Adiabatic and nonadiabatic Berry phases for two-level atoms

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(Received 20 March 1991)

A formalism for calculating Berry phases for nonadiabatic systems containing an additional adiabatic time dependence is developed. This is then applied to a two-level atom in a strong resonant polarized laser, where the laser polarization provides the adiabatic parameter. It is found that, when the laser polarization is kept constant, judicious polarization choices allow one to eliminate either the rotating-wave or non-rotating-wave part of the interaction. Furthermore, the adiabatic Berry phasor obtained when the laser polarization is varied is found to be complex. This can be compared to the case of purely adiabatic time-reversal even systems, which can only have real Berry phasors.

PACS number(s): 42.50.−p, 03.65.−w

I. INTRODUCTION

The Berry phase for periodic systems was originally discussed in the adiabatic context [1]. This was soon extended to nonadiabatic motion [2] and in fact nonperiodic motion [3]. The purpose of this paper is to analyze the effect of superimposing an adiabatic variation onto an already nonadiabatically periodic motion. That is, we will look at a system containing an underlying nonadiabatic evolution with period $\tilde{t}_N$ modified by some adiabatically varying parameters with period $\tilde{t}_A$ much larger than $\tilde{t}_N$. We find that the Berry phase splits into adiabatic and nonadiabatic parts.

A parallel analysis has been done by Ellinas, Barnett, and Dupertuis, [4], who used the split in total Berry phase between the purely adiabatic phase and the phase that persists from the nonadiabatic evolution that one gets when the parameters are kept constant.

We also discuss an interesting two-level atomic system, which results when an atomic $s \rightarrow p_{-1}$ transition, split from the other p states by, say, a magnetic field, is irradiated by a strong polarized laser. When the laser polarization (the external parameter) is kept constant, the Berry phases can be experimentally investigated by noting that the difference in overall phases is just the splitting of the Mollow triplet, and is simply related to the frequency of the Rabi oscillation [5,6].

Particularly interesting results are gained when the laser is restricted to have circular polarization. With right-circular polarization the non-rotating-wave coupling is identically zero and so we see only the rotating-wave part. More importantly, however, when left-circular polarization is used, the usually dominant rotating-wave interaction vanishes and so we can see the usually negligible non-rotating-wave coupling.

II. AN EXACTLY SOLVABLE MODEL

We wish to be able to analyze systems that have an underlying nonadiabatic periodicity but which also depend on parameters that are varied adiabatically. The normal adiabatic theorem cannot be used as the Hamiltonian contains a nonadiabatic time dependence, and so we need a modified ansatz. To motivate the ansatz that we will propose, in this section we discuss a simple exactly solvable model, the semiclassical two-level atom in a circularly polarized radiation field. This system clearly displays the fact that the Berry phase for a system with combined adiabatic and nonadiabatic time dependencies can be naturally split into its adiabatic and nonadiabatic components. Furthermore, it points to a method for writing the generalized adiabatic ansatz in such a way that this split is transparent for all systems.

The semiclassical two-level atom in a circularly polarized radiation field has Hamiltonian

$$H = \begin{bmatrix} \omega/2 & k e^{-i\Theta(t)} e^{-i\omega t} \\ k e^{i\Theta(t)} e^{i\omega t} & -\omega/2 \end{bmatrix} . \tag{2.1}$$

This contains two time dependencies. The first, embodied in the factors $\exp(\pm i\omega t)$, is nonadiabatic and arises from the sinusoidal time dependence of the semiclassical radiation field. The second, contained in the factors $\exp[\pm i\Theta(t)]$, describes the adiabatic variation of an external parameter. For convenience, we have chosen the laser to be exactly resonant with the atomic transition.

As the system has two sources of time dependence it also has two natural periods. The nonadiabatic period $\tilde{t}_N = 2\pi/\omega$ is fixed by the frequency of the atomic transition, while the adiabatic period $\tilde{t}_A$ may be chosen almost at will. The "almost" proviso is because we must choose $\tilde{t}_A$ to be much larger than $\tilde{t}_N$ for the parameter variation to be adiabatic. Of course, we want the entire Hamiltonian (2.1) to be periodic, and not merely some of its components. Hence we need the two periods to be commensurable: $\tilde{t}_A = N\tilde{t}_N$ for some integer N. As the adiabatic period is much larger than the nonadiabatic one, N must be large.

We now turn to calculating the cyclic initial states and Berry phases for the system. To this, we could directly solve the time-dependent Schrödinger equation; however, it turns out that an indirect approach is more illuminating. This method is based on the fact that the Hamiltonian (2.1) is the semiclassical limit of the joint electron-photon Hamiltonian

$$H_j=\frac{\omega}{2}\sigma_z+\omega a^*a+\nu(e^{-i\Theta(t)}\sigma_+a+e^{i\Theta(t)}\sigma_-a^*)\ , \quad (2.2)$$

where the σ's are the usual Pauli matrices.

