Emission from atoms in linear superpositions of center-of-mass wave packets

Robert Graham,* Daniel F. Walls, and Peter Zoller†
Department of Physics, University of Auckland, Auckland, New Zealand
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We study theoretically the electromagnetic field emitted by atoms prepared in linear superpositions of several internal states, each of which is attached to a different center-of-mass wave packet by the preparation process. It is shown that the motion and mutual separation of the wave packets can be monitored either by observing the coherent spontaneous emission in a heterodyne experiment or by measuring the energy-absorption rate from a weak probe-laser beam. We also show that a three-level system involving three wave packets coherently emits photons, forming a linear superposition of states of opposite wave vectors. A heterodyne scheme for detecting the photons in this state is proposed.

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I. INTRODUCTION

Deflection of atoms by laser light has been the subject of considerable theoretical and experimental interest during the past few years [1–6]. For sufficiently short interaction times of the atom with the radiation field, spontaneous emission can be neglected during excitation and the scattering process becomes coherent [2,3]. Thus the wave packet of a single atom incident on a light wave will be transformed by the radiative interaction into a *superposition* of product states of internal atomic levels and center-of-mass wave packets. According to the momentum transferred by the light field these wave packets will propagate in different directions and finally separate, forming a macroscopic superposition of quantum states.

Apart from the intrinsic interest in the study of effects of macroscopic atomic superposition states, coherent scattering of atomic beams by traveling and standing laser-light waves is one of the key elements in the optical realization of a beam splitter and mirror in wave-matter interferometry with atoms [7–11] (see also Refs. [12,13]). In comparison with neutron interferometery [14–16] the atomic interferometer promises considerably enhanced resolution [8] in both a new generation of fundamental physics experiments (in, for example, gravitation physics) and a new class of accelerometers. In addition, advances in cavity QED have stimulated recent theoretical studies of coherent atomic scattering by few-photon standing-wave fields which point out interesting perspectives related to quantum effects of the light field and measurements of photon statistics [4,5].

In the present paper we study the scattered radiation and weak-field absorption of a stationary atomic beam in an atomic-superposition state formed in coherent scattering by light waves. We are specifically interested in monitoring the atomic motion and mutual separation of the center-of-mass wave packets in the coherent spontaneous emission and energy-absorption rate from a weak laser beam.

In what follows we consider a stationary beam of atoms of mass m emerging from their source in a well-defined internal state, which we shall assume as their ground state $|g\rangle$, and with a given narrow distribution $W(E)$ of their kinetic energy. In the plane $z=0$, the beam passes the hole in a screen in the z direction. After passing the hole the atomic beam traveling mainly in the z direction passes a laser beam in the x direction with axis $y=0$, $z=z_0$ and frequency $\omega_1=ck_1$ such that it can excite one or several internal states $|e\rangle$ of the atom. After passing the laser field the atom is in a coherent superposition of excited states and the ground state, each of which is attached to its own wave packet traveling with its own mean velocity fixed by the energy and momentum (and eventually angular momentum) conservation of the excitation process.

The wave packets propagate into the region $z>0$, overlapping at first, but eventually separating. One or two detectors pick up the electromagnetic field emitted by the excited atoms when they spontaneously return to their ground state.

The field emitted from the wave packets in the region where they still overlap contains interferences which are absent in the radiation emitted from regions where the wave packets have no overlap. These interferences can be used to monitor the separation of the wave packets and their calculation is a goal of the present paper. Alternatively, the absorption from a weak probe-laser beam passing through the atomic beam can be used to monitor the linear superposition, and is also calculated here.

We shall consider two situations distinguished by their simplicity. As the first and simplest case, we consider a beam of two-level atoms, whose dipole matrix elements are aligned in the y direction, excited by a running-wave laser propagating in the x direction and linearly polarized in the y direction. As the second case, we consider a beam of atoms with a $J=0$ ground state and a $J=1$ excited state, excited by a standing-wave laser such that the degenerate $M_J=+1,-1$ sublevels $|e_+\rangle,|e_-\rangle$ of the excited state are coherently excited together by the σ_+ (σ_-) component of the standing wave traveling in the x $(-x)$ direction.

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II. PREPARATION OF THE EXCITED STATE

A. Two-level atoms excited by a running wave

The purpose of this section is to provide a brief summary of the basic equations for coherent scattering of a two-level system from a running light wave as well as the approximations and assumptions involved in deriving these equations [2,3]. We shall emphasize in our discussion the derivation of these results from the point of view of a *stationary ensemble* of atoms in a beam, as this corresponds most closely to the situation in present experiments.

The Hamiltonian may be written as

$$H(t)=\frac{\hat{p}^2}{2m}+H_A+H_F+H_I(t)\ , \tag{1}$$

where $\hat{p}^2/2m$ is the kinetic-energy operator containing the momentum operator $\hat{p}$, $H_A=\hbar\omega_0|e\rangle\langle e|$ is the free atomic Hamiltonian for the internal (electronic) degree of freedom containing the excited state $|e\rangle$ and the excited energy $\hbar\omega_0$, and H_F is the Hamiltonian of the free electromagnetic field. The interaction Hamiltonian in the dipole and rotating-wave approximation is

$$H_1(t)=-\int d^3x\,\mathbf{D}^{(-)}(\mathbf{x})\cdot[\mathcal{E}_C^{(+)}(\mathbf{x},t)+\mathcal{E}_R^{(+)}(\mathbf{x},t)]+\text{H.c.}\ , \tag{2}$$

with the negative-frequency part of the dipole operator density

$$\mathbf{D}^{(-)}(\mathbf{x})=d\epsilon^*|e\rangle\langle g|\otimes|\mathbf{x}\rangle\langle\mathbf{x}|\ , \tag{3}$$

where d is the dipole matrix element, ϵ is a unit vector in the direction of the dipole, and $|\mathbf{x}\rangle$ is the atomic center-of-mass position eigenstate. The positive-frequency part of the quantized electric field at position $\mathbf{x}$ is

$$\mathcal{E}_R^{(+)}(\mathbf{x})=i\sum_\lambda\int d^3k\left[\frac{\hbar\omega}{4\pi^2}\right]^{1/2}e^{i\mathbf{k}\cdot\mathbf{x}}\epsilon_{\mathbf{k}\lambda}b_{\mathbf{k}\lambda}\ ; \tag{4}$$

$b_{\mathbf{k}\lambda}$ is the annihilation operator and $\epsilon_{\mathbf{k}\lambda}$ is the polarization vector for the mode $\mathbf{k}\lambda$, respectively. The positive-frequency part of the field traveling in the x direction and polarized in the y direction is written as

$$\mathcal{E}_C^{(+)}(\mathbf{x},t)=\epsilon_y\mathcal{E}_G(\mathbf{x})e^{ik_Lx-i\omega_Lt}\ , \tag{5}$$

with amplitude $\mathcal{E}_G(\mathbf{x})$ changing in its transverse y and z directions slowly on the scale of the laser wavelength.

We are interested in a *stationary* solution of the Schrödinger equation

$$i\hbar\frac{d}{dt}|\psi_E(t)\rangle=H(t)|\psi_E(t)\rangle \tag{6}$$

describing a stationary atomic-beam configuration which enters the interaction region in the ground state. As the boundary condition at $z=0$ we choose

$$\langle x,y,0|\psi_E(t)\rangle=\sqrt{N_E}e^{-(i/\hbar)Et}f(x,y)|g\rangle\otimes|0\rangle\ , \tag{7}$$

where $|0\rangle$ denotes the vacuum of the electromagnetic field, and $f(x,y)$ describes the (coherent) spatial distribution of the atoms at the slit: $f(x,y)=1/\sqrt{A}$ with A the area for (x,y) inside the hole, and $f(x,y)=0$ outside. We choose the normalization factor $N_E^{-1}=\sqrt{2E/m}\equiv v_E$ to normalize the average beam intensity to one atom per unit time.

