Nature of quantum jumps

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Recent experiments have found abrupt changes in the intensity of laser light scattered off single ions and atoms. These abrupt changes have been interpreted as resulting from Bohr's "quantum jumps" of the ion or atom from one state to another. In Bohr's description, such quantum jumps are not predicted by the Schrödinger equation and are introduced by additional postulates into quantum theory. We consider here an alternative explanation of these experiments using only properties of solutions of the Schrödinger equation and avoiding Bohr's added postulate.

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I. INTRODUCTION

A number of experiments on trapped single ions or atoms have been performed in recent years [1,2]. Monitoring the intensity of scattered laser light off of such systems has shown abrupt changes (see Fig. 1) that have been cited as evidence of "quantum jumps" between states of the scattered ion or atom [3]. The existence of such jumps was required by Bohr in his theory of the atom. He assumed that an atom remained in an atomic eigenstate until it made an instantaneous jump to another state with the emission or absorption of a photon. Since these jumps do not appear to occur in solutions of the Schrödinger equation, something similar to Bohr's idea has been added as an extra postulate in modern quantum mechanics. The question arises whether an explanation of these jumps can be found to result from a solution of the Schrödinger equation alone without additional postulates. The interpretation of one of these experiments given below shows that the answer is "yes."

The linearity of the Schrödinger equation will play an essential role in this analysis. von Neumann [4] based his "theory of measurement" on this linearity. We shall make use of some of his work as developed by London and Bauer and by Wigner [4] to demonstrate properties of the solutions of the Schrödinger equation. It will be apparent that an understanding of the property of equation linearity provides sufficient information to allow a qualitative description of the wave function from which the form of the experimental results can be understood. For this purpose, it is unnecessary to find a detailed solution of the Schrödinger equation.

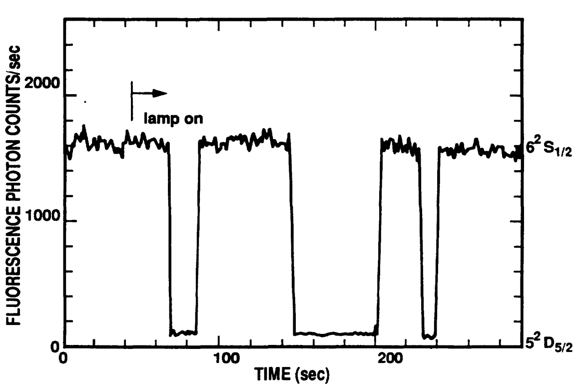

FIG. 1. A typical trace of the light scattered off of a Ba^+ ion involving the $6\,^2S_{1/2}$–$6\,^2P_{1/2}$ transition in the experiments reported by Nagourney, Sandberg, and Dehmelt [1]. It shows "quantum jumps" after the hollow cathode lamp is turned on.

To illustrate this method of attack, we shall consider an experiment by Nagourney, Sandberg, and Dehmelt [1]. They scattered laser light off of a trapped Ba^+ ion to obtain the plot of intensity versus time shown in Fig. 1. In order to tie this experiment in with the previous work in Ref. [4], we shall analyze a simple model based on the Stern-Gerlach experiment that has all the essential characteristics needed to produce quantum jumps. It will become clear that an intensity plot such as that in Fig. 1 is to be expected from a straightforward solution of the Schrödinger equation describing the scattering of light off of a Ba^+ ion.

In order to understand the use of equation linearity in von Neumann's work, the reader is advised to study the writings of London and Bauer and of Wigner [4]. They make a thorough analysis of the Stern-Gerlach measurement of the spin component of an atom. A summary of their approach is presented here in Sec. II with the addition to the apparatus of a detection device that measures the amplitude of the wave function. In Sec. III a sequence of Stern-Gerlach devices is considered. The Stern-Gerlach apparatus is slightly modified in order to make the analysis of the sequence similar to that of the measurements of Nagourney, Sandberg, and Dehmelt [1]. In Sec. IV an examination of the Ba^+ scattering experiments shows that an intensity plot like that in Fig. 1 is to be expected from a solution of the Schrödinger equation. Some implications of the elimination of Bohr's postulate from the interpretation of quantum phenomena are discussed in Sec. V.

II. THE STERN-GERLACH EXPERIMENT

The Stern-Gerlach apparatus includes a magnet that produces an inhomogeneous field through which the wave function to be analyzed passes as shown in Fig. 2.