The connections between the two Hamiltonians is made by considering the joint initial state $\Phi(0)=\phi(0)\otimes|z\rangle$, where $\phi(0)$ is an electronic state and $|z\rangle$ is a photon coherent state:

$$|z\rangle=\sum_{n=0}^{\infty}e^{-|z|^2/2}\frac{z^n}{\sqrt{n!}}|n\rangle\ . \quad (2.3)$$

Then the evolutions of $\phi(0)$ under H_j and H coincide in the asymptotic limit: $\nu\to 0$ and $|z|\to\infty$ with $k=\nu|z|$ kept constant [7].

The reason we want to infer the evolution under H from that under H_j is that the joint Hamiltonian is a normal adiabatic Hamiltonian, its only time dependence arising from the parameter $\Theta(t)$. Therefore we can use the normal adiabatic theorem on H_j. To generate the evolution of the atomic component of the system we use the reduced density operator method. First we evaluate the joint electron-photon density operator

$$\rho_j=|\Phi(t)\rangle\langle\Phi(t)| \quad (2.4)$$

for the initial state $\Phi(0)$. This is easily done by decomposing the initial state in an eigenbasis of $H_j(0)$ and using the adiabatic theorem.

One then traces out the unwanted photon degrees of freedom to get the atomic reduced density operator $\rho(t)$. Now, in general a reduced density operator need not describe a pure state. However, in this case we find that it does and so we can write

$$\rho(t)=|\zeta(t)\rangle\langle\zeta(t)|\ , \quad (2.5)$$

giving the electronic evolution. Note that this procedure cannot determine the phase of the electronic state. It is for this reason that we write $\zeta(t)$ in Eq. (2.5) instead of $\phi(t)$; a given choice of $\zeta(t)$ need only equal the true evolving state $\phi(t)$ modulo a multiplicative phase. However, this does not cause any problems due to the geometric nature of the Berry phase. As the Berry phase only depends on the shadow of the evolution on the projective Hilbert space, and $\phi(t)$ and $\zeta(t)$ case the same shadow, both vectors must generate the same Berry phase.

For the system under study, the application of this algorithm is straightforward and so we will only quote the results. Let the initial electronic state be

$$\phi(0)=\begin{bmatrix}a_+\\a_-\end{bmatrix}\ . \quad (2.6)$$

Then we can take $\zeta(t)$ to be

$$\zeta(t)=\frac{1}{2}\begin{bmatrix}e^{-i\Theta(t)}e^{-i\omega t}[(a_++a_-)e^{-ikt}+(a_+-a_-)e^{ikt}]\\(a_++a_-)e^{-ikt}-(a_+-a_-)e^{ikt}\end{bmatrix}\ . \quad (2.7)$$

There is one slight subtlety. As we only know the evolving state up to an overall multiplicative phase, it follows that we only know the Hamiltonian up to an overall (time-dependent) energy normalization. This normalization does not interest us here, as it only affects the dynamical phase and not the Berry phase. Hence it is convenient to take the Hamiltonian to be traceless. It is then easy to see that we do indeed recover the Hamiltonian (2.1).

Note that the independence of the Berry phase on energy normalization helps to motivate the splitting of the overall phase into geometric and dynamic components. We would indeed expect that any change in energy normalization should only affect the dynamical part of the phase. This fact is a simple corollary of the normal justification for this splitting, as the Berry phase only depends on the path followed by the system in projective Hilbert space, it is truly a geometric phase.

We now need to find the cyclic initial states and Berry phases. For convenience we drive Θ in such a way that $\Theta(0)=0$ and $\Theta(\tilde{t}_A)=2\pi$. Thus as $\omega\tilde{t}_A=2N\pi$, we have that

$$\zeta(\tilde{t}_A)=\tfrac{1}{2}e^{-ik\tilde{t}_A}(a_++a_-)\begin{bmatrix}1\\1\end{bmatrix}+\tfrac{1}{2}e^{ik\tilde{t}_A}(a_+-a_-)\begin{bmatrix}1\\-1\end{bmatrix}\ . \quad (2.8)$$

As $\phi(t)$ and $\zeta(t)$ only differ by a multiplicative phase, $\phi(0)$ can be a cyclic initial state if and only if $\zeta(\tilde{t}_A)=\exp(i\alpha)\zeta(0)$. Hence, using the fact that the two exponentials in Eq. (2.8) will not be equal in general, we can see that the cyclic initial states are simply

$$\phi_\pm=\sqrt{1/2}\begin{bmatrix}\pm1\\1\end{bmatrix}\ . \quad (2.9)$$

To calculate the Berry phases corresponding to these two initial states, we invoke the concept of a single-valued vector [8]. This is a state $\psi(t)$, equal to the evolving state $\phi(t)$ to within a multiplicative phase, such that $\psi(\tilde{t}_A)=\psi(0)$. For then the cyclic initial state $\phi(0)$ can be shown to have Berry phase

$$\gamma=i\int_0^{\tilde{t}_A}\langle\psi(t)|\dot{\psi}(t)\rangle dt\ . \quad (2.10)$$

This equation generalizes the adiabatic result of Berry, with the position of the single-valued eigenvectors being taken by the single-valued vectors.