The state vector $|\psi_E(t)\rangle$ can be written as a sum of terms describing the presence of $n=0,1,2,\ldots$ scattered photons in the field,

$$|\psi_E(t)\rangle=|\psi_{E_0}(t)\rangle+\sum_{\{n_{\mathbf{k}\lambda}\}}{}'\,|\psi_{\{n_{\mathbf{k}\lambda}\}}(t)\rangle\ , \tag{8}$$

where $|\psi_{E_0}(t)\rangle$ is the state where no photon has yet been emitted (vacuum amplitudes) and $|\psi_{\{n_{\mathbf{k}\lambda}\}}(t)\rangle$ corresponds to the one-, two-, etc., photon contribution [17]. For $z>0$ we solve the Schrödinger equation with the ansatz of stationary oscillations with the frequency ω_L

$$|\psi_{E_0}(t)\rangle=e^{-(i/\hbar)Et}|0\rangle\otimes\int d^3x[\phi_g(\mathbf{x})|g\rangle+e^{-i\omega_Lt}\phi_e(\mathbf{x})|e\rangle]\otimes|\mathbf{x}\rangle\ . \tag{9}$$

Its insertion in the Schrödinger equation (6) with the Hamiltonian equation (1) leaves us with the two coupled *time-independent* Schrödinger equations for the vacuum amplitudes

$$E\phi_g(\mathbf{x})=-\frac{\hbar^2}{2m}\nabla^2\phi_g(\mathbf{x})-d^*\mathcal{E}_g^{(-)}(\mathbf{x})e^{-ik_Lx}\phi_e(\mathbf{x})\ , \tag{10}$$

$$(E+\hbar\omega_L)\phi_e(\mathbf{x})=-\frac{\hbar^2}{2m}\nabla^2\phi_e(\mathbf{x})-d\,\mathcal{E}_G^{(+)}(\mathbf{x})e^{ik_Lx}\phi_g(\mathbf{x})+\left[\hbar\omega_0-\frac{i}{2}\hbar\gamma\right]\ . \tag{11}$$

Here γ is the spontaneous-decay rate of the excited level. Once Eqs. (10) and (11) have been solved, the statistical operator of the atomic beam for $z=0$ in the subspace of the photon vacuum is known,

$$\rho^{(0)}(t)=\int dE\ W(E)|\psi_{E_0}(t)\rangle\langle\psi_{E_0}(t)|\ , \tag{12}$$

where $W(E)$ is the energy distribution of the atomic beam.

Equations (10) and (11) can be simplified under the assumption of large kinetic energy $E\gg\hbar\delta_L,\hbar\gamma,\hbar^2k^2/2m$ [for all E for which $W(E)$ is appreciable]. We write $\phi_{g,e}(\mathbf{x})=e^{imv_Ez/\hbar}\tilde{\phi}_{g,e}(\mathbf{x})$ with $\tilde{\phi}_{g,e}$ slowly varying on the scale of the de Broglie wavelength $\lambda_E=2\pi\hbar/(mv_E)$. Then Eqs. (10) and (11) can be simplified by dropping the second-order derivatives with respect to z, but keeping the first-order derivative in z.

For simplicity we shall work in the Raman-Nath limit where the interaction time is assumed to be short compared to the spontaneous-decay time and the inverse recoil frequency, that is $W_0/v_E\ll 1/\gamma, 1/(\hbar k_L^2/2m)$, $1/(\hbar k_L/mL)$ with W_0 the laser beam waist of a Gaussian laser beam [18]. We further assume the beam width W_0 is much greater than L and the optical wavelength $\gg L, 2\pi/k_L$ and $z_0\ll\sqrt{L/k}$ so that diffraction effects may be neglected. Then the Schrödinger equation can be simplified further in the region inside the laser beam by neglecting γ and the spatial derivatives with respect to x and y. Furthermore, because $L \ll W_0$, the laser field can be considered to be constant over the cross section of the atomic beam $\mathcal{E}_G(\mathbf{x}) \simeq \mathcal{E}_0$.

The resulting equations are

$$iv_E \frac{\partial}{\partial z}\tilde{\phi}_g = -\tfrac{1}{2}\Omega(z) e^{-ik_L x}\tilde{\phi}_e \ , \tag{13}$$

$$iv_E \frac{\partial}{\partial z}\tilde{\phi}_e = -\delta_L \tilde{\phi}_e - \tfrac{1}{2}\Omega(z) e^{ik_L x}\tilde{\phi}_g \ , \tag{14}$$

with the Rabi frequency $\Omega(z) = \Omega_0 e^{-(z-z_0)^2/W_0^2}$; $\Omega_0 = 2|d\mathcal{E}_0|$; $z = z_0$, the beam axis; and $\delta_L = \omega_L - \omega_0$, the detuning. Equation (13) has the form of a Schrödinger equation for a two-level system with the z coordinate playing the role of an "effective interaction time" z/v_E. This equation has been the starting point of essentially all previous theoretical work on coherent atomic-beam scattering [2,3].

Simple analytical and qualitatively correct solutions are obtained by approximating the z dependence of the Rabi frequency by the step function

$$\Omega(z) \rightarrow \begin{cases} \Omega_0 & \text{if } |z - z_0| < \sqrt{\pi} W_0 \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

Imposing the boundary condition

$$\tilde{\phi}_g(x,y,0) = f(x,y) \ , \quad \tilde{\phi}_e(x,y,0) = 0 \ , \tag{16}$$

we see that the solution of Eqs. (13) and (14) is

$$\begin{aligned} &\tilde{\phi}_g(x,y,z_0 + \tfrac{1}{2}\sqrt{\pi}W_0) = C_g f(x,y), \\ &\tilde{\phi}_e(x,y,z_0 + \tfrac{1}{2}\sqrt{\pi}W_0) = C_e f(x,y) e^{ik_L x} \end{aligned} \tag{17}$$

with

$$\begin{aligned} C_g &= \frac{1}{\sqrt{v_E}} e^{i\delta_L \sqrt{\pi} W_0/(2v_E)} \left[\cos\left[\frac{\sqrt{\pi}W_0}{2v_E} (\delta_L^2 + \Omega_0^2)^{1/2} \right] - \frac{i\delta_L}{(\delta_L^2 + \Omega_0^2)^{1/2}} \sin\left[\frac{\sqrt{\pi}W_0}{2v_e} (\delta_L^2 + \Omega_0^2)^{1/2} \right] \right] \\ C_e &= \frac{1}{\sqrt{v_E}} \frac{i\Omega_0}{(\delta_L^2 + \Omega_0^2)^{1/2}} e^{i\delta_L \sqrt{\pi} W_0/(2v_E)} \sin\left[\frac{\sqrt{\pi}W_0}{2v_E} (\delta_L^2 + \Omega_0^2)^{1/2} \right] . \end{aligned} \tag{18}$$

To summarize, the atomic beam, immediately behind the exciting laser field, is described by the statistical operator

$$\begin{aligned} \langle z_0 + \tfrac{1}{2}\sqrt{\pi}W_0 | \rho^{(0)}(t) | z_0 + \tfrac{1}{2}\sqrt{\pi}W_0 \rangle &= \int dE\ W(E) \langle z_0 + \tfrac{1}{2}\sqrt{\pi}W_0 | \psi_{E_0}(t) \rangle \\ &\quad \times \langle \psi_{E_0}(t) | z_0 + \tfrac{1}{2}\sqrt{\pi}W_0 \rangle \ , \end{aligned} \tag{19}$$

with

$$\begin{aligned} \langle z_0 + \tfrac{1}{2}\sqrt{\pi}W_0 | \psi_{E_0}(t) \rangle &= e^{-(i/\hbar)Et} \int d^2x \left(C_g |g\rangle + e^{ik_L x - i\omega_L t} C_e |e\rangle \right) \\ &\quad \otimes f(x,y) |x,y\rangle \otimes |0\rangle \ , \end{aligned} \tag{20}$$

where C_g and C_e are given by Eqs. (18), and $f(x,y)$ is the slit function. Thus the atom is prepared in a linear superposition of the ground and excited states.

B. $J = 0 \leftrightarrow J = 1$ transition excited by $\sigma_+ \leftrightarrow \sigma_-$ standing wave

We now shall consider a three-level atom in a V configuration interacting with a standing-wave field as shown in Fig. 1. Due to angular momentum conservation the σ_+ and σ_- components of the exciting laser beam separately drive the $m = 0 \leftrightarrow m = 1$ and $m = 0 \leftrightarrow m = -1$ transitions, respectively, and this excitation scheme reduces to that of two independent two-level systems with a common ground state. Thus the analysis of Sec. II A can be used with only a few changes and we can therefore be brief.