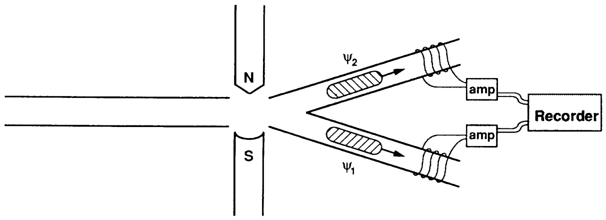

FIG. 2. Stern-Gerlach apparatus where a neutron wave function passes through an inhomogeneous magnetic field and splits into spin-up and spin-down components indicated by ψ_1 and ψ_2. These components pass through coils in which currents are induced by the neutron magnetic moment. These currents are greatly amplified and information about their amplitudes is stored in a recorder.

We shall take this wave function to be a neutron so that it will, in general, separate into spin-up and spin-down components that will be represented by ψ_1 and ψ_2. This magnetic field has the effect of correlating the spin projections with the subsequent spatial displacements of the two components of the neutron wave function.

After the wave-function components are given impulses in opposite directions by the magnetic field, they follow two divergent channels. A wire coil is wrapped around a section of each of these channels in such a way that the neutron wave component passing down that channel will induce a current by means of its magnetic moment. See Fig. 2. Each coil is connected to an amplifier that provides a much increased output current. The wave function for the coils and amplifiers will be represented by χ. The initial wave function with no current flowing is χ_i. A subscript 1 indicates that a current was induced by the ψ_1 neutron wave component but not by the ψ_2 component. A 2 indicates the state with a current induced by the ψ_2 component alone.

The currents from the two amplifiers pass on into an attached recording apparatus whose state is represented by ϕ. A subscript i labels the initial state with no current recorded. A subscript 1 indicates that a current originating from a ψ_1 was recorded alone while a subscript 2 shows the recording of the passage of a ψ_2 component alone.

If the original neutron wave function incident on the magnetic field involves only a ψ_1 component, then it will proceed down its channel and be recorded as such. Before this neutron wave function reaches the magnetic field, there will be no induced current in the coil so that the wave function for the system will be

$$\Psi_i = \psi_1 \chi_i \phi_i \ . \tag{1}$$

After the neutron wave function has passed through the coil around the ψ_1 channel, and the currents have made their recording, the wave function takes the form

$$\Psi_f = \psi_1 \chi_1 \phi_1 \ . \tag{2}$$

If, on the other hand, the original neutron wave function consists entirely of a ψ_2 component, the initial wave function for the system will be

$$\Psi_i = \psi_2 \chi_i \phi_i \ . \tag{3}$$

The magnetic field will impart an impulse to the neutron wave function in the opposite direction, and the wave function will proceed down its channel and be so recorded. The final wave function will then be

$$\Psi_f = \psi_2 \chi_2 \phi_2 \ . \tag{4}$$

In general, however, the initial neutron wave function will be a superposition of a spin-down and a spin-up component. The initial wave function for the system is then

$$\Psi_i = (\psi_1 + \psi_2) \chi_i \phi_i \ . \tag{5}$$

Because of the linearity of the Schrödinger equation for the system, the development of the system due to the incident ψ_1 component as given by Eq. (1) will be unaffected by the presence of the ψ_2 component and vice versa. As a result, the final wave function can be written as the sum of the two final-state wave functions in Eqs. (2) and (4). Thus, in general,

$$\Psi_f = \psi_1 \chi_1 \phi_1 + \psi_2 \chi_2 \phi_2 \ . \tag{6}$$

This superposition property is the characteristic of quantum fields that distinguishes them from classical fields and, as we shall see, produces solutions to the Schrödinger equation that appear to have quantum jumps. It is important to note that there are two recorder states, one remembering a spin-up measurement result and the other a spin-down measurement. There are no recordings of *both* a spin-up and a spin-down state. Such a result would require an initial wave function involving two or more neutron coordinates. A classical field behaves like a many-neutron wave function.

If first-order perturbation or semiclassical radiation theory is used, the neutron wave component passing through a coil will induce a current as though the magnetic moment were distributed over it in proportion to $|\psi|^2$. Any portion of the neutron wave function will induce a flow of charge in the coil whose magnitude is proportional to $\int |\psi|^2 d^3x$ over this portion. Since the current induced in the coil must be extremely small, the amplification requirement on the amplifier circuits must be very stringent. We shall conjecture that in order to provide the amplification to produce a macroscopic current from a single-quantum one, a circuit will have properties similar to a Geiger counter in that the tiny quantum current will precipitate a cascade of a macroscopic number of electrons. Such a cascade and the subsequent recovery of the device requires a very small interval of time and sends a pulse of current into the attached amplification stages and finally into the recorder.