For our problem, a suitable single-valued vector is

$$\psi_\pm(t)=\sqrt{1/2}\begin{bmatrix}\pm e^{-i\Theta(t)}e^{-i\omega t}\\1\end{bmatrix}\ , \quad (2.11)$$

giving, using the fact that $\Theta(\tilde{t}_A)-\Theta(0)=2\pi$, the Berry phases $\gamma_\pm=(N+1)\pi$.

We now attempt to split the Berry phase into its adiabatic and nonadiabatic constituents. To do this, imagine that we allowed the system to evolve without varying Θ. Then any Berry phase that is generated must be purely nonadiabatic. By an analogous derivation to that presented above, we find that the nonadiabatic Berry phase is just $\Gamma_\pm^N=N\pi$. This arises as a sum of N terms of π, one for each nonadiabatic period $\tilde{t}_N$ that goes into the single adiabatic period $\tilde{T}_A$.

Hence the adiabatic part of the Berry phase is just $\Gamma_{\pm}^{A}=\pi$. This conclusion can be strengthened by noting that we could have considered the normal adiabatic Berry phases for then joint Hamiltonian H_j as in Andreev, Klimov, and Lerner [9]. In this case the cyclic initial states are just the instantaneous eigenvectors of $H_j(0)$,

$$\phi_{\pm,n}(0)\pm\sqrt{1/2}|+,n-1\rangle+\sqrt{1/2}|-,n\rangle\ , \tag{2.12}$$

and have Berry phases $\gamma_{\pm,n}=\pi$, exactly the adiabatic contribution $\Gamma_{\pm}^{A}$.

To recapitulate, the Berry phases can be exactly computed for the simple model considered here. Furthermore, by analyzing the purely adiabatic Hamiltonian H_j and the purely nonadiabatic Hamiltonian that arises when Θ in H is not varied, this Berry phase can be split into its adiabatic and nonadiabatic constituents.

Note that this approach is be no means the most elegant way of treating the problem under study. The reason we use it is twofold. First it shows us exactly where the adiabatic theorem is used in the derivation: as the joint electron-photon Hamiltonian's time dependence is purely adiabatic, we can solve for its evolution using the normal adiabatic theorem. To deduce the corresponding electronic evolution we then use the reduced density operator.

Second, and perhaps more importantly, there is a direct analogy to this procedure for the general case. Using Floquet theory, we can reduce a periodic system to an equivalent time-independent form. This form corresponds to the joint Hamiltonian above. We can then superimpose the adiabatic time variation using the normal adiabatic theorem. Projecting back to the original Hilbert space then gives us a precise statement of the adiabatic theorem for the case of a Hamiltonian with a preexisting nonadiabatic periodicity. This is the approach that we will take in Sec. III.

III. THE MODIFIED ADIABATIC ANSATZ

The approach we use here is based on that of Breuer, Deitz, and Holthaus [10], which is based on the identification of the purely nonadiabatic problem with an equivalent time-independent one. This time-independent treatment uses an expanded Hilbert space, embedding the periodic time dependence into the Hilbert space structure and was first discussed by Shirley [11], being applied to the calculation of Berry phases by Moore [12].

To motivate this approach, consider the time-dependent Schrödinger equation satisfied by a given $\tilde{t}$-periodic and nonadiabatic Hamiltonian $H(t)$:

$$[H(t)-i\partial_t]\phi(t)=0\ . \tag{3.1}$$

This can be written as $K\phi=0$, where K is called the Floquet Hamiltonian. Later we will superimpose an adiabatic variation onto $H(t)$ and so onto K.

To find the Berry phase for this Hamiltonian we need the single-valued vectors. As before, these are vectors $\psi(t)$ following the evolution of the cyclic initial state $\phi(0)$ modulo a multiplicative phase, that satisfy the boundary condition $\psi(\tilde{t})=\psi(0)$. The simplest such vectors are simply given by

$$\psi(t)=e^{-\chi t/\tilde{t}}\phi(t)\ , \tag{3.2}$$

where χ is the overall corresponding to $\phi(0)$.

