation (2.3) indicates that for a large degree of squeezing, phase effects can become important, even if the coupling between the squeezed vacuum and the atoms is weak.

The $\langle\sigma_z\rangle$-dependent nonlinear terms in (2.3) are due to NDD interaction between the atoms, with coupling strengths given by Eqs. (2.5) and (2.6). The term proportional to ϵ in Eq. (2.3), leads to atomic excitation-dependent frequency shifts, and can cause intrinsic optical bistability in steady state [7] and dynamical frequency chirp for pulse excitation [12]. The terms in Eqs. (2.3) and (2.4) proportional to $1/\tau_R$, Eq. (2.5), give rise to superradiant decays [7,13]. Also note that the nonlinear frequency renormalization, proportional to ϵ, Eq. (2.6), is independent of the film thickness d and is proportional to the square of the matrix element of the transition dipole moment u and to the atomic density n, whereas the cooperative superradiant decay rate given by $1/\tau_R$, Eq. (2.5), depends upon the film thickness d, through the factor kd. For $kd\ll 1$, corresponding to $d\ll\lambda$, the terms in $1/\tau_R$ can be neglected; however, for $kd\approx 1$, they contribute to the same order as ϵ.

III. INTRINSIC OPTICAL BISTABILITY AND CRITICAL POINTS

The steady-state behavior of the system can be determined by setting the time derivatives to zero in Eqs. (2.3) and (2.4). The steady-state atomic polarization thus obtained is

$$\langle\sigma_-\rangle^{\rm SS}=\frac{i}{2}\langle\sigma_z\rangle^{\rm SS}\cdot\frac{\Omega\mathcal{D}-\gamma_1 M\Omega^*}{|\mathcal{D}|^2-\gamma_1^2|M|^2}\ , \quad (3.1a)$$

with

$$\mathcal{D}=i(\Delta+\epsilon\langle\sigma_z\rangle^{\rm SS})+\left[\frac{\langle\sigma_z\rangle^{\rm SS}}{\tau_R}-\gamma_2(2N+1)\right]\ , \quad (3.1b)$$

where the superscript SS indicate evaluation in steady state. The elimination of $\langle\sigma_-\rangle^{\rm SS}$ in Eq. (3.1), using (2.3) and (2.4), leads to the equation

$$[\gamma_1+\gamma_1(2N+1)\langle\sigma_z\rangle^{\rm SS}](|\mathcal{D}|^2-\gamma_1^2|M|^2)^2+|\Omega|^2\langle\sigma_z\rangle^{\rm SS}[\gamma_1|M|\cos\phi+\gamma_2(2N+1)](|\mathcal{D}|^2-\gamma_1^2|M|^2)$$
$$+\frac{|\Omega|^2}{\tau_R}(2\gamma_1|M|)\left[\gamma_1|M|+(\epsilon\langle\sigma_z\rangle^{\rm SS}+\Delta)\sin\phi+\left[\gamma_2(2N+1)-\frac{\langle\sigma_z\rangle^{\rm SS}}{\tau_R}\cos\phi\right]\right]=0\ , \quad (3.2)$$

which has the form of a fifth-order polynomial in the steady-state population excitation $\langle \sigma_z \rangle^{SS}$. This is the state equation describing the bistable behavior, as we shall show later, of the system in terms of the population excitation, the external field intensity, the SV parameters, and the relative phase ϕ, between the external field and the SV. For normal vacuum $N=|M|=0$, the last term in Eq. (3.2) vanishes and we obtain the result previously derived by Ben-Aryeh, Bowden, and Englund [7]. These authors have shown that for normal vacuum the state equation, Eq. (3.2), is a cubic in $\langle \sigma_z \rangle^{SS}$ and leads to intrinsic bistability effects as a function of the input field intensity $|\Omega|^2$. It should be noted that SV introduces relative phase dependence in the state equation, (3.2). By solving Eq. (3.2) numerically, we have verified that the fifth-order polynomial can very well be approximated by three real roots in certain parameter regions, causing the usual intrinsic optical bistability effect [7]. The numerical solution of the full state equation and the influence of squeezing on optical bistability are discussed in Sec. IV.