After the amplification device has recovered, it is ready to repeat its cycle. If a single-quantum wave function interacts with the device for a period much longer than the recovery time, then the cascade cycle will repeat over and over. For example, if a Geiger counter is set up to record the γ rays from radioactive nuclei, a number of years may be required for a nucleus to emit a γ-ray wave while a Geiger counter and its control circuit can cycle in a small fraction of a second. However, we know that a single-quantum state will be recognized by a recorder by a single isolated event (a single click in an attached speaker). It must be true, therefore, that different states of the recorder are associated with different Geiger-counter cycles. Each state recognizes only one cycle at some particular time. The large number of clicks that we ordinarily hear from a Geiger counter detecting γ rays must be from a large number of different γ-ray wave functions emitted by many nuclei.

We may imagine the required amplification for our Stern-Gerlach experiment to be provided by a single-cycle multivibrator. In this circuit, a very small charge on the initial condensor of the circuit is required to trigger a cycle with a relatively large charge output. This output is passed on to the recorder to produce a state with a memory of an event at this time. We assume that the multivibrator cycle time is negligible, but significant time is required to build sufficient charge on the initial condensor to initiate a cycle. We assume that this cycle time is much smaller than the time required for the neutron wave function to pass through the coil.

During the buildup time of the charge on the initial multivibrator condensor for one cycle, a required quantity of magnetic moment will pass through the center of the coil. After one multivibrator cycle, a second portion of the neutron wave function will begin to build up the condensor charge until another cycle occurs. Thus we can think of the neutron wave-function component as being chopped up into a superposition of packets, each with the same magnetic moment or, equivalently, the same value of $\int|\psi|^2d^3x$. Each packet will be associated with its own recorder state.

If we solve the Schrödinger equation backwards in time, we can identify each of these packets in the initial neutron wave function before it enters the magnetic field. Thus, in analogy to Eq. (4), the initial wave function of the system takes the form

$$\Psi_i=\sum_{s,t}\psi_{st}\chi_i\phi_i \ , \tag{7}$$

where s is 1 or 2 to indicate the spin component, and t numbers the packets in order as they pass the center of a coil.

Each of the packets ψ_{st} can be followed through the apparatus to produce a term in the system wave function of the form $\psi_{st}\chi_{st}\phi_{st}$ where ϕ_{st} represents a recorder state that remembers a signal from the s channel generated by the pulse numbered t. Since the Schrödinger equation for the system is linear, the final wave function of the system can be constructed by adding together the solutions associated with the packets to produce

$$\Psi_f=\sum_{s,t}\psi_{st}\chi_{st}\phi_{st} \ . \tag{8}$$

Again it is important to remember that each recorder state ϕ_{st} remembers only one signal pulse, that induced by the neutron wave packet in channel s numbered t.

The apparatus described above has the kind of elements essential to the prediction of the abrupt changes in intensity appearing in Fig. 1 as a consequence of properties of the Schrödinger equation as we shall see in the following sections. However, the question may now be asked, "Is there any other experimental evidence for the decomposition of a neutron wave function into packets associated with recorder states?" The answer is "yes" in that the above apparatus will lead to the accepted postulate of quantum mechanics that declares that the probability of obtaining a given spin projection result is proportional to $\int|\psi|^2d^3x$ over the component of the wave function with that spin projection. This postulate is in agreement with experiment.

To show this, we consider the result of applying repeated identical incident neutron wave functions to the Stern-Gerlach apparatus. The initial state of the system, when the second neutron wave function $\psi^{(2)}$ is incident, is the product of the wave function in Eq. (8) and that of the incident neutron, namely,

$$\Psi_i^{(2)}=\psi^{(2)}\sum_{s^1,t^1}\psi^{(1)}_{s^1t^1}\chi_{s^1t^1}\phi_{s^1t^1} \tag{9}$$

where superscripts (1) or (2) have been placed on labels and wave functions on the right side of the equation to indicate that they are associated with the first or second neutron wave function, respectively. As noted above, the linearity of the Schrödinger equation allows us to solve for each term in the last equation separately as the second neutron wave function proceeds through the apparatus. This second neutron wave function will be broken up into packets in the same way that the first one was, and each state of the recorder will remember two signals, each induced by one of the packets from each wave function. Thus the wave function of the system after the second neutron wave function has passed through will be of the form

$$\Psi_f^{(2)}=\sum_{s^2,t^2}\psi^{(2)}_{s^2t^2}\sum_{s^1,t^1}\psi^{(1)}_{s^1t^1}\chi_{s^1t^1s^2t^2}\phi_{s^1t^1s^2t^2} \ . \tag{10}$$

After n neutron wave functions have passed through the apparatus, the system wave function is

$$\Psi_f^{(n)}=\sum_{s^1,t^1,\ldots,s^n,t^n}\prod_{r=1}^{n}\psi^{(r)}_{s^rt^r}\chi_{s^1t^1,\ldots,s^nt^n}\phi_{s^1t^1,\ldots,s^nt^n} \ . \tag{11}$$

Each recorder state remembers n signals, each one induced by one packet from each neutron wave function.