The changes in the Hamiltonian affect the free atomic Hamiltonian for the internal degrees of freedom, which now reads $H_A = \hbar\omega_0 (|e_-\rangle\langle e_-| + |e_+\rangle\langle e_+|)$, where $|e_\pm\rangle$ denote the degenerate $m = \pm 1$ sublevels of the excited state (we drop the state $|e_{m=0}\rangle$ which does not couple in the present laser configuration). The positive-frequency part of the atomic dipole operator density now reads

$$\mathbf{D}^{(+)}(\mathbf{x}) = d(\boldsymbol{\epsilon}_+ |g\rangle\langle e_+| + \boldsymbol{\epsilon}_- |g\rangle\langle e_-|) \otimes |\mathbf{x}\rangle\langle \mathbf{x}| \ . \tag{21}$$

Here $\epsilon_\pm$ are the complex unit vectors describing right and left circular polarization. We shall take the quantization axis in the x direction. The exciting laser mode with σ_+ and $\sigma-$ counterpropagating components has a positive-frequency part of the electric field of the form

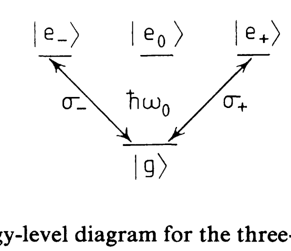

FIG. 1. Energy-level diagram for the three-level system.

$$\mathcal{E}_C^{(+)}(\mathbf{x},t)=\mathcal{E}_+^{(+)}(\mathbf{x},t)+\mathcal{E}_-^{(+)}(\mathbf{x},t)\ , \tag{22}$$

with

$$\mathcal{E}_\pm^{(+)}(\mathbf{x},t)=\frac{\epsilon_\pm}{\sqrt{2}}\mathcal{E}_G^{(+)}(\mathbf{x})e^{\pm ik_Lx-i\omega_Lt}\ . \tag{23}$$

We can now repeat the analysis of Sec. II A and find that the three-level problem reduces to a two-level problem between the levels $|g\rangle$ and $|e_1\rangle=2^{-1/2}(|e_+\rangle e^{ik_Lx}+|e_-\rangle e^{-ik_Lx})$. The result for the statistical operator of the atomic beam, immediately behind the exciting laser beam (i.e., for $z=z_0+\sqrt{\pi}W_0/2$), is therefore given by Eq. (19) with

$$\langle z_0+\tfrac{1}{2}\sqrt{\pi}W_0|\psi_{E_0}(t)\rangle=e^{-(i/\hbar)Et}\int d^2x\left[C_g|g\rangle+\frac{C_e}{\sqrt{2}}e^{-i\omega_Lt}(|e_+\rangle e^{ik_Lx}+|e_-\rangle e^{-ik_Lx})\right]\otimes f(x,y)|x,y\rangle\otimes|0\rangle \tag{24}$$

where C_g and C_e are still given by Eqs. (18). Thus the atom is now prepared in a linear superposition of the ground state and two excited states.

III. PROPAGATION OF THE SUPERPOSITION OF WAVE PACKETS

After passing the laser beam the atoms are left in linear superpositions of excited states and the ground state, traveling as free wave packets with differing mean center-of-mass velocities, and spreading in directions transverse to the atomic beam. We shall calculate the propagation of the wave function below.

A. Two-level atoms

The wave function in the region $z>z_0+(\sqrt{\pi}/2)W_0=\bar{z}_0$ is still of the form

$$|\psi_{E_0}(t)\rangle=e^{-(i/\hbar)Et}|0\rangle\otimes\int d^3x\, e^{i(\sqrt{2mE}/\hbar)z}[\tilde{\phi}_g(\mathbf{x})|g\rangle+e^{-i\omega_Lt}\tilde{\phi}_e(\mathbf{x})|e\rangle]\otimes|\mathbf{x}\rangle\ , \tag{25}$$

where $\tilde{\phi}_g$ and $\tilde{\phi}_e$ satisfy

$$i\hbar v_E\frac{\partial}{\partial z}\tilde{\phi}_g=-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\tilde{\phi}_g\ , \tag{26}$$

$$i\hbar v_E\frac{\partial}{\partial z}\tilde{\phi}_e=\left[-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]-\hbar\left[\delta_L+i\frac{\gamma}{2}\right]\right]\tilde{\phi}_e\ , \tag{27}$$

with the boundary conditions

$$\tilde{\phi}_g(z=\bar{z}_0)=C_gf(x,y)\ ,\qquad \tilde{\phi}_e(z=\bar{z}_0)=C_ee^{ik_Lx}f(x,y)\ . \tag{28}$$

For simplicity we now redefine z so that $\bar{z}_0=0$.

The solutions of Eqs. (26) and (27) are

$$\tilde{\phi}_g(\mathbf{x})=C_gf_E(x,y,z)\ ,\qquad \tilde{\phi}_e(\mathbf{x})=C_ee^{i(\delta_L'+i\gamma)z/v_E+ik_Lx}f_E\left[x-\frac{\hbar k_Lz}{mv_E},y,z\right]\ , \tag{29}$$

where

$$\delta_L'=\omega_L-\omega_0-\frac{\hbar k_L^2}{2m} \tag{30}$$

and

$$f_E(x,y,z)=\frac{-imv_E}{(\pi L^2)^{1/2}2\pi\hbar z}\int_{x'^2+y'^2<L^2}dx'dy'e^{(imv_E/2\hbar z)[(x-x')^2+(y-y')^2]}\ . \tag{31}$$

It follows from Eq. (31) that

$$f_E(x,y,z)\simeq\begin{cases}f(x,y) & \text{if } z\ll L^2mv_E/\hbar\\[4pt] \dfrac{-i}{2\pi(\pi L^2)^{1/2}}\displaystyle\int_0^{[(mv_E/\hbar z)^{1/2}]L}\xi\mathcal{J}_0\left[\left[\frac{mv_E}{\hbar z}\right]^{1/2}(x^2+y^2)^{1/2}\xi\right]d\xi & \text{if } z\gg L^2mv_E/\hbar\ ,\end{cases} \tag{32}$$

where we assumed a circular hole $x^2+y^2\le L^2$. In the following we shall only be concerned with the region $z\ll L^2mv_E/\hbar$. Thus the ground and excited states propagate with different k vectors as shown in Fig. 2.

B. $J=0\leftrightarrow J=1$ superposition

The wave function in the region $z>z_0+(\sqrt{\pi}/2)W_0$ (or, after redefinition of z, $z>0$) can be written in the form

$$|\psi_{E_0}(t)\rangle=e^{-(i/\hbar)Et}\int d^3x\, e^{i(\sqrt{2mE}/\hbar)z}\left[\tilde{\phi}_g(\mathbf{x})|g\rangle+\frac{1}{\sqrt{2}}e^{-i\omega_Lt}[\tilde{\phi}_+(\mathbf{x})|e_+\rangle+\tilde{\phi}_-(\mathbf{x})|e_-\rangle]\right]\otimes|\mathbf{x}\rangle\otimes|0\rangle\ , \tag{33}$$

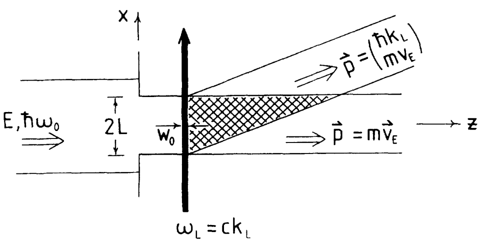

FIG. 2. Scattering of a two-level atom from a traveling-wave laser field. (Note the width of the laser is not drawn to scale; actually $W_0 \gg 2L$.) The shaded area represents the overlap region.

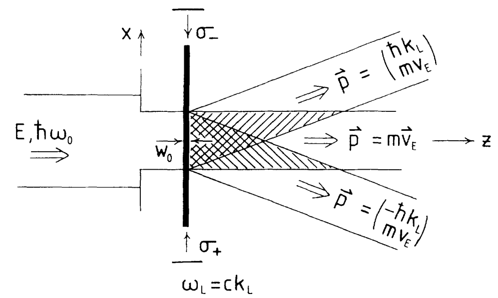

FIG. 3. Scattering of a three-level atom from a standing-wave laser field with $\sigma_+ - \sigma_-$ counterpropagating (again the width of the laser is not drawn to scale; actually $W_0 \gg 2L$). The shaded area represents the overlap region.

where $\tilde{\phi}_g(\mathbf{x})$ and $\tilde{\phi}_\pm(\mathbf{x})$ are determined as for the two-level atom in Sec. IV A. We find, using the notation of Sec. IV A

$$\tilde{\phi}_g(\mathbf{x}) = C_g f_E(x,y,z) \,,$$
$$\tilde{\phi}_\pm(\mathbf{x}) = \frac{C_e}{\sqrt{2}} e^{i(\delta'_L + i\gamma)z/v_E \pm ik_L x} f_E\left[x \mp \frac{\hbar k_L z}{m v_E}, y, z\right] . \tag{34}$$

In this case the atom propagates with the different k vectors for the ground state and two excited states as shown in Fig. 3.