Now let us see how the Floquet Hamiltonian K acts on $\psi(t)$. It is easy to see that

$$K\psi(t)=\epsilon\psi(t)\ , \tag{3.3}$$

where $\epsilon=-\chi/\tilde{t}$. Thus in some sense the single-valued vectors are eigenvectors of the Floquet Hamiltonian. To strengthen this conclusion, we note that the set of $\tilde{t}$-periodic vectors forms a Hilbert space $\mathcal{H}$ with inner product [13]

$$\langle\langle\cdots|\cdots\rangle\rangle=\frac{1}{\tilde{t}}\int_0^{\tilde{t}}\langle\cdots|\cdots\rangle dt\ . \tag{3.4}$$

The Hilbert space $\mathcal{H}$ absorbs the periodic time dependence of the single-valued vectors $\psi(t)$ and the Floquet Hamiltonian K, which is then effectively time independent. Hence on $\mathcal{H}$, the single-valued vectors are simply the eigenvectors of the (time-independent) Floquet Hamiltonian K. Furthermore, the corresponding eigenvalues are proportional to the overall phases. In the literature, these eigenvectors and eigenvalues are often called the quasienergy states and quasienergies, respectively [14].

Note that by writing the periodic Hamiltonian in terms of its Fourier coefficients,

$$H=\sum_{n=-\infty}^{\infty}H^{[n]}e^{in\omega t}\ ,$$

where

$$H^{[n]}=\int_0^{2\pi}He^{-in\omega t}dt\ , \tag{3.5}$$

the Floquet Hamiltonian can be seen to have matrix elements

$$\langle\langle\alpha n|K|\beta m\rangle\rangle=H_{\alpha\beta}^{[n-m]}+n\omega\delta_{\alpha\beta}\delta_{\alpha\beta}\ . \tag{3.6}$$

Here α and β represent electronic basis vectors and n and m are integers. If one applies this process to the semiclassical Jaynes-Cummings Hamiltonian (2.1), with Θ supposed time independent, we find that K is essentially just the joint Hamiltonian (2.2), where n and m are interpreted as labeling photon-number states.

To see how Breuer, Dietz, and Holthaus proceed, we note that when an extra adiabatic time dependence is superimposed onto the system, the Floquet Hamiltonian K, as an operator on $\mathcal{H}$, becomes time dependent. As the variable t has already been used, I will parametrize this dependence by τ: $K=K(\mathbf{R}(\tau))$, where $\mathbf{R}$ represents the adiabatic parameter. Furthermore, as this is a "normal" adiabatic Hamiltonian, we can apply the normal adiabatic theorem (as modified by Berry, of course).

Hence the initial state $\phi(0)=\psi(\mathbf{R}(0))$ on $\mathcal{H}$ will evolve into

$$\phi(\tau)=\exp^{-i\int_0^{\tau}\epsilon(\mathbf{R}(\tau'))d\tau'}e^{i\Gamma_A(\tau)}\psi(\mathbf{R}(\tau))\ , \tag{3.7}$$

where Γ_A is the adiabatic Berry phase. Now take the parameter $\mathbf{R}$ around a closed path: $\mathbf{R}(\tilde{t}_A)=\mathbf{R}(0)$. Then, choosing the phases of the eigenstates $\psi(\mathbf{R}(\tau))$, to be single valued, we find that the initial state $\psi(\mathbf{R}(0))$ is cyclic in $\mathcal{H}$ with adiabatic Berry phase

$$\Gamma_A = i\int_0^{\tilde{t}_A} \langle\langle \psi(\mathbf{R}(\tau))|\partial_\tau \psi(\mathbf{R}(\tau))\rangle\rangle d\tau \ . \tag{3.8}$$

To get back to the problem in the original Hilbert space $\mathcal{H}$, we just reinstate the t dependence of the single-valued vectors $\psi(\mathbf{R}(\tau))$ and set $\tau = t$ [10]. For the evolving state $\phi(t)$ to be cyclic in $\mathcal{H}$ we require that $\psi(\mathbf{R}(t),t)$ to be single valued; that is that $\psi(\mathbf{R}(\tilde{t}_A),\tilde{t}_A) = \psi(\mathbf{R}(0),0)$. Now we know that $\mathbf{R}(\tilde{t}_A) = \mathbf{R}(0)$. Furthermore, for each $\mathbf{R}$, $\psi(\mathbf{R},\tilde{t}_N) = \psi(\mathbf{R},0)$. Hence $\psi(0)$ is a cyclic initial state if and only if $\tilde{t}_A = N\tilde{t}_N$ so that the adiabatic and nonadiabatic periods match.

Note that the Floquet approach is effectively the method we used in solving the quantum optical problem in Sec. II. There the extended Hilbert space $\mathcal{H}$ has the physical interpretation of the tensor product of the atomic Hilbert space $\mathcal{H}$ and the photon Hilbert space. This allows one to interpret the Rabi oscillations and Mollow triplet splitting of quantum optics in terms of atomic Berry phases [5].