The sequences of recorder measurements labeling the recorder states are those that would be drawn from an ensemble whose elements are the combinations of s and t. As noted above, all of the combinations of s and t originate from packets with equal values of $\int|\psi|^2d^3x$. For this ensemble, the probability of obtaining a given value of s is proportional to the number of elements of the ensemble with that value of s and, therefore, to the sum of $\int|\psi|^2d^2x$ over all of the packets in the component of the neutron wave function with the value s. This is another way of saying that the probability of obtaining s is proportional to $\int|\psi|^2d^3x$ over the entire component of the wave function with the label s. This is, as noted above, a postulate of quantum mechanics and is consistent with many experiments. Thus the assumption that the measuring apparatus decomposes the wave function into packets of the neutron wave function of equal values of $\int |\psi|^2 d^3x$ is consistent with experiment. We shall see in the next two sections how it leads to an explanation of the jumps in Fig. 1.

III. REPEATED STERN-GERLACH MEASUREMENTS WITH TRANSITIONS

The analysis of a Stern-Gerlach device can be used to understand how an intensity plot like Fig. 1 can result from the scattering of light off of a single ion. For this purpose we shall consider a sequence of identical neutron spin measurements, each measurement, with one exception, being like that described in the preceding section. These Stern-Gerlach devices are connected one after the other in the manner shown in Figs. 3 and 4. The neutron detection coils, however, will be located only on the ψ_2 channels. Each successive device will be placed so that the ψ_2 wave function from the preceding apparatus will enter its magnetic field to be analyzed as shown in Fig. 3. However, this state will not be represented by ψ_2 alone because the currents induced in the detector coils of the first apparatus will react back on the neutron to rotate the spin by a small amount. Thus, although the wave function entering each coil consists of ψ_2 alone, the one leaving will have a small admixture of ψ_1.

These sets of apparatus can be numbered in order starting from the one entered by the original neutron wave function as indicated by the numbers in Fig. 4. This order number will be represented by the letter r. The wave function entering the first device will be from a coil identical to the others into which is also introduced a pure ψ_2 wave function. Thus the wave function entering each of the Stern-Gerlach devices will consist principally of ψ_2 but with a small admixture of ψ_1.

It will be assumed that the amplifiers are so effective

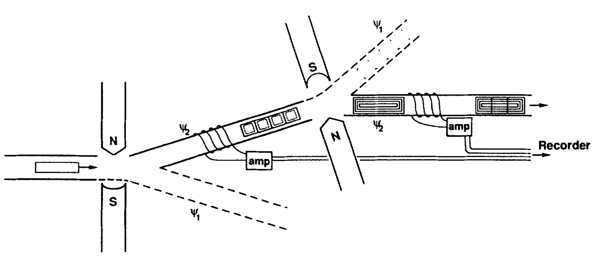

FIG. 3. A sequence of identical Stern-Gerlach devices are connected as shown. The ψ_2 component from the magnet of one feeds into its detection coil. The coil admixes a small amount of ψ_1 component by rotating the spin. The amplifier requires a given amount of energy to actuate one of its stages. This required energy is supplied by packets of the neutron wave function so that each packet (four are shown) is associated with a recorder state. The resulting wave function then passes through the magnetic field of the next device, and the ψ_1 component is separated out. The ψ_2 component (shown with four nested packets) then passes into the next coil and is again divided into packets (three shown), each associated with one of the recorder states.