IV. COHERENT SPONTANEOUS EMISSION

A. General expressions

We now wish to evaluate the coherent electromagnetic radiation spontaneously emitted by the atoms after their coherent excitation. To this end we consider the expectation value of the positive-frequency part of the emitted quantized electromagnetic field

$$\langle \mathcal{E}_R^{(+)}(\mathbf{x},t)\rangle = i \sum_\lambda \int d^3k \left[\frac{\hbar\omega}{4\pi^2}\right]^{1/2} e^{i\mathbf{k}\cdot\mathbf{x}} \epsilon_{\mathbf{k}\lambda} \langle b_{\mathbf{k}\lambda}\rangle e^{-i\omega_L t} . \tag{35}$$

The equation of motion of $\mathcal{E}_R^{(+)}(\mathbf{x},t), \mathcal{E}_R^{(-)}(\mathbf{x},t)$ in the Heisenberg picture follows from the Hamiltonian (1). It can be put into the form of the inhomogeneous wave equation

$$\left[\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right] \mathcal{E}_R^{(\pm)}(\mathbf{x},t) = -4\pi \frac{\partial^2}{\partial t^2} \mathbf{D}_\perp^{(\pm)}(\mathbf{x},t) \,, \tag{36}$$

where $\mathbf{D}_\perp^{(+)}(\mathbf{x})$ is the transverse projection of the positive frequency part $\mathbf{D}^{(+)}(\mathbf{x},t)$ of the dipole operator density $\mathbf{D}(\mathbf{x},t)$ in the Heisenberg picture. The transverse projection of a vector field $\mathbf{A}(x)$ is defined by

$$\mathbf{A}_\perp(\mathbf{x},t) = \mathbf{A}(\mathbf{x},t) - \frac{1}{4\pi}\nabla \int \frac{\nabla\cdot\mathbf{A}(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|} d^3x' \tag{37}$$

and satisfies $\nabla \mathbf{A}_\perp = 0$, $\nabla\times\mathbf{A}_\perp = \nabla\times\mathbf{A}$. We note that the operations $[A(\mathbf{x})]_\perp$ and $\int d^3x' |\mathbf{x}-\mathbf{x}'|^{-1}\mathbf{A}(\mathbf{x}')$ commute. The emitted quantum field is given by the retarded solution of Eq. (36) which reads

$$\mathcal{E}_R^{(\pm)}(\mathbf{x},t) = -\frac{1}{c^2}\int d^3x' \frac{\frac{\partial^2}{\partial t^2}\mathbf{D}_\perp^{(+)}\left[\mathbf{x}', t - \frac{|x-x'|}{c}\right]}{|\mathbf{x}-\mathbf{x}'|} . \tag{38}$$

Taking expectation values and evaluating the retarded time-dependent expectation value on the right-hand side in the Schrödinger picture using the density matrix (19) and (20) we obtain

$$\langle \mathcal{E}_R^{(\pm)}(\mathbf{x},t)\rangle = -\frac{1}{c^2}\int dE\, W(E) \int d^3x' \frac{1}{|\mathbf{x}-\mathbf{x}'|} \frac{\partial^2}{\partial t^2} \left\langle \psi_{E_0}\left[t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right] \middle| \mathbf{D}_\perp^{(\pm)}(\mathbf{x}') \middle| \psi_{E_0}\left[t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right]\right\rangle . \tag{39}$$

B. Two-level system

Inserting the wave function (25) and the dipole density (3) in the general expression (39) we find for the positive-frequency part

$$\langle \mathcal{E}_R^{(+)}(\mathbf{x},t)\rangle = k_L^2 d e^{-i\omega_L t} \int dE\, W(E) C_g^* C_e \int d^3x' \frac{1}{|\mathbf{x}-\mathbf{x}'|} \times \left[\epsilon_y e^{ik_L(|\mathbf{x}-\mathbf{x}'|+x') + i(\delta'_L + i\gamma/2)z'/v_E} f_E^*(\mathbf{x}') f_E\left[\mathbf{x}' - \frac{\hbar k_L z'}{m v_e}\mathbf{e}_x\right]\right]_\perp . \tag{40}$$

It is clear from this result that only the spatially overlapping parts of the wave packets $f_E(\mathbf{x})$ and $f_E(\mathbf{x}-(\hbar k_L z/mv_E)\mathbf{e}_x)$ of the ground state and the excited state contribute to this coherently radiating dipole density. As soon as the two components of the center-of-mass wave function are completely separated spatially, the coherent emission stops and the excited state can only decay via incoherent emission. Indeed, after complete spatial separation, the two linearly superposed components of the atomic beam are nonoverlapping both in center-of-mass coordinate and momentum and behave indistinguishably from two mutually independent and incoherent atomic beams of corresponding intensity. However, if the two separated components of the beam are somehow brought to overlap again, as is done in an atomic interferometer, interference effects and coherent emission will reappear, provided the decay due to spontaneous emission is not yet completed, i.e., $\gamma x'/2v_E$ is not too large.

The remaining task of this subsection is the evaluation of the integral (40). For simplicity we consider only a limiting case defined by the following conditions: (i) $k_L L \gg 1$ (the hole is large compared to the laser wavelength), (ii) $z' \ll L^2/\lambda_E$ (only the near zone of the hole is considered where diffraction of the atomic beam is negligible), (iii) $x \gg L, (x^2+y^2)^{1/2} \gg (k_L L)L$ (the radiation field is only considered at large distances from the atomic beam), (iv) furthermore, we consider a square hole $|x|<L, |y|<L$. As we show in Appendix A, we obtain as our final result for $\langle \mathcal{E}_R^{(+)} \rangle$

$$
\begin{aligned}
\langle \mathcal{E}_R^{(+)}(\mathbf{x},t)\rangle = {} & \frac{(2\pi)^{1/2}}{L^2}\frac{d}{\sqrt{k_L}}e^{-i\omega_L t + i\pi/4}\int dE\, W(E) C_g^* C_e \\
& \times \left\{ \epsilon_y \frac{(x^2+y^2)^{3/4}}{(\cos\alpha)^{1/2} y[(x^2+y^2)^{1/2}-x\cos\alpha]} \right. \\
& \quad \times \exp\left[ik_L \left[\cos\alpha - \frac{\hbar k_L}{2mv_E}\tan\alpha \right](x^2+y^2)^{1/2} + ik_L z\sin\alpha \right] \\
& \quad \times \exp\left[ik_L x \frac{\hbar k_L}{2mv_E}\sin\alpha + i\frac{\hbar k_L^2}{2mv_E} z \right] \exp\left[-ik_L\frac{\hbar k_L}{2mv_E}\frac{xz}{(x^2+y^2)^{1/2}}\cos\alpha \right] \\
& \quad \times \exp\left[-\frac{\gamma}{2v_E}[z-(x^2+y^2)^{1/2}\tan\alpha] \right] \sin\left[k_L L \frac{y\cos\alpha}{(x^2+y^2)^{1/2}} \right] \\
& \quad \times \sin\left[k_L L \left[1 - \frac{x\cos\alpha}{(x^2+y^2)^{1/2}} \right] \left[1 - \frac{\hbar k_L (z-(x^2+y^2)^{1/2}\tan\alpha)}{2mv_E L} \right] \right] \\
& \quad \left. \times \Theta\left[2L - \frac{\hbar k_L}{mv_E}(z-(x^2+y^2)^{1/2}\tan\alpha) \right] \Theta\left[z-(x^2+y^2)^{1/2}\tan\alpha \right] \right\}_\perp ,
\end{aligned}
\tag{41}
$$

where we defined the angle α by

$$
\sin\alpha = \frac{\delta_L'}{k_L v_E} , \tag{42}
$$

$(\pi/2-\alpha)$ is the mean angle between the z axis and the mean direction of propagation of the coherently emitted field as shown in Fig. 4; and Θ is the step function. This expression is complicated because of the somewhat complex geometry of the system which lacks simple symmetries.

A major source of complication is the presence of detuning $\delta_L' \neq 0$ giving rise to $\alpha \neq 0$. Physically this is easily understood: the coherent emission from a stationary beam of atoms must occur with the frequency ω_L of the driving laser frequency. On the other hand, the excited-state wave packet of the atom in the laboratory frame differs in energy from the ground-state wave packet by $\hbar\left[\omega_0+(\hbar k_L^2/2m)\right]$, which is different from $\hbar\omega_L$ if $\delta_L'=0$. In order to make up for this difference the atoms have to make use of a Doppler shift by emitting at an angle $\alpha>0$ (blueshift) if $\omega_L > \omega_0 + \hbar k_L^2/2m$ or $\alpha<0$ (red-

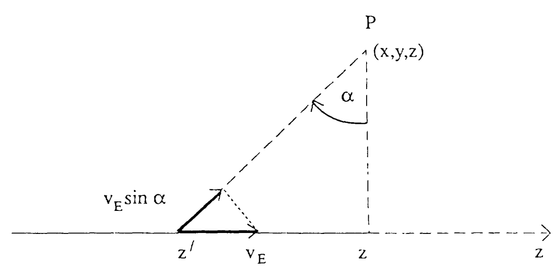

FIG. 4. V_E is the velocity of the atom in the z direction. The component of the velocity in the direction of the coherent radiation is $V_E\sin\alpha$. P is the position of the detector. The frequency of the light emitted by the atom in its rest frame is ω_0. However, due to the Doppler shift, the frequency seen by the vector is the laser frequency W_L.

shift) if $\delta_L' < 0$ (see Fig. 4). Indeed this simple physical consideration leads to Eqs. (A6) and (42) relating the positions where the coherent radiation is emitted and received, and it can be seen from Eq. (41) that, apart from corrections of order $(\hbar k_L / m v_E)$, the field is emitted along a cone of angle $(\pi/2-\alpha)$ around the z axis as shown in Fig. 5.