To recapitulate, the procedure discussed above gives us a rigorous proof that the initial state $\phi(0) = \psi(\mathbf{R}(0),0)$ is cyclic in $\mathcal{H}$ with overall phase

$$\chi = -i\int_0^{\tilde{t}_A} \epsilon(\mathbf{R}(t))dt + \Gamma_A \ . \tag{3.9}$$

Breuer, Dietz, and Holthaus [10] go further and state that Γ_A is the Berry phase for the atomic system. However, this phase has no nonadiabatic part, in direct contradiction to the results derived earlier in this section. The reason for this is that the quasienergy ϵ gives the nonadiabatic overall phase and not merely the dynamical phase. In other words, the integral of $\epsilon(\mathbf{R}(t))$ gives the sum of the true dynamical phase

$$\delta = -\int_0^{\tilde{t}_A} \langle \phi(t)|H(t)|\phi(t)\rangle dt \ , \tag{3.10}$$

and the nonadiabatic Berry phase

$$\Gamma_N = i\int_0^{\tilde{t}_A} \langle \psi(\mathbf{R}(t),t)|\partial_t \psi(\mathbf{R}(t),t)\rangle dt \ . \tag{3.11}$$

Hence Breuer, Dietz, and Holthaus are only partially correct; while they derive the adiabatic part of the Berry phase, they leave the nonadiabatic part intertwined with the nongeometrical dynamical phase. We can go a little further and show that the nonadiabatic phase (3.11) is in fact the sum of the nonadiabatic Berry phases for each nonadiabatic period. To do this we break the integration in Eq. (3.11) into the sum of integrations from $(n-1)\tilde{t}_N$ to $n\tilde{t}_N$ for $n = 1,2,\ldots,N$. Furthermore, assuming that N is large and $\mathbf{R}$ slowly varying, we can ignore the variation of $\mathbf{R}$ in each of these integrations. This allows us to replace $\mathbf{R}(t)$ by its value at the end of the integration range of interest, namely $\mathbf{R}(n\tilde{t}_N)$. Hence

$$\Gamma_\alpha^N = \sum_{n=1}^{\infty} \gamma_\alpha^N(n\tilde{t}_N) \ , \tag{3.12}$$

where γ_α^N is the nonadiabatic Berry phase for the $\tilde{t}_N$-periodic Hamiltonian $H(\mathbf{R}(n\tilde{t}_N),t)$.

Thus in general we can decompose the Berry phase into a sum of nonadiabatic phases and a phase due to the adiabatic variation of some parameter $\mathbf{R}$ upon which the Hamiltonian depends. This result can be recast into several different forms. For example, Eq. (3.7) shows that the single-valued vector $\psi(t)$ for the system is just the instantaneous single-valued vector $\psi(\mathbf{R}(t),t)$ appropriate to the Hamiltonian $H(\mathbf{R},t)$ with $\mathbf{R}$ taken at time t. This allows us to directly use the formalism of Aharonov and Anandan [2].

Furthermore, we can also derive the generalization of the decomposition scheme of Moore and Stedman [8]. Here the evolution operator for a nonadiabatic and period Hamiltonian is written $U(t) = Z(t)\exp(iMt)$, where Z is unitary and periodic and M is self-adjoint and subject to a simple spectral constraint. This decomposition can be shown to be strongly linked to Floquet theory [12]. A quick calculation shows that this method can be extended to the case of a superimposed adiabatic time variation by putting

$$U(t) = Z(\mathbf{R}(t),t)e^{iM(\mathbf{R}(t))t} \ , \tag{3.13}$$

where $Z(\mathbf{R}(t),t)$ and $M(\mathbf{R}(t))$ are the operators corresponding to the instantaneous Hamiltonian $H(\mathbf{R},t)$, where once again $\mathbf{R} = \mathbf{R}(t)$.

Evolution-operator decompositions can also be used to discuss the generalization of Berry phase to nonperiodic systems. For example Anandan [3] shows that for certain Hamiltonians (which do not have to be periodic) the evolution operator can be written in the form

$$U(t) = P\exp\left(-i\int_0^t K\,dt\right)B(t) \ , \tag{3.14}$$

where the first factor generalizes the dynamical phase and $B(t)$ is purely geometrical. This approach focuses on the relationship between the Berry phase and the fiber bundle structure of the Hilbert space. Hence, when restricted to the case of periodic Hamiltonians, it is complementary to that of Moore and Stedman, which focuses on the periodic nature of the Hamiltonian.

IV. EXAMPLE

We now apply the theory developed above to a model two-level atom. Consider a two-level atom in a resonant laser where the atomic transition is between an s state and a p_{-1} state split from the other p states by, say, a magnetic field. This transition is chosen as the definite angular momenta of the two states lead to the applicability of simple selection rules. Of course the experimental realization of such a system is difficult due to interference from the other p states, among other things. This point will be touched on at the end of Sec. IV A.