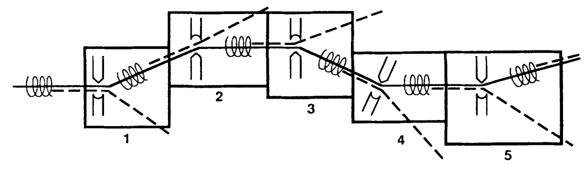

FIG. 4. Each Stern-Gerlach device of the sequence is numbered (five devices are shown). The neutron wave function feeding into the first device comes from a coil identical to the others into which a ψ_2 neutron wave function (indicated by a solid line) is fed. The magnetic field of each device separates off the ψ_1 component (indicated by a dashed line) leaving a ψ_2 component to enter its coil. Each coil rotates the spin slightly to mix in some ψ_1 component and the mixture then proceeds to the next magnet.

that the coils draw a negligible amount of energy from the neutron wave function. A similar statement holds for the magnetic fields so that the energy of the complete neutron wave function is approximately conserved as it passes through the entire system of Stern-Gerlach devices. This is equivalent to the conservation of $\int |\psi|^2 d^3x$ over this wave function since we assume an almost monochromatic neutron wave function. All of the amplifiers are attached to the same recorder that remembers the channel and time of each arriving signal.

The amplifier in each device divides up the ψ_2 component of the neutron wave function into packets of equal size and associates each packet with a recorder state. As noted above, the back magnetic field generated by each coil will rotate the neutron spins by a small amount decreasing the ψ_2 component and creating a small ψ_1 component. As a result, the next inhomogeneous magnetic field will separate out a small amount from the main ψ_2 neutron wave component. This erodes this component as it passes through succeeding devices. Since all of the devices are identical, each coil will reduce the $\int |\psi|_2|^2 d^3x$ for the wave function that enters it by the same factor $(1-f)$ while generating a ψ_1 component with

$$\int |\psi_1|^2 d^3x = f \int |\psi_2|^2 d^3x \tag{12}$$

to be split off by the next magnet. See Fig. 4. After r devices, the $\int |\psi_2|^2 d^3x$ will be reduced by a factor $(1-f)^r$. In the limit of large r and small f, this factor can be approximated by e^{-fr} since

$$\lim_{n\to\infty} (1 - fr/n)^n = e^{-fr} \; . \tag{13}$$

Since each device associates neutron wave packets with recorder states, all with the same values of $\int |\psi_2|^2 d^3x$, the number of packets generated by the rth coil will be

$$N(r) = \int |\psi_2^{(r)}|^2 d^3x \Big/ \int |\psi_2^0|^2 d^3x \propto e^{-fr} \tag{14}$$

where $\psi_2^{(r)}$ is the neutron wave component that enters the coil of the rth device, and ψ_2^0 is an elementary packet to be associated with a recorder state.

If there are n devices in the sequence, the final wave function of the system will consist of terms involving all of the n ψ_1 neutron wave components separated out by these devices as well as the final, attenuated ψ_2 component that passes out of the last one. The contribution to the total wave function associated with the ψ_1 component leaving the device numbered r will have the form

$$\Psi^{(r)}=\sum_{t^1,\ldots,t^{r-1}} \psi_{1,t^1,\ldots,t^{r-1}}\chi_{1,t^1,\ldots,t^{r-1}}\phi_{1,t^1,\ldots,t^{r-1}} \tag{15}$$

where t^j numbers the packets associated with recorder states in the jth device. This component has passed through the coils of all of the previous devices and become a superposition of packets (each represented by $\psi_{1,t^1,\ldots,t^{r-1}}$) that are associated with recorder states.

Although each of these ψ_1 neutron packets contributes to a signal in each of the first $r-1$ coils, it has no interaction with the remaining Stern-Gerlach devices. See Fig. 4. The associated recorder state remembers that it was in the ψ_2 state in the device numbered $r-1$ and each of the preceding devices, but will have no record of it for greater r's. The recorder can represent each of its states using a graph with a scale of r values on the horizontal axis. It draws a horizontal line out to the value of r for the device that emitted the associated neutron wave component. A horizontal line one unit up vertically is then plotted from this value of r to the final value n. Each recorder state is then represented by a graph like that in Fig. 5. Such a plot exhibits a jump from the ψ_2 state to the ψ_1 state at r. We shall see in the next section how an intensity plot like that in Fig. 1 is related to such a graph.

The question remains as to the probability of a jump occurring at a given value of r. To answer this question, we note that, if the number of devices gets very large, the term in the final wave function associated with the ψ_2 neutron state will become negligible, while the remainder of the wave function will consist of the sum over r of the ψ_1 wave functions in the last equation. The recorder states in this wave function form an ensemble. The probability of obtaining a particular plot like that in Fig. 5 is proportional to the number of elements in this ensemble (terms in the final wave function) with a jump at a given value of r.