We now determine the critical points for $\langle \sigma_z \rangle^{SS}$ for the special case of a thin film such that $d \ll \lambda$, corresponding to $kd \ll 1$. This implies from Eqs. (2.5) and (2.6) that compared to ϵ, the effect of the terms proportional to $1/\tau_R$ can be neglected. The state equation (3.2) then reduces to the cubic form

$$[\gamma_1+\gamma_1(2N+1)\langle \sigma_z \rangle^{SS}]\left[(\Delta+\epsilon\langle \sigma_z \rangle^{SS})^2 + \left[\frac{\langle \sigma_z \rangle^{SS}}{\tau_R}-\gamma_2(2N+1)\right]^2-\gamma_1^2|M|^2\right] = -|\Omega|^2[\gamma_2(2N+1)+\gamma_1|M|\cos\phi]\langle \sigma_z \rangle^{SS} . \quad (3.3)$$

Thus we explicitly observe that a major effect of the SV is to replace the input intensity $|\Omega|^2$ by an effective intensity $I_{\text{eff}}=|\Omega|^2[\gamma_2(2N+1)+\gamma_1|M|\cos\phi]$, which depends on the relative phase ϕ between the externally applied field and the SV. For ordinary vacuum, the optical bistability effects are attained by varying the external field intensity $|\Omega|^2$. Interestingly, we can now observe optical-bistability effects by varying the input effective intensity in two ways: either by varying $|\Omega|^2$ for a fixed ϕ or by varying the relative phase ϕ for a fixed $|\Omega|^2$ while keeping the other parameters for the SV, N, and $|M|$, constant. The latter is an entirely new effect and is the main result of the present paper. The critical points for $\langle \sigma_z \rangle^{SS}$ as a function of either $|\Omega|^2$ or ϕ may be determined by taking the derivative of Eq. (3.3) with respect to $\langle \sigma_z \rangle^{SS}$ and setting $d|\Omega|^2/d\langle \sigma_z \rangle^{SS}=d\phi/d\langle \sigma_z \rangle^{SS}=0$. The resulting expression is

$$(\langle \sigma_z \rangle^{SS})^2+F\langle \sigma_z \rangle^{SS}+G=0 , \quad (3.4)$$

$$F=\frac{2[\epsilon+2\Delta(2N+1)]}{3\epsilon} , \quad (3.5)$$

$$G=\frac{\gamma_1(2N+1)[\Delta^2+\gamma_2^2(2N+1)^2-\gamma_1^2|M|^2]+2\gamma_1\epsilon\Delta+|\Omega|^2[\gamma_2(2N+1)+\gamma_1|M|\cos\phi]}{3\epsilon^2\gamma_1(2N+1)} \quad (3.6)$$

which must be satisfied simultaneously with Eq. (3.3). The roots of the above quadratic determine the turning points of the bistability curve. The threshold condition is obtained by setting the discriminant equal to zero, i.e.,

$$\Delta^2+\gamma_2^2(2N+1)^2-\gamma_1^2|M|^2+\frac{2\epsilon\Delta}{(2N+1)} +|\Omega|^2\left[\frac{\gamma_2}{\gamma_1}+\frac{|M|}{2N+1}\cos\phi\right] =\frac{\epsilon^2}{3}\left[\frac{\gamma_1^2}{\gamma_2^2(2N+1)^2}+\frac{4\Delta^2}{\epsilon^2}+\frac{4\Delta}{(2N+1)\epsilon}\right] . \quad (3.7)$$

Assuming $\epsilon \gg (2N+1)\gamma_2, |M|, \Delta$, we obtain the result

$$\frac{|\Omega|^2}{\epsilon}\frac{\gamma_2(2N+1)+\gamma_1|M|\cos\phi}{\gamma_1(2N+1)}=\frac{1}{3} \quad (3.8)$$

as the threshold condition. Hence, as the effective intensity I_{eff} is increased compared to the NDD coupling constant ϵ by either varying $|\Omega|^2$ or ϕ, we find that the two-level system switches from the low- to the high-transmission branch.