The electron-photon interaction has the electric dipole form $\mathbf{e}\cdot\mathbf{r}$, where $\mathbf{e}$ is the laser polarization [15]. Thus, if the laser has polarization vector

$$\mathbf{e} = \sqrt{1/2}(1 = i\cos\Theta, -i\sin\Theta, 0) \ , \tag{4.1}$$

then, noting that $p_{-1} \sim |x\rangle - i|y\rangle$, the semiclassical electronic Hamiltonian is given by

$$H = \begin{bmatrix} \omega/2 & k(\bar{\lambda}e^{i\omega t} + \bar{\mu}e^{i\omega t} \\ k(\lambda e^{i\omega t} + \mu e^{-i\omega t}) & -\omega/2 \end{bmatrix} \ , \tag{4.2}$$

where $\lambda=1+i\exp(i\Theta)$ and $\mu=1-i\exp(i\Theta)$. We can identify the rotating-wave coupling constant λ and the non-rotating-wave coupling constant μ.

A. The nonadiabatic Berry phase

First we calculate the nonadiabatic Berry phase, keeping the parameter Θ in the polarization vector $\mathbf{e}$ constant. To do this we calculate the eigenvalues ϵ_{an} and eigenvectors $|\epsilon_{an}\rangle$ of the Floquet Hamiltonian K. As this cannot be done exactly for general Θ we proceed perturbatively. Now the rotating-wave part of the interaction is usually dominant. Hence we treat it exactly, taking the non-rotating-wave coupling to be a perturbation. Putting $\mu=0$ into Eq. (4.2) we find that the rotating-wave Floquet Hamiltonian K^R has eigenvalues

$$\epsilon^R_{\pm n}=(n-\tfrac{1}{2})\omega\pm k|\lambda| \tag{4.3}$$

and eigenvectors

$$|\epsilon^R_{\pm n}\rangle=\sqrt{1/2}|-,n\rangle\pm\sqrt{1/2}\frac{\bar{\lambda}}{|\lambda|}|+,n-1\rangle\ . \tag{4.4}$$

We note that $|\epsilon^R_{\pm n}\rangle$ is indeterminate at $\lambda=0$. This is a consequence of the fact that K^R is degenerate at $\lambda=0$ and will be dealt with later.

Reinstating μ we find that, due to the degeneracy at $\lambda=0$, we must use degenerate perturbation theory [16]. After considerable labor we find that

$$\epsilon_{\pm,n}=(n-\tfrac{1}{2})\omega\pm(a^2+b^2)^{1/2}\ , \tag{4.5}$$

$$\begin{aligned}\frac{1}{N}|\epsilon_{\pm,n}\rangle=&\alpha_\pm|-,n\rangle+\frac{\bar{\lambda}}{|\lambda|}\beta_\pm|+,n-1\rangle+\frac{k\lambda\bar{\mu}}{4|\lambda|}\left[\alpha_\pm\frac{k|\lambda|}{\omega^2-k^2|\lambda|^2}-\beta_\pm\left[\frac{1}{\omega}-\frac{\omega}{\omega^2-k^2|\lambda|^2}\right]\right]|-,n+2\rangle\\&+\frac{k\bar{\mu}}{4}\left[-\beta_\pm\frac{k|\lambda|}{\omega^2-k^2|\lambda|^2}-\alpha_\pm\left[\frac{1}{\omega}+\frac{\omega}{\omega^2-k^2|\lambda|^2}\right]\right]|+,n+1\rangle\\&+\frac{k\bar{\lambda}\mu}{4|\lambda|}\left[-\alpha_\pm\frac{k|\lambda|}{\omega^2-k^2|\lambda|^2}+\beta_\pm\left[\frac{1}{\omega}=\frac{\omega}{\omega^2-k^2|\lambda|^2}\right]\right]|-,n-2\rangle\\&+\frac{k\mu\bar{\lambda}^2}{4|\lambda|^2}\left[\beta_\pm\frac{k|\lambda|}{\omega^2-k^2|\lambda|^2}+\alpha_\pm\left[\frac{1}{\omega}-\frac{\omega}{\omega^2-k^2|\lambda|^2}\right]\right]|+,n-3\rangle\ ,\end{aligned} \tag{4.6}$$

where

$$N=\left[1+\frac{k^2|\mu|^2}{8\omega^2}\left[1+\frac{1+k^2|\lambda|^2/\omega^2}{(1-k^2|\lambda|^2/\omega^2)^2}\right]\right]^{-1/2}\ , \tag{4.7}$$

$$\alpha_+=\beta_-=\tfrac{1}{2}(a^2+b^2)^{-1/4}\{[(a^2+b^2)^{1/2}+a]^{1/2}+b[(a^2+b^2)^{1/2}+a]^{-1/2}\}\ , \tag{4.8}$$

$$\beta_+=-\alpha_-=\tfrac{1}{2}(a^2+b^2)^{-1/4}\{[(a^2+b^2)^{1/2}+a]^{1/2}-b[(a^2+b^2)^{1/2}+a]^{-1/2}\}\ , \tag{4.9}$$

$$a=k|\lambda|1-\frac{k^2|\mu|^2}{4\omega^2}\left[1-\frac{k^2|\lambda|^2}{\omega^2}\right]^{-1}\ , \tag{4.10}$$

$$b=-\frac{k^2|\mu|^2}{4\omega}1+\left[-\frac{k^2|\lambda|^2}{\omega^2}\right]^{-1}\ . \tag{4.11}$$