Since $N(r)$ is the number of packets correlated with recorder states by the equipment in the rth device, then it is also the number of recorder states associated with a plot with a "jump" at a greater value of r. Thus the number of terms in the final wave function with a jump at r is

$$N(r+1)-N(r)\approx e^{-fr}(e^{f}-1)\underset{f\to 0}{\sim} fe^{-fr}\,. \tag{16}$$

Thus the probability of a jump occurring at a given value of r decays, at least approximately, exponentially with r. This is interpreted as the probability of the ion remaining in ψ_2. Nagourney, Sandberg, and Dehmelt [1] found that the probability of a jump up in intensity after a given time after a jump down also decayed exponentially with that time.

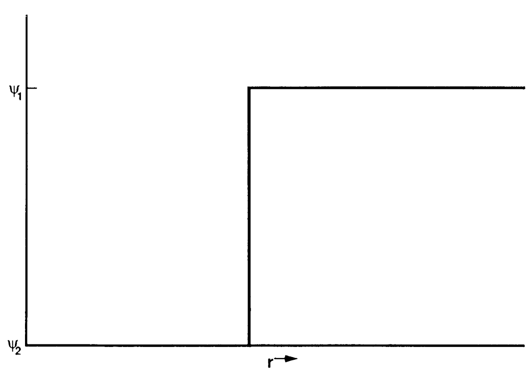

FIG. 5. A plot identifying the sequence of signals from the coils to the recorder that labels a typical recorder state. Each ψ_1 component separated off by the magnet of the device numbered r (see Fig. 4) is associateed with a recorder state registering signals from all of the previous coils but none from the latter coils. The plot is constructed by drawing a horizontal line on the axis up to the r of separation and then a horizontal line at unit height beyond. The associated neutron wave packet is in state ψ_2 for devices numbered smaller than r and in state ψ_1 for later devices. An apparent "quantum jump" between states is evident.

IV. SCATTERING FROM AN ISOLATED Ba^+ ION

Nagourney, Sandberg, and Dehmelt [1] have measured the scattering of laser light of an isolated Ba^+ ion. The levels of the ion involved are shown in Fig. 6. Two laser beams were scattered from the ion, and the transition induced by them are marked with heavy lines. The scattered light producing the transitions between the $6\,^2S_{1/2}$ and the $6\,^2P_{1/2}$ levels was monitored, and its intensity for one experiment is plotted in Fig. 1. These two laser beams provide a strong coupling between the levels shown in heavy lines in Fig. 6 so that they are grouped together and represented by the symbol ψ_1.

Occasionally, a photon wave arrives at the ion from a filtered barium hollow cathode lamp. This wave converts the ψ_1 wave function to a superposition of ψ_1 and an increasing wave function for the $6\,^2P_{3/2}$ excited state as shown by the dashed arrow on the left in Fig. 6. This ex-

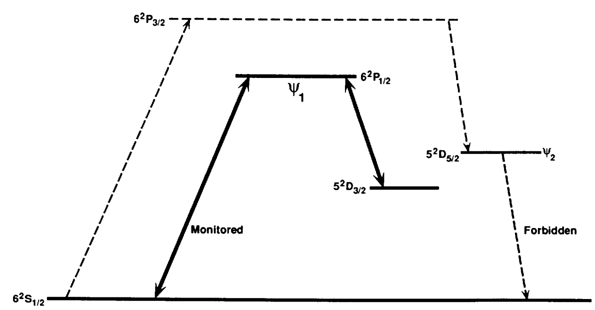

FIG. 6. The level diagram of the Ba^+ ion showing the grouping into ψ_1 (heavy lines) and into ψ_2 (the $5\,^2D_{5/2}$ level). The slanted heavy lines indicate laser-induced transitions. The scattered light from the $6\,^2S_{1/2}$–$6\,^2P_{1/2}$ levels was detected by a photomultiplier tube. The forbidden transition from the $5\,^2D_{5/2}$ to the $6\,^2S_{1/2}$ provides the ψ_2 to ψ_1 leakage analogous to that from the coils in Fig. 4.

cited state loses amplitude with time as it radiates to an increasing metastable $5\,^2D_{5/2}$ wave function leaving a superposition largely of the latter state and ψ_1. The amplitude of the wave function of the $5\,^2D_{5/2}$ level (that we shall call ψ_2) decays slowly by a forbidden transition back to ψ_1. This forbidden transition corresponds to the transitions from ψ_2 to ψ_1 induced by the detector coils in the model described in the last section. The superposition of ψ_1 and ψ_2 produced by the light wave from the hollow cathode lamp corresponds to the initial neutron wave function that enters the first Stern-Gerlach device shown in Fig. 4.