IV. NUMERICAL RESULTS FOR IOB

In this section we present and discuss numerical results for IOB [calculated from Eq. (3.2)] for various system parameters. The stability condition, Eq. (A7), is also verified numerically. In Fig. 1 we illustrate the optical bistable behavior of the steady-state inversion $\langle \sigma_z \rangle^{SS}$ as a function of the input intensity $|\Omega|^2$ [calculated using Eq. (3.2)]. Interestingly, the SV shifts the turning points to lower values. This is not surprising, as there are additional photons present in the SV resulting in a lower driving field intensity to switch the system from the lower to the upper branch, as well as nonvanishing excitation at zero input intensity. The effect of the coherence terms, terms proportional to $1/\tau_R$, on the IOB is depicted in Fig. 2. These terms are seen to produce only the small effect of slightly increasing the IOB thresholds, as compared to the results shown in Fig. 1. However, the most interesting aspect is the dependence of $\langle \sigma_z \rangle^{SS}$ on the relative phase ϕ, as discussed in the preceding section. Figure 3 exhibits the phase-induced IOB for a given input intensity $|\Omega|^2$. Finally, in Fig. 4 we show the changes in the bistable behavior of $\langle \sigma_z \rangle^{SS}$ as a function of $|\Omega|^2$ for

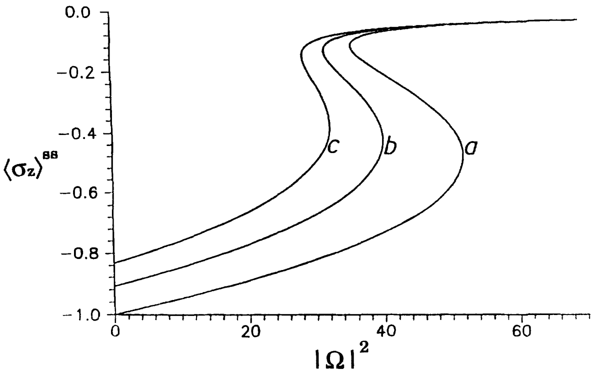

FIG. 1. Illustration of the optical bistable behavior of the inversion $\langle \sigma_z \rangle^{SS}$ as a function of the input intensity $|\Omega|^2$ for $\epsilon=10$, $1/\tau_R=\Delta=0$, relative phase $\phi=0$, $N=|M|$. Curve a shows the bistable behavior in normal vacuum. Curves b and c are the results for squeezed vacuum corresponding to $N=0.05$ and 0.1, respectively. All the parameters are scaled in terms of the homogeneous linewidth γ_2.

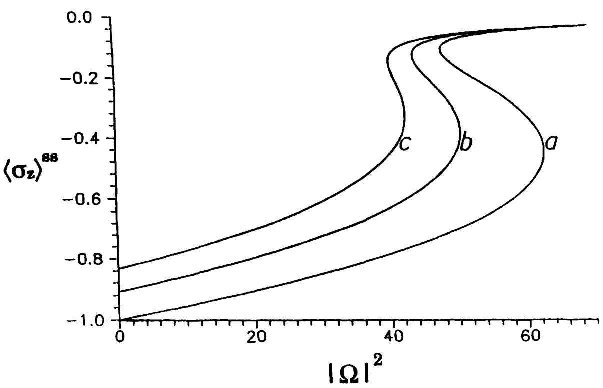

FIG. 2. Same as in Fig. 1 except that $1/\tau_R$ is nonzero and is equal to 3.

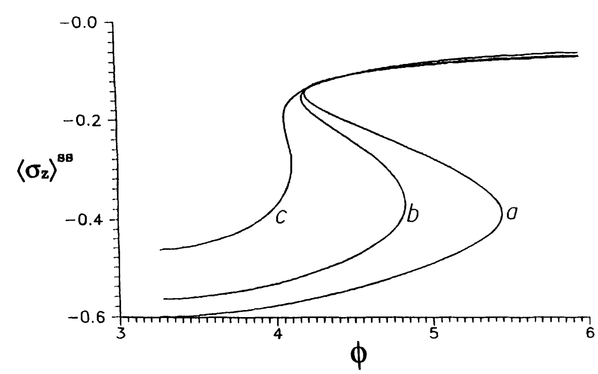

FIG. 3. Bistable behavior of $\langle \sigma_z \rangle^{SS}$ as a function of the relative phase ϕ (in radians) between the input field and the squeezed vacuum at fixed input field intensity $|\Omega|^2=32$ and for $\epsilon=9$, $1/\tau_R=\Delta=0$, and $N=|M|$. Curves a, b, and c correspond to the values of $N=0.065$, 0.085, and 0.15, respectively. All parameters are scaled in terms of the homogeneous linewidth γ_2.