Thus we find that the cyclic initial states $\phi_\pm(0)$ are given by

$$\phi_\pm(0)=N\begin{bmatrix}l_\pm\\ m_\pm\end{bmatrix}\ , \tag{4.12}$$

where

$$m_+=\frac{\lambda}{|\lambda|}\left[1+\frac{ik^2\cos\Theta}{\omega^2-k^2|\lambda|^2}\right]\alpha_+-\frac{ik\cos\Theta\lambda}{\omega|\lambda|^2}\beta_+\ , \tag{4.13}$$

$$l_+=\left[1+\frac{ik^2\cos\Theta}{\omega^2-k^2|\lambda|^2}\right]\beta_+-\frac{ik\cos\Theta}{\omega|\lambda|}\alpha_+\ , \tag{4.14}$$

$$l_-=\frac{\bar{\lambda}}{|\lambda|}\left[1+\frac{ik^2\cos\Theta}{\omega^2-k^2|\lambda|^2}\right]\beta_--\frac{ik\cos\Theta\bar{\lambda}}{\omega|\lambda|^2}\alpha_-\ , \tag{4.15}$$

$$m_-=\left[1+\frac{ik^2\cos\Theta}{\omega^2-k^2|\lambda|^2}\right]\alpha_--\frac{ik\cos\Theta}{\omega|\lambda|}\beta_-\ . \tag{4.16}$$

We note that the singularity in $\lambda/|\lambda|$ is not a problem. This is because at $\Theta=\pi/2$, which is where $\lambda/|\lambda|$ is indeterminate, α_+ and β_- vanish so that the offending terms do not contribute to m_+ or l_-. Furthermore, the terms in $\lambda\cos\Theta/|\lambda|^2$ are well behaved in the limit $\Theta\to\pi/2$.

Finally, applying Eq. (2.10), the nonadiabatic Berry phases are given by

$$\gamma^N_\pm=-2\pi N^2\left[-\beta_\pm^2+\frac{k^2|\mu|^2}{16(\omega^2-k^2|\lambda|^2)}X\right]\ , \tag{4.17}$$

where X is a constant of order unity.

This result is interesting in its own right, especially if we consider the special cases of $\Theta=\pi/2$, corresponding to left-circularly polarized light, and $\Theta=-\pi/2$, corresponding to right-circularly polarized light. For a right-circularly polarized laser, the non-rotating-wave coupling μ is zero and so we see only the rotating-wave interaction. This is not particularly useful as the rotating-wave contribution is dominant anyway. Of more interest is the case of a left-circularly polarized laser, for which the rotating-wave coupling λ is zero and we see the non-rotating-wave coupling which is usually hidden.

In theory the non-rotating-wave coupling could be observed by measuring the splitting of the Mollow triplet in left polarization [15], a quantity that is proportional to the difference in overall phases of the two electronic cyclic initial states [5,6]. In practice the situation is complicated by the presence of magnetic dipole and other terms, which are likely to be of at least the same order of magnitude as the non-rotating-wave term.

Few other authors have considered the effect of the non-rotating-wave coupling. For example, Phoenix [17] shows that the counterrotating term can induce a small phase-dependent term into the expression for the atomic inversion, but also notes that one would expect the two-level approximation to fail before the rotating-wave approximation. Thus, in some sense, the arguments about the measurability of non-rotating-wave terms in the two-level model are academic. We now allow the polarization parameter to vary, giving an adiabatic Berry phase.

B. The adiabatic Berry phase

We can also calculate the adiabatic Berry phase given by

$$\Gamma_{\pm}^{A}=i\int_0^{2\pi}\left\langle\phi_{\pm}\left|\frac{d}{d\Theta}\right|\phi_{\pm}\right\rangle d\Theta\ . \tag{4.18}$$

To do this we take only the leading terms in the expansion of $\phi_{\pm}(0)$. For $\phi_{-}(0)$,

$$l_{-}=\frac{\bar{\lambda}}{|\lambda|}\beta_{-},\quad m_{-}=\alpha_{-}\ . \tag{4.19}$$

Now as $\bar{\lambda}/|\lambda|$ has unit magnitude it is a phase, $e^{-if(\Theta)}$. At $\Theta=\pi/2$, $\bar{\lambda}/|\lambda|$ jumps from $-i$ to i and so f jumps from $\pi/2$ to $-\pi/2$. Thus $f(\Theta+2\pi)=f(\Theta)+\pi$. Also, using the fact that α_{-} and β_{-} are both real, we find that Eq. (48) gives

$$\Gamma_{\pm}^{A}=\int_{\pi/2}^{5\pi/2}\dot{f}\beta_{\pm}^{2}d\Theta\ . \tag{4.20}$$

Thus all we need to do is find β_{-}.