The model considered in the last section measures $\int|\psi_2|^2d^3x$ over the ψ_2 component of the neutron wave function as it passes through successive devices while, instead, Nagourney, Sandberg, and Dehmelt monitored ψ_1 with passing time. See Fig. 6. For simplicity, we can think of the atomic transitions as due to one electron in the Hartree-Fock approximation. As the forbidden radiative transition proceeds, $\int|\psi_2|^2d^3x$ decreases exponentially with time as it did in the Stern-Gerlach model as the neutron wave function passed through successive devices. The $\int|\psi|^2d^3x$ over the entire wave function is conserved so that the loss in $\int|\psi_2|^2d^3x$ is equal to the increase in $\int|\psi_1|^2d^3x$ with the result that the sum of the two remains constant.

A photomultiplier tube and attached circuits detect the scattered light wave from the transition between the $6\,^2S_{1/2}$ and $6\,^2P_{1/2}$ states. Even though the wave trains of the scattered laser light may be long, a state of the photomultiplier tube records only one sharp pulse for each photon wave train. We may ask when, in the long wave train, the tube, circuits, and recorder choose the particular instant to record the pulse. We have seen from previous sections that they actually produce a large number of pulses and are recording a pulse at each instant of time as the wave train passes into the tube, but they associate each pulse at a given instant with its own recorder state. In this way, the entire photon wave train is divided into packets of equal energy, and each packet is associated with a recorder state remembering one pulse and its arrival time t.

Each photon packet from a given photon wave train scatters off of a small portion of the state ψ_1 with a value of $\int|\psi_1|^2d^3x$ equal to the value for each of the other portions. In this way, a wave train breaks ψ_1 up into equal packets, each of which can be labeled by the recorded time t. A packet can then be represented by ψ_{1t}. Although a packet ψ_{1t} can scatter a given photon wave train only once, it can participate in the scattering of a second wave train. The scattered wave from this second photon wave train will again be broken up into equal packets by the detection apparatus so that the ψ_1 wave function goes into packets $\psi_{1t^1t^2}$ associated with states recording pulses at t^1 and t^2.

As we have noted, there is a continual flow of $\int|\psi|^2d^3x$ from the ψ_2 state to the ψ_1 state. If we solve the Schrödinger equation for all times for each packet, we will see a flow of packets through the forbidden transition from ψ_2 to ψ_1 as shown in Fig. 6. A packet starting in the ψ_2 state cannot contribute to the scattering of the laser light, and therefore cannot affect the recorder, until it has reached the ψ_1 state. If the time at which the transition takes place for a particular packet is r and there are n photon wave trains being scattered, then it can be represented by $\psi^r_{1,t^1,\ldots,t^n}$ where the times $t^1,\ldots,t^n$ of the recorded pulses are restricted to being greater than r. It is the passage of one of these packets from state ψ_2 to ψ_1 as predicted by the Schrödinger equation that produces the upward jumps in the scattered intensity shown in Fig. 1.

After a long time, the wave function of the system will have the form

$$\Psi_f=\sum_r\sum_{t^1,\ldots,t^n}\psi^r_{1,t^1,\ldots,t^n}\chi^r_{t^1,\ldots,t^n}\phi^r_{t^1,\ldots,t^n}\,. \tag{17}$$

If the n-photon wave trains are identical and exactly overlap, then the recorded states will have pulse arrival times distributed uniformly at random following the time r and no recordings prior to this time. If the density of light pulses in time is plotted for each recorded state, the result will look like Fig. 5. As noted above, this accounts for the upward jumps in intensity shown in Fig. 1.

The rate of flow of $\int|\psi|^2d^3x$ from ψ_2 to ψ_1 decreases exponentially with time for the same reason that the rate of jumps decreases with r as shown in Eq. (16). The rate of decay of this exponential with time is the lifetime of ψ_2. Thus the distribution of time intervals at background intensity of length r in Fig. 1 is proportional to this same exponential. This was the finding of Nagourney, Sandberg, and Dehmelt [1].

Nagourney, Sandberg, and Dehmelt [1] noted that the frequency of the downward jumps increased when the intensity of the barium hollow cathode lamp was increased. This is because its wave trains provided a path from the ψ_1 state to the ψ_2 state. Thus a packet that was part of state ψ_1, where it could participate in the scattering of the monitored laser light, could be boosted up to state ψ_2 where it could not scatter. As a result the recorded state associated with it would show no light pulses after the transition time so that its light intensity plot would drop to background.

From Fig. 1, it is clear that the scattered intensity has the same value and remains at that value after each upward jump until a downward jump occurs. This is because the detecting-recording apparatus always selects portions of ψ_1 that have equal amounts of $\int|\psi_1|^2d^3x$ and, therefore, the same light-scattering capacity.