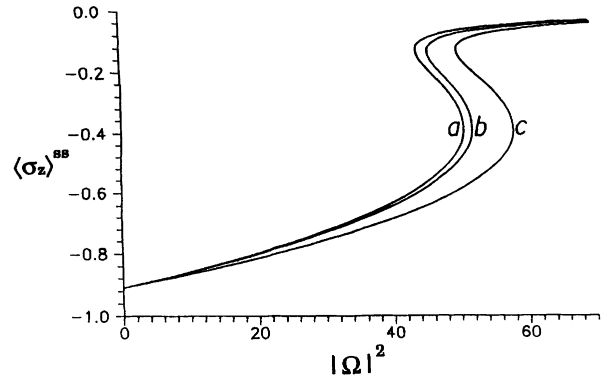

FIG. 4. Effect of the relative phase ϕ on the bistable behavior of $\langle \sigma_z \rangle^{SS}$ as a function of the input intensity $|\Omega|^2$ for $\epsilon=10$, $1/\tau_R=3$, $\Delta=0$, and $N=|M|=0.05$. Curves a, b, and c correspond to the values of $\phi=0$, $\pi/2$, and π, respectively. All parameters are scaled in terms of the homogeneous linewidth γ_2.

several different values of the relative phase. The stability conditions, Eq. (A7), verified numerically for each of the curves in the various figures, revealed, as usual, stability for the upper and lower branches, whereas the middle branch was found to be unstable. For nonzero detunings $\Delta\neq0$, the curves in Figs. 1 and 2 are simply shifted horizontally to the right or left, depending upon the sign of Δ. The threshold condition, Eq. (3.8) derived in Sec. IV, agrees reasonably well with that deduced from the numerical plots. Thus, with SV we have been able to demonstrate the existence of a type of bistable behavior that arises due to phase alone.

V. CONCLUSION

We have considered a collection of dense two-level atoms interacting with each other via the quantized radiation field which is initially in a squeezed-vacuum state. The system, in addition, is driven by an externally applied classical field. The dipole-dipole interactions in a dense atomic medium cause atomic-excitation-dependent renormalization of the atomic resonance frequency. Due to this renormalization, the optical Bloch equations become nonlinear in the atomic variables, and the steady-state equation for the atomic excitations turns into a fifth-order polynomial which can lead to optical bistability in dense two-level systems. For normal vacuum, the state equation for the atomic excitation is a cubic polynomial and depends only on the amplitude—and not on the phase—of the input field. Thus IOB is a phase-insensitive process and it has been shown that the quantum noise in the reaction field is above the standard quantum limit [14]. However, we have shown that in the presence of a squeezed vacuum, the phenomenon of IOB becomes phase sensitive and that switching action can be achieved by the variation of the relative phase between the driving field and the squeezed vacuum. This leads to the possibility of an optical switch controlled by the phase of the input field. The proposed phase-controlled switch could also be operated by placing the small sample of dense two-level atoms in a broadband squeezed light, which has in recent years been generated in a number of ways, using four-wave mixing in atomic vapors [15], in optical fibers [16], and in optical parametric oscillators [17].