A major feature of the result (41) is the fact that the coherent radiation can only be observed in a certain region of space given by

$$(x^2+y^2)^{1/2}\tan\alpha < z < (x^2+y^2)^{1/2}\tan\alpha + \frac{2 m v_E L}{\hbar k_L} \tag{43}$$

and $x>0$, that is, in the upper half cone of angle $(\pi/2-\alpha)$ around the z axis whose vertex lies at the point where the wave packets separate. For $\alpha<0$ the lower boundary for z should be replaced by 0.

Having discussed the effects of detuning $\delta_L' \neq 0$, we may now restrict our attention to the case $\delta_L'=0$, $\alpha=0$, where Eq. (41) reduces to

$$\begin{aligned}\langle \mathcal{E}_R^{(+)}(\mathbf{x},t)\rangle = {} & \frac{(2\pi)^{1/2}}{L^2}\frac{d}{\sqrt{k_L}} e^{-i\omega_L t + i\pi/4}\int dE\, W(E) C_g^* C_e \\ & \times \left\{ \epsilon_y \frac{(x^2+y^2)^{3/4}}{y[(x^2+y^2)^{1/2}-x]} e^{ik_L[(x^2+y^2)^{1/2}+(\hbar k_L/2mv_E)z]} \right. \\ & \times \exp\left[-ik_L \frac{\hbar k_L}{2mv_E}\frac{xz}{(x^2+y^2)^{1/2}} - \frac{\gamma}{2v_E}z\right] \sin\left[k_L L \frac{y}{(x^2+y^2)^{1/2}}\right] \\ & \left. \times \sin\left[k_L L \left(1-\frac{x}{(x^2+y^2)^{1/2}}\right)\left(1-\frac{\hbar k_L z}{2mv_E L}\right)\right] \Theta\left[2L - \frac{\hbar k_L}{m v_E} z\right] \right\}_{\perp} . \end{aligned} \tag{44}$$

Because $k_L L \gg 1$, the radiation is emitted into a region $y/(x^2+y^2)^{1/2} \sim 1/(k_L L) \ll 1$. The strongest radiation is received in the (x,z) plane, $y=0$. In that case Eq. (44) reduces to

$$\begin{aligned}\langle \mathcal{E}_R^{(+)}(x,0,z)\rangle = {} & \epsilon_y \frac{(2\pi)^{1/2}}{|x|^{1/2}} d k_L^{3/2} e^{-i\omega_L t + i\pi/4}\int dE\, W(E) C_g^* C_e \Theta\left[1-\frac{\hbar k_L z}{2mv_E L}\right] e^{-(\gamma/2v_E)z} \\ & \times \left[\left(1-\frac{\hbar k_L z}{2mv_E L}\right) e^{ik_L x}\Theta(x) + e^{-ik_L[x-(\hbar k_L/mv_E)z]} \right. \\ & \left. \times \frac{\sin\left[2k_L L\left(1-\dfrac{\hbar k_L z}{2mv_E L}\right)\right]}{2k_L L}\Theta(-x) \right] . \end{aligned} \tag{45}$$

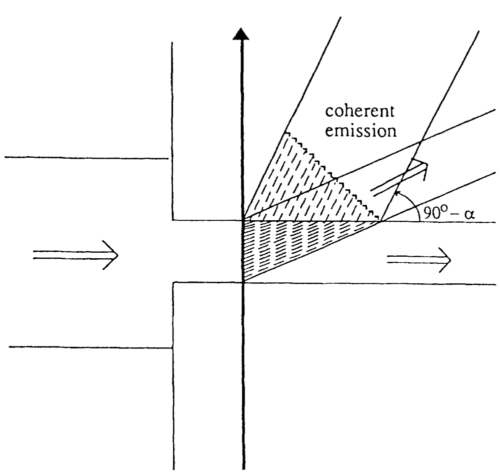

FIG. 5. The coherent radiation emitted from the two-level atom is depicted. The lengths of the arrows represent the amplitude of the radiation.

This expression shows that the amplitude of the radiation goes to zero continuously as one approaches the border of the region where it is no longer seen due to the separation of the wave packets. It also shows a pronounced asymmetry between the radiation emitted in the upward x direction (the direction of propagation of the exciting laser field) and the radiation emitted in $(-x)$ direction which is strongly suppressed [on the order of $(k_L L)^{-1}$] except in a domain of thickness of the order of $(m v_E/\hbar k_L^2)$ near the boundary of the region where the coherent radiation disappears. Within that domain, the radiation emitted in the upward and downward directions is of about the same magnitude.

C. $J=0 \leftrightarrow J=1$ superposition

The calculation presented in Sec. IV B can be repeated in a similar fashion for the two-level transitions $|g\rangle \leftrightarrow |e_+\rangle$, $|g\rangle \leftrightarrow |e_-\rangle$. The interesting feature is that the coherent spontaneous emission observable in the cone

$$(x^2+y^2)^{1/2}\tan\alpha < z < (x^2+y^2)^{1/2}\tan\alpha + \frac{2mv_E L}{\hbar k_L} \quad (46)$$

consists now of photons in linear-superposition states of opposite circular polarization traveling in different directions as shown in Fig. 6.

As the effects of detuning and the preferential emission at small angles [of order $(k_L L)^{-1}$] against the (x,z) plane are exactly the same as before, we only state the result for $y=0$ and $\alpha=0$:

$$\langle \mathcal{E}^{(+)}(x,0,z)\rangle = \frac{(2\pi)^{1/2}}{|x|^{1/2}} dk_L^{3/2} e^{-i\omega_L t + ik_L|x| + i\pi/4} \int dE\, W(E) C_g^* C_e \Theta\left[1-\frac{\hbar k_L z}{2mv_E L}\right] e^{-(\gamma/2v_E)z}$$
$$\times \left[\left[1-\frac{\hbar k_L z}{2mv_E L}\right] \frac{1}{\sqrt{2}}[\epsilon_+\Theta(x)+\epsilon_-\Theta(-x)] + e^{ik_L(\hbar k_L/mv_E)z} \frac{\sin\left[2k_L L\left[1-\frac{\hbar k_L z}{2mv_E L}\right]\right]}{2k_L L} \times \frac{1}{\sqrt{2}}[\epsilon_+\Theta(x)+\epsilon_-\Theta(-x)] \right] . \quad (47)$$

Thus in the (x,z) plane, except in a domain of thickness $\Delta z \sim (mv_E/\hbar k_L^2)$ near the boundary of the region in Eq. (46) the coherent emission in that region consists of linear superposition states

$$\langle \mathcal{E}^{(+)}(x,0,z)\rangle \sim \frac{1}{\sqrt{2}|x|}[\epsilon_+\Theta(x)+\epsilon_-\Theta(-x)] \times e^{-i\omega_L t + ik_L|x|} , \quad (48)$$

as is seen in Fig. 6 for the case of $\alpha > 0$.

D. Homodyne detection of coherent spontaneous emission

The coherent emission from the atomic beam may be detected via homodyning the emitted field with the field of a local classical oscillator of the same frequency which has a well-defined phase ϕ with respect to the driving laser field. Most conveniently the local oscillator is derived directly from the driving laser. The photocurrent generated in the detector is then proportional to

$$I \sim |\mathbf{E}_{\mathrm{LO}}|^2 + \mathbf{E}_{\mathrm{LO}}\langle \mathcal{E}_R^{(+)}\rangle e^{-i\phi} + \langle \mathcal{E}_R^{(-)}\rangle \mathbf{E}_{\mathrm{LO}} e^{i\phi} , \quad (49)$$

where $\mathbf{E}_{\mathrm{LO}}e^{i\phi}$ is the amplitude of the positive-frequency part of the local-oscillator electric field. Subtracting out the contribution of the local oscillator alone, which is done automatically in a balanced homodyne scheme, the photocurrent becomes a direct measure of the desired expectation value. [By varying the phase ϕ of the local oscillator with respect to the driving laser, the contributions from the positive- and the negative-frequency parts $\langle \mathcal{E}^{(\pm)}\rangle$ can be distinguished.]