V. DISCUSSION

We have seen that Fig. 1, a plot of the intensity of light scattered off of a single Ba^+ ion, can be explained in terms of solutions of the Schrödinger equation without adding additional assumptions to the theory. It is necessary, however, to assume that a solution of the Schrödinger equation would show that the apparatus that converts the tiny electrical currents induced by single-quantum wave functions to recordable macroscopic currents must associate macroscopic recorded states with packets of equal values of $\int|\psi|^2 d^3x$ in the original single-quantum wave function. This is presumably the result of an amplification process like that in a Geiger counter where a small amount of the single-quantum wave function produces the signal for a recorder state. The analysis of such experiments makes use of the linearity of the Schrödinger equation. This analysis can be made in a standardized form [4] making use of wave functions for the system, the apparatus, and the recorder or the observer.

The assumption that the detecting equipment for single-quantum events must break up the wave function into a superposition of equal packets, each to be associated with a recorded macroscopic state, should be subjected to more direct experimental test. It may be possible to experimentally chop the single-quantum wave function into pieces smaller than the equal packets suggested above and thereby find their size. For example, the incident laser beam whose scattering is monitored could be reduced in length until the scattered wave has a length of the order of the size of a pulse generated by the detection. This would affect the rate of detection in a characteristic manner. A determination of the parameters on which this packet size depends should further verify their existence. Similar information might be obtained from a study of the structure of the jumps like those in Fig. 1 since each is the result of the transition of a packet from one atomic state to another. Under some conditions, the recorder and apparatus are replaced by a human observer's brain and eyes or ears. In such cases, it would be useful to identify and understand the operation of the biological elements that provide the generation of an electron cascade by a small charge to produce the amplification from a current at the quantum level to the macroscopic level.

The elimination of the need for Bohr's postulate from quantum theory has much broader implications than those for single-ion experiments. The "collapse of the wave function" is very similar to Bohr's quantum jump. Everett [5] showed that postulating such a "collapse" can be avoided if many states of an observer's brain are allowed. The introduction of particles into quantum mechanics also becomes superfluous since localized particle phenomena can be explained in terms of fields (wave functions). For example, Rutherford's α-particle experiments where point scintillations appeared on a screen even though the wave function extended over a large region of space are explained in the above manner in Ref. [6]. A similar explanation of the sharp clicks produced by Geiger counters from photon waves with considerable spatial extent is presented in Ref. [7]. These explanations involve dividing the space over which a single-quantum wave function ranges into very small regions and associating each portion of the wave function, contained in each small region of space, with different observer states. An observer is then only aware of one localized region which he interprets as a click or a scintillation.

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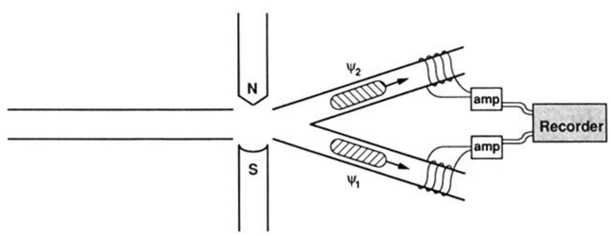

FIG. 2. Stern-Gerlach apparatus where a neutron wave function passes through an inhomogeneous magnetic field and splits into spin-up and spin-down components indicated by ψ_1 and ψ_2. These components pass through coils in which currents are induced by the neutron magnetic moment. These currents are greatly amplified and information about their amplitudes is stored in a recorder.

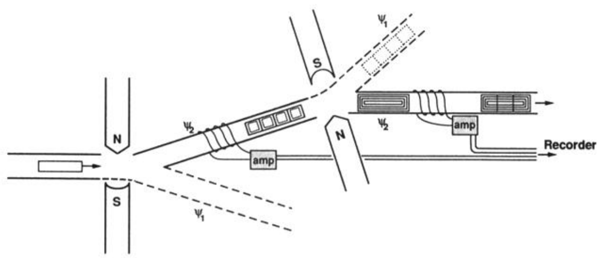

FIG. 3. A sequence of identical Stern-Gerlach devices are connected as shown. The ψ_2 component from the magnet of one feeds into its detection coil. The coil admixes a small amount of ψ_1 component by rotating the spin. The amplifier requires a given amount of energy to actuate one of its stages. This required energy is supplied by packets of the neutron wave function so that each packet (four are shown) is associated with a recorder state. The resulting wave function then passes through the magnetic field of the next device, and the ψ_1 component is separated out. The ψ_2 component (shown with four nested packets) then passes into the next coil and is again divided into packets (three shown), each associated with one of the recorder states.