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APPENDIX: LINEAR STABILITY ANALYSIS

Here we analyze the stability of the stationary solutions of the state equation for IOB, Eq. (3.2). Adopting the usual procedure for linear stability analysis [5] we denote infinitesimal deviations of the system variable from the steady state in terms of the relations

$$\langle \sigma_j \rangle = \langle \sigma_j \rangle^{SS} + \delta\langle \sigma_j \rangle \ , \quad j = \pm \text{ or } z \tag{A1}$$

where $\langle \sigma_j \rangle^{SS}$ is the steady-state solution of Eqs. (2.3) and (2.4) and $\delta\langle \sigma_j \rangle$ is the infinitesimal deviation from the stationary solution. From (A1) and Eqs. (2.3) and (2.4), we obtain the linearized set of equations in the following matrix form:

$$\dot{\psi}(t) = -M\psi(t) \ , \tag{A2}$$

where M is a 3×3 square matrix whose elements are given by

$$\begin{aligned}
M_{11} &= M_{22}^* = -i(\Delta + \epsilon\langle \sigma_z \rangle^{SS}) + \left[\frac{\langle \sigma_z \rangle^{SS}}{\tau_R} - \gamma_2(2N+1) \right] , \\
M_{33} &= -\gamma_1(2N+1) \ , \\
M_{12} &= M_{21} = -\gamma_1 M \ , \\
M_{13} &= M_{23}^* = -i\left[\epsilon\langle \sigma_- \rangle^{SS} + \frac{\Omega}{2} \right] + \frac{\langle \sigma_- \rangle^{SS}}{\tau_R} \ , \\
M_{31} &= M_{32}^* = -i\Omega - \frac{4}{\tau_R}\langle \sigma_+ \rangle^{SS}
\end{aligned} \tag{A3}$$

and ψ is a vector with components, $\psi_1 = \delta\langle \sigma_z \rangle$, $\psi_2 = \delta\langle \sigma_- \rangle$, $\psi_3 = \delta\langle \sigma_+ \rangle$. The asterisk denotes complex conjugate. Equation (A2) has the nontrivial solution

$$\psi(t) = \psi(0)\exp(\lambda t) \ . \tag{A4}$$

Here the values λ are the complex eigenvalues that are to be determined from the solution of the polynomial

$$\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 = 0 \ , \tag{A5}$$

obtained from the characteristic equation $\det(M - \lambda 1) = 0$, for the matrix M. The various coefficients of the polynomial equation are

$$p_2 = \gamma_1(2N+1) + 2\left[\gamma_2(2N+1) - \frac{\langle \sigma_z \rangle^{SS}}{\tau_R} \right] , \tag{A6a}$$

$$p_1 = -\frac{|\Omega|^2}{2}\left\{ \left[1 + \frac{2}{\tau}\mathcal{L} \right] \left[\left[i\epsilon + \frac{1}{\tau_R} \right] \mathcal{L}^* - 1 \right] + \text{c.c.} \right\} + |\mathcal{D}|^2 - \gamma_1^2|M|^2 + 2\gamma_1(2N+1)\left[\gamma_2(2N+1) - \frac{\langle \sigma_z \rangle^{SS}}{\tau_R} \right] , \tag{A6b}$$

$$\begin{aligned}
p_0 = \frac{|\Omega|^2}{2} & \left[\left[1 - \frac{2}{\tau_R}\mathcal{L}^* \right] \left\{ \mathcal{D}\left[\left[\frac{1}{\tau_R} - i\epsilon \right] \mathcal{L} - 1 \right] - \gamma_1|M|e^{-i\phi}\left[\left[\frac{1}{\tau_R} + i\epsilon \right] \mathcal{L}^* - 1 \right] \right\} + \text{c.c.} \right] \\
& + \gamma_1(2N+1)(|\mathcal{D}|^2 - \gamma_1^2|M|^2) \ ,
\end{aligned} \tag{A6c}$$

with

$$\mathcal{L} = \langle \sigma_z \rangle \frac{\mathcal{D} - \gamma_1|M|e^{-i\phi}}{|\mathcal{D}|^2 - \gamma_1^2|M|^2} \ , \tag{A6d}$$

where $\mathcal{D}$ and $|\Omega|^2$ are given by Eqs. (3.1b) and (3.2), respectively.

For the system to be stable under small deviations from the stationary state, it is necessary that the real part of all the roots of Eq. (A5) be negative. Using the Hurwitz theorem [18] for roots of a polynomial we find the stability conditions for the present system as

$$p_2 > 0, \quad p_2 p_1 - p_0 > 0, \quad p_0 > 0 \ . \tag{A7}$$

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