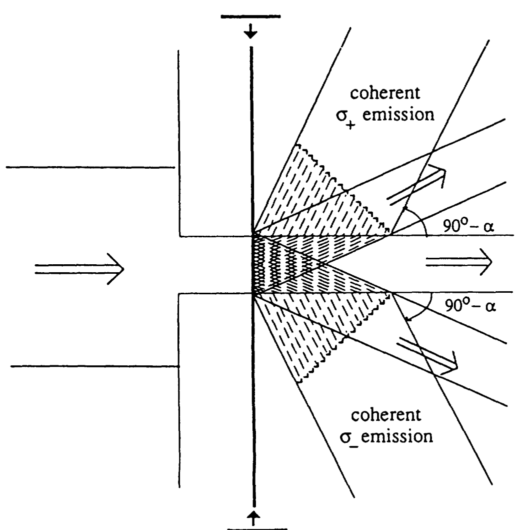

FIG. 6. The coherent radiation from the three-level atom is depicted. The lengths of the arrows represent the amplitude of the radiation of the two components corresponding to σ_+ and σ_- polarizations.

V. DETECTION OF THE SUPERPOSITION BY ABSORPTION

Instead of observing the coherent spontaneous emission one can observe the overlap of the wave packets and its disappearance by measuring the absorption rate of energy from a weak probe beam $[\mathbf{E}_p^{(+)}(\mathbf{x})e^{-i\omega_L t - i\Delta\omega t + i\mathbf{k}\cdot\mathbf{x}} + \mathrm{c.c.}]$ at the frequency $\omega_L+\Delta\omega$ passed through the wave packet. The probe field is assumed to have a fixed phase relation to the exciting laser field. The energy-absorption rate is given by

$$W = \int d^3x \left[\mathbf{E}_p^{(-)}(\mathbf{x})\cdot\frac{\partial}{\partial t}\langle \mathbf{D}^{(+)}(\mathbf{x},t)\rangle e^{-i\mathbf{k}\cdot\mathbf{x}+i\Delta\omega t+i\omega_L t} + \mathrm{c.c.} \right] \quad (50)$$

and oscillates with the frequency $\Delta\omega$. By this oscillation and its proportionality to the probe-beam amplitude, this rate can be distinguished from the incoherent absorption rate. Evaluating the expectation value using the results of Secs. IV A and IV B, we obtain for the two-level atom

$$W=-ick_L d\int dE\, W(E)\left[C_g^* C_e e^{-i\Delta\omega t}\int d^3x\, E_{py}^{(-)}(\mathbf{x})e^{ik_L x-i\mathbf{k}\cdot\mathbf{x}+i(\delta_L'+i\gamma/2)z/v_E} f_E^*(\mathbf{x})f_E\left[\mathbf{x}-\frac{\hbar k_L z}{mv_E}\mathbf{e}_x\right]-\text{c.c.}\right]. \quad (51)$$

We shall assume that the probe beam has constant amplitude E_{py} over the atomic beam in the x and y directions and illuminates a very small section of the atomic beam in the z direction.

Introducing the Fourier transform of the overlap

$$g_E(\mathbf{k},z_p)=\int d^3x\, f_E^*(\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}} f_E\left[\mathbf{x}-\frac{\hbar k_L z}{mv_E}\mathbf{e}_x\right]\times\frac{1}{\sqrt{\pi}W_P}e^{-(\gamma/2v_E)z}e^{-(z-z_p)^2/W_p^2}, \quad (52)$$

where the normalized Gaussian factor describes the profile of the probe beam, we may rewrite Eq. (51) as

$$W=\frac{\hbar c k_L}{2i}\Omega_p\int dE\, W(E)\left[C_g^* C_e e^{-i\Delta\omega t}\times g_E\left[\mathbf{k}-\mathbf{e}_x k_L-\mathbf{e}_z\frac{\delta_L'}{v_E},z_p\right]-\text{c.c.}\right], \quad (53)$$

where $\Omega_p=(dE_p/2\hbar)$ is the Rabi frequency of the probe beam. We note that W, for sufficiently small E_p, is linear in E_p.

Using the wave packets in the limit (A1) and assuming, for convenience, that the width W_p of the probe beam in the z direction satisfies $W_p \ll v_E/\gamma, mv_E/\hbar k_L k_x, (1/k_z)e$, we have explicitly, for $0\le z_p \le 2mv_E/\hbar k_L$,

$$g_e(\mathbf{k},z_p)=\frac{\sin k_y L}{k_y L}e^{i(\hbar k_x k_z z_p)/2mv_E}\times\frac{\sin k_x\left[L-\dfrac{\hbar k_z z_p}{2mv_E}\right]}{k_x L}e^{-(\gamma/2v_E)z_0}. \quad (54)$$

For z_p outside the overlap interval $g_E(\mathbf{k},z_p)$ vanishes, i.e., the absorption rate W is a very good indicator for the overlap of the wave packets. We again find a pronounced directional characteristic, because $g_E(\mathbf{k},z_p)$ is small unless $|k_y L|\le 1$, $|k_x[L-\hbar k_z z_p/(2mv_E)]|\le 1$. As long as $0<z_p\ll 2mv_E L/\hbar k_z$, this condition implies that W remains small unless the wave vector $\mathbf{k}$ of the probe beam satisfies $|k_y L|\le 1$, $|(k_x-k_y)L|\le 1$, $k_z L\le\sqrt{2k_L L}$, where the third inequality follows from $k_x^2+k_y^2+k_z^2=k_L^2$. The maximum absorption rate is obtained for $k_x=k_L$, $k_y=0$, $k_z=\delta_L'/v_E$ and is given by

$$W_{\max}=\tfrac{1}{2}\hbar c k_L\Omega_p\int dE\, W(E)\left[1-\frac{\hbar k_z z_p}{2mv_E L}\right]\times e^{-(\gamma/2v_E)z_p}(C_g^* C_e e^{-i\Delta\omega t}+\text{c.c.}). \quad (55)$$

For the $J=0\leftrightarrow J=1$ transition, very similar results are obtained. Even more crucially than in the preceding example, the absorption rate depends on the polarization of the probe beam. A probe beam that is circularly polarized with respect to the z direction predominantly couples only to the upward or downward moving component of the excited-state superposition. A beam linearly polarized in the z or y direction couples to both components, and we shall therefore assume polarization in y direction. Equation (53) is then generalized and now reads

$$W=\frac{\hbar c k_L}{2i}\Omega_p\int dE\, W(E)\left\{\frac{1}{\sqrt{2}}C_g^* C_e e^{-i\Delta\omega t}\left[g_{E_+}\left[\mathbf{k}-\mathbf{e}_x k_L-\mathbf{e}_z\frac{\delta_L'}{v_E},z_p\right]+g_{E_-}\left[\mathbf{k}+\mathbf{e}_x k_L-\mathbf{e}_z\frac{\delta_L'}{v_E},z_p\right]\right]+\text{c.c.}\right\}, \quad (56)$$

with

$$g_{E_\pm}(\mathbf{k},z_p)=\int d^3x\, f_E^*(\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}} f_E\left[\mathbf{x}\mp\frac{\hbar k_L z}{mv_E}\mathbf{e}_x\right]\frac{1}{\sqrt{\pi}W_p}e^{-(\gamma/2v_E)z}e^{-(z-z_p)^2/W_p^2}. \quad (57)$$

The conclusions concerning the importance of the overlap of the wave packets of the ground state and the excited states, and of the directional characteristics of the probe beam for a nonvanishing absorption rate $W\neq 0$ remain the same, except that now k_x may have a positive and negative sign, $|(k_x\pm k_L)L|\le 1$.