To leading order we have

$$a=k|\lambda|,\quad b=\frac{-k^2|\mu|^2}{4\omega}\ . \tag{4.21}$$

Now $|a|\gg|b|$ for all Θ except for a small interval around $\Theta=\pi/2$. Further this small interval does not contribute to the adiabatic Berry phase in lowest order and can be ignored, allowing us to take $\beta_{-}=1/\sqrt{2}$ from Eq. (4.8). Hence the adiabatic Berry phase is given by $\Gamma_{-}^{A}=\pi/2$. We can similarly show that $\Gamma_{+}^{A}=-\pi/2$. Thus we see a complex phasor change. This contrasts, for example, with the case of a time reversal even adiabatic Hamiltonian. As elegantly shown by Kivelson and Rokhsar [18], these systems can only have Berry phases of 0 or π.

This behavior is due to the fact that our chosen polarization path takes the system through a point ($\lambda=0$) where the rotating-wave eigenfunctions of the joint electron-photon Hamiltonian are degenerate. To see why this is so we use the fact that the adiabatic part of the Berry phase for the atomic system is the same as the Berry phase for the joint electron-photon system itself. As the only time dependence of this Hamiltonian comes from the adiabatic variation of Θ we may use Berry's original result, expressing the Berry phase as [1]

$$\gamma_A=i\int_0^{\tilde{t}}\left\langle\epsilon_{an}\left|\frac{d}{dt}\right|\epsilon_{an}\right\rangle dt\ . \tag{4.22}$$

As the non-rotating-wave coupling has negligible effect we can take the $|\epsilon_{an}\rangle$ to be the eigenvectors $|\epsilon_{an}^{R}\rangle$ of the rotating-wave electron-photon Hamiltonian

$$H^{R}=\frac{\omega}{2}\sigma_z+\omega a^{*}a+k(\bar{\lambda}a^{*}\sigma_{+}+\lambda a\sigma_{-})\ . \tag{4.23}$$

The relevant eigenvectors are then

$$|\epsilon_{an}^{R}\rangle=\sqrt{1/2}|-,n\rangle\pm\sqrt{1/2}\frac{\bar{\lambda}}{|\lambda|}|+,n-1\rangle\ . \tag{4.24}$$

For completeness we allow λ to vary over an arbitrary path, not merely the path $\lambda=1+i\exp\{i\Theta\}$ used above. We write $\bar{\lambda}/|\lambda|=e^{-if}$ as before. Now is λ is never zero along our chosen path, then, as the path is closed, $f(\tilde{t})-f(0)=2m\pi$. But then the Berry phase from Eq. (4.22) is simply $m\pi$. Of course this result is not due to time reversal invariance. The Hamiltonian (4.23) is not time reversal even, as can be seen from the fact that the eigenvectors are not real. To get a complex phasor the path must be taken through the degeneracy at $\lambda=0$ as is done above. At first sight this causes problems as the degeneracy means that the adiabatic assumptions should not be tenable; however the non-rotating-wave contribution splits the two states restoring its validity. Traveling through the degeneracy also means that $\bar{\lambda}/|\lambda|$ need no longer be single valued. This is because near the degeneracy the magnitude of the coefficients of the basis elements $|-,n\rangle$ and $|+,n-1\rangle$ is no longer constant as in (4.24), so that the coefficient of $\bar{\lambda}/|\lambda|$ could vanish at the degeneracy. If this is the case, any jump in $\bar{\lambda}/|\lambda|$ at the degeneracy will not spoil the necessary single valuedness of the basis. This is exactly what happens in our example. At $\Theta=\pi/2$ we find that f jumps by π, generating the Berry phase $\pi/2$.

V. DISCUSSION

In this paper we have discussed a modified adiabatic ansatz for systems with both adiabatic and nonadiabatic time dependences. We find that, as expected, the Berry phase for the combined system contains easily discrim-

inated adiabatic and nonadiabatic components. This contrasts with the work of other authors, who are content to leave the nonadiabatic part of the Berry phase entangled with the dynamical phase.

Finally, we explicitly calculated both the adiabatic and nonadiabatic Berry phases for an interesting quantum optical problem; the case of an $s \rightarrow p_{-1}$ transition irradiated by a resonant polarized laser. The adiabatic Berry phasor of this system is complex (in the sense of not being an integral multiple of π)—a property due to its lack of time-reversal invariance.

ACKNOWLEDGMENT

The authors would like to thank R. Ballagh for useful discussions.

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