VI. INCOHERENT SPONTANEOUS EMISSION

A. General expressions

For the sake of completeness we also consider the incoherent spontaneous emission from the atomic beam after passing the exciting laser beam. This is most conveniently represented by the correlation function

$$G_{ij}(\mathbf{x},\mathbf{x}';t-t')=\langle \mathcal{E}^{\dagger}_{R,i}(\mathbf{x},t)\mathcal{E}_{R,j}(\mathbf{x}',t')\rangle \tag{58}$$

(i,j refer to polarization indices), which in the Heisenberg picture can be written as

$$G_{ij}(\mathbf{x},\mathbf{x}';t-t')=k_L^4\int dE\, W(E)\int d^3\bar{x}\int d^3\bar{x}'\frac{1}{|\mathbf{x}-\bar{\mathbf{x}}||\mathbf{x}'-\bar{\mathbf{x}}'|}\left\langle\psi_{E_0}\left|\mathbf{D}_{\perp i}^{(-)}\left[\bar{\mathbf{x}},t-\frac{|\mathbf{x}-\bar{\mathbf{x}}|}{c}\right]\right.\right.$$
$$\left.\left.\times\mathbf{D}_{\perp j}^{(+)}\left[\bar{\mathbf{x}}',t'-\frac{|\mathbf{x}'-\bar{\mathbf{x}}'|}{c}\right]\right|\psi_{E_0}\right\rangle . \tag{59}$$

Inserting a complete set of plane-wave states $|\mathbf{p}\rangle$ with momentum $\mathbf{p}$ and turning to the Schrödinger picture, we can rewrite Eq. (59) in the form

$$G_{ij}(\mathbf{x},\mathbf{x}';t-t')=\int dE\, W(E)G_{ij}(\mathbf{x},\mathbf{x}';t-t';E)\ , \tag{60}$$
$$G_{ij}(\mathbf{x},\mathbf{x}';t-t';E)=\frac{1}{(2\pi\hbar)^3}\int d^3p\, F_i^*(\mathbf{p},\mathbf{x},E)F_j(\mathbf{p},\mathbf{x}',E)\times e^{i[\omega_L+E/\hbar-p^2/(2m\hbar)](t-t')}\ ,$$

$$F_i(\mathbf{p},\mathbf{x},E)=k_L^2\int d^3x'\frac{e^{i[\omega_L+E/\hbar-p^2/(2m\hbar)]|\mathbf{x}-\mathbf{x}'|/c}}{|\mathbf{x}-\mathbf{x}'|}\times\langle g|\otimes\langle\mathbf{p}|\mathbf{D}_{\perp i}^{(+)}(\mathbf{x}')|\psi_{E_0}(0)\rangle\ , \tag{61}$$

where we made use of the time dependence of the wave function in the explicit form given by Eqs. (25) and (33).

Thus the emitted field is an incoherent mixture of the amplitudes $F_i(\mathbf{p},\mathbf{x},E)$ for different $\mathbf{p}$ and E, oscillating with frequencies $\omega_E+[E-(\hbar^2p^2/2m)]/\hbar$, respectively. The fact that the atomic beam is in a linear superposition state with different center-of-mass components attached to the excited state and the ground state has no influence on the amplitudes $F_i(\mathbf{p},\mathbf{x},E)$ and the correlation function G_{ij}. The spectrum of incoherent spontaneous emission

$$S_{ij}(\mathbf{x},\mathbf{x}';\omega)=\int dt\, e^{-i\omega t}\int dE\, W(E)G_{ij}(\mathbf{x},\mathbf{x}';t;E) \tag{62}$$

is given by

$$S_{ij}(\mathbf{x},\mathbf{x}';\omega)=\frac{1}{(2\pi\hbar)^2}\int d^3p\, W(E(p))F_i^*(\mathbf{p},\mathbf{x},E(p))\times F_j(\mathbf{p},\mathbf{x}',E(p))\ , \tag{63}$$

with

$$E(p)=\hbar(\omega-\omega_L)+\frac{p^2}{2m}\ . \tag{64}$$

B. Two-level atoms

For two-level atoms we obtain from Eqs. (25) and (29)

$$\langle g|\otimes\langle\mathbf{p}|\mathbf{D}_{\perp i}^{(+)}(\mathbf{x})|\psi_{E_0}(0)\rangle$$
$$=C_e d\left\{\epsilon_y f_E\left[\mathbf{x}-\frac{\hbar k_L z}{mv_E}\mathbf{e}_x,y,z\right]\times\exp-i\left[\frac{\mathbf{p}}{\hbar}-\left[\frac{mv_E}{\hbar}+\frac{\delta_L'+i\gamma/2}{v_E}\right]\mathbf{e}_z-k_L\mathbf{e}_x\right]\cdot\mathbf{x}\right\}_\perp . \tag{65}$$

An approximate evaluation of the integral (61) is summarized in Appendix B. We obtain

$$F_i(\mathbf{p},\mathbf{x},E)=\frac{2\pi^2}{L}\hbar^2k_L^2C_e d\left[\frac{2\pi i}{k'\cos\beta}\right]^{1/2}$$
$$\times\left[\frac{\epsilon_{yi}}{(x^2+y^2)^{1/4}}\exp\left[-\frac{\gamma}{2v_E}[z-(x^2+y^2)^{1/2}\tan\beta]+ik'[(x^2+y^2)^{1/2}\cos\beta+z\sin\beta]\right]\right.$$
$$\left.\times\delta\left[p_x+\hbar k'\frac{x\cos\beta}{(x^2+y^2)^{1/2}}-\hbar k_L\right]\delta\left[p_y+\hbar k'\frac{y\cos\beta}{(x^2+y^2)^{1/2}}\right]\right]_\perp , \tag{66}$$

where we have defined for fixed intermediate momentum $\mathbf{p}$, and beam center-of-mass energy E the angle $\beta(p_z,E)$ by

$$\sin\beta=\frac{(\delta_L'/v_E)+(mv_E-p_z)\hbar}{k'}\ . \tag{67}$$

We can now return to Eq. (60). For $\mathbf{x}\neq\mathbf{x}'$ we can do the integration over the intermediate momentum using the δ functions, a stationary-phase approximation. The resulting correlation function $G_{ij}(\mathbf{x},\mathbf{x}',t-t',E)$ is nonvanishing only at space-time points that are connected by an emitted photon.

For $\mathbf{x}=\mathbf{x}'$ the approximation (B3) yields a diverging result and we have to return to the expression (B1) to resolve this divergency. We obtain, taking the trace over i,j,

$$\sum_i S_{ii}(\mathbf{x},\mathbf{x};\omega)=2\pi\int dE\, W(E)\frac{(k_L^2d|C_e|)^2}{v_E\omega/c}\times\frac{x^2+y^2\sin^2\beta(\omega)}{(x^2+y^2)^{3/2}\cos\beta(\omega)}\times e^{-(\gamma/v_E)[z-(x^2+y^2)^{1/2}\tan\beta]} \tag{68}$$

with

$$\sin\beta(\omega)=\frac{c}{v_E}\frac{\omega-\omega_0-\hbar k_L^2/2m}{\omega} . \quad (69)$$

The condition $|\sin\beta(\omega)|\le 1$ limits the frequency of the emitted photons to a small neighborhood of $\omega_0+\hbar k_L^2/2m$.

C. $J=0\leftrightarrow J=1$ system

The calculation of Sec. VI B may be repeated for the $J=0\leftrightarrow J=1$ system with only minor changes.

One feature is the possibility to do coincidence measurements on photons emitted in opposite directions. We therefore consider

$$\bar{S}_{yy}(x,z,\omega)=S_{yy}(x,0,z,-x,0,z;\omega)+\text{H.c} . \quad (70)$$

and obtain

$$\bar{S}_{yy}(x,z,\omega)=-\int dE\; W(E)\frac{(\pi k_L^2 d|C_e|)^2}{L\cos\beta(\omega)v_E\omega/c^2}\frac{1}{|x|} \times e^{-(\gamma/v_E)[z-|x|\tan\beta(\omega)]} \times\delta(\omega_L-\omega\cos\beta(\omega)) , \quad (71)$$

where $\sin\beta(\omega)$ is given by Eq. (69).

This correlation function is measured in a heterodyne experiment where the emitted field is first added to the field of two mutually coherent local oscillators with frequency ω at the two points $(x,0,z)$ and $(-x,0,z)$ and then measured by two photodetectors whose intensity-intensity correlation is recorded. The negative sign of $\bar{S}_{yy}$ reflects the anticorrelation for the emission of photons in opposite directions; but the linear superposition in the excited state, or between the excited state and the ground state, has no influence on this correlation function.

VII. CONCLUSIONS

We have analyzed the coherent and incoherent emission of photons from a stationary atomic beam placed in a linear superposition of excited states and the ground state attached to different center-of-mass wave packets. The examples of two-level atoms excited by a running wave laser, and of $J=0\leftrightarrow J=1$ level atoms excited by a $\sigma_+-\sigma_-$ standing-wave laser, have been considered explicitly. For convenience the excitation process was treated in the Raman-Nath limit, which requires that the interaction time be short compared to the spontaneous-decay time and the inverse recoil frequency. However, these limitations are a matter of computational convenience only and not of any fundamental importance to our results. Within the Raman-Nath approximation the wave packet created in the excited state is identical in form to the ground-state wave packet, but its momentum is displaced by that of the absorbed photon. In the $\sigma_+-\sigma_-$ configuration the exciting laser mode is a standing wave and hence a linear superposition of the photon momenta $\pm\hbar k_L$. This linear superposition is transferred to the excited state in the case of $J=0\leftrightarrow J=1$ transitions.

The two examples we considered were made particularly simple by angular momentum conservation. In the more general case of a standing-wave field without angular momentum conservation the atoms, after passing the exciting laser beam, are in a linear superposition of the ground state and the excited state, each consisting of a linear superposition of several center-of-mass wave packets. These have average momenta $2n\hbar k_L$ and $(2n+1)\hbar k_L$ for the ground state and the excited state, respectively, where n assumes positive and negative integer values. Each such component of the total wave function then carries a phase factor $\exp[-il(\omega_L t-k_L x)]$, with $l=2n$, $2n+1$, respectively.

In Secs. V and VI we have analyzed two ways to detect and monitor linear superpositions of the ground state and the excited state, provided their wave packets have spatial overlap. The most promising experimental signature is provided by the energy absorption rate from a weak probe-laser beam, which is proportional to the probing-field strength, oscillates with the difference of the probing- and exciting-laser frequencies, and, for a sufficiently monoenergetic atomic beam, exhibits a pronounced directional characteristic. The directional characteristic arises because the spatial extension of the atomic beam in all directions is assumed to be large compared to the laser wavelength, and because of the spatial coherence of the atomic beam over its cross section. Another experimentally detectable signature is provided by the coherent spontaneous emission from the superposition state at the frequency of the exciting laser which has also, and for the same reasons, pronounced directional characteristics and could be measured by a homodyne scheme.

Both experimental tests require a second laser field in a fixed phase relation to the exciting laser, and most easily derived directly from the driving laser field. In the case of the absorption experiment, the frequency shift $(\Delta\omega)$ could be generated via sidebands using a modulation technique. In order that the finite-energy spread ΔE of the atomic beam does not destroy the phase coherence of the linear superposition at position z with the exciting laser at $z=0$ the inequality $\Delta E/E \ll v_E/z'\delta'_L, v_E/W_0\Omega_0$ must be satisfied, as is most easily seen directly from Eqs. (40) and (18). To satisfy the first of these conditions over the entire overlap domain of Eq. (A2) the condition $\Delta E/E \ll \hbar k_L/mL\delta'_L$ must be satisfied. Interestingly, only the detuning δ'_L, which can be made very small, and the Rabi frequency appear in these conditions, but not the laser frequency ω_L, because the coherent oscillation in time at frequency ω_L of the atomic dipole moment is imposed by the driving laser field, and is only limited by the phase coherence of that field, independent of the velocity distribution of the atoms in the beam.

Note added. Realization of an atomic interferometer has been reported by O. Carnal and J. Mlynek, Phys. Rev. Lett. **66**, 2689 (1991); W. Keith, C. R. Ekstrom, Q. A. Turchette, and D. E. Pritchard, Phys. Rev. Lett. **66**, 2693 (1991); M. Kasevich and S. Chu, Phys. Rev. Lett. **67**, 181 (1991); and, F. Riehle, Th. Kisters, A. Witte, J. Helmcke, and Ch. J. Borde, Phys. Rev. Lett. **67**, 177 (1991).

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APPENDIX A: EVALUATION OF THE INTEGRAL (40)

In this appendix we evaluate the integral (40) under the following assumptions for a square hole $|x|<L, |y|<L$ (see Sec. IV B): (i) $k_L L \gg 1$, (ii) $z' \ll L^2/\lambda_E$, (iii) $x \gg L$, $(x^2+y^2)^{1/2} \gg (k_L L)L$.

Due to the second condition, the function $f_E(\mathbf{x})$ can be approximated by

$$f_E(\mathbf{x}) \simeq f(x,y) = \begin{cases} (2L)^{-1}, & |x|,\ |y|<L\ , \\ 0 & \text{elsewhere}\ . \end{cases} \tag{A1}$$

The region of overlap of the two wave packets is then restricted to

$$0 \le z' \le 2\frac{mv_E}{\hbar k_L}L, \quad \frac{\hbar k_L z'}{mv_E} - L \le x' \le L \tag{A2}$$

and is located entirely within the near zone of the hole because of the first condition $k_L L \gg 1$. The condition $x \gg L$ permits us to neglect x' in the integrand of Eq. (40), except in the phase factor which varies rapidly. The integrations over x',y'

$$F_E(\mathbf{x},z') = \int dx'dy' \frac{1}{|\mathbf{x}-\mathbf{x}'|} e^{ik_L(|\mathbf{x}-\mathbf{x}'|+x')} \times f(x',y') f\left[x' - \frac{\hbar k_L z'}{mv_E}, y'\right] \tag{A3}$$

can then be performed in the limit $(x^2+y^2)^{1/2} \gg k_L L^2$, where we can use the fact that the integrand of Eq. (A3) is cut off by the functions f rather than by the oscillations of the phase factor, because in that limit we have $k_L L \ll \sqrt{R k_L}$ with $R=|\mathbf{x}-z'\mathbf{e}_z|$. Expanding therefore the phase to first order in x',y' and performing the integrals we get

$$F_E(\mathbf{x},z') = \frac{1}{k_L^2 L^2 R} e^{ik_L[R+(\hbar k_L z')/(2mv_E)(1-x/R)]} \frac{\sin(k_L y L/R) \sin\left[k_L(1-x/R)\left[L - \frac{\hbar k_L z'}{2mv_E}\right]\right]}{(y/R)(1-x/R)} \Theta\left[\frac{2mv_E}{\hbar k_L}L - z'\right] \tag{A4}$$

where we defined the step function $\Theta(x) = \frac{1}{2}(x+|x|)/x$.

The remaining space integration

$$G_E(\mathbf{x}) = \int dz' F_E(\mathbf{x},z') e^{i(\delta_L' + i\gamma/2)z'/v_E} \tag{A5}$$

can be carried out in stationary-phase approximation, because the phase factor $e^{ik_L R + i\delta_L' z'/v_E}$ varies much more rapidly than the rest of the integrand. Its phase is stationary for

$$z - z' = (x^2+y^2)^{1/2}\tan\alpha, \quad R = \frac{(x^2+y^2)^{1/2}}{\cos\alpha}\ , \tag{A6}$$

where the angle α is defined in Eq. (42). This gives Eq. (41).

APPENDIX B: EVALUATION OF THE INTEGRAL (61)

We sketch the further steps by which the integral (61) can be evaluated in stationary-phase approximation.

(i) Equation (64) is inserted in Eq. (61) and the phase of the rapidly varying phase factor is expanded to first order in x',y'.

(ii) Using the approximation (A1) for $f_E(\mathbf{x})$ and assuming $R \gg L$, $R \gg k_L L^2$ where $R=[x^2+y^2+(z-z')^2]^{1/2}$ the x',y' integrals can be done, yielding the factor

$$2e^{i(k'x/R + p_x/\hbar - k_L)(\hbar k_L/mv_E)z'} \times \frac{\sin\left[\left[k'\frac{x}{R} + \frac{p_x}{\hbar} - k_L\right]L\right] \sin\left[\left[k'\frac{y}{R} + \frac{p_y}{\hbar}\right]L\right]}{\left[k'\frac{x}{R} + \frac{p_x}{\hbar} - k_L\right]\left[k'\frac{y}{R} + \frac{p_y}{\hbar}\right]L}\ , \tag{B1}$$

where

$$k' = \frac{1}{c}\left[\omega_L + \frac{E}{\hbar} - \frac{p^2}{2m\hbar}\right] \simeq \frac{1}{c}\left[\omega_L - \frac{v_E}{\hbar}(p_z - mv_E)\right]. \tag{B2}$$

We shall assume that $k_L L \gg 1$ and approximate (B1) by

$$\frac{2\pi^2}{L}\delta\left[k'\frac{x}{R} + \frac{p_x}{\hbar} - k_L\right]\delta\left[k'\frac{y}{R} + \frac{p_y}{\hbar}\right]. \tag{B3}$$

We note that this factor appears under the integral over the intermediate center-of-mass momentum $\mathbf{p}$. Therefore the factor (B1) does not imply any directional characteristics of the emitted incoherent radiation.

(iii) The integral over z' can be done in stationary-phase approximation. This approximation yields simpler expressions under the assumption $\hbar k_L \ll m v_E$. For fixed intermediate momentum **p**, and beam center-of-mass energy E, we define the angle $\beta(p_z, E)$ by Eq. (67). In terms of β, the stationary phase condition assumes the form

$$z - z' = R \sin\beta = (x^2 + y^2)^{1/2} \tan\beta \ . \tag{B4}$$

The condition $|\sin\beta| \leq 1$ limits the z component of the momentum which may appear in the intermediate state. As our final result we obtain Eq. (66).

*Permanent address: Fachbereich Physik, Universität GHS Essen, D-4300 Essen, Germany.

†Permanent address: Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria. Present address: Joint Institute for Laboratory Astrophysics and Department of Physics, University of Colorado, Boulder, CO 80309.

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