Numerical study of the linewidth of a semiconductor laser: Effect of saturation

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In this paper we calculate the intensity and carrier-density distribution inside a Fabry-Pérot-type semiconductor laser with arbitrary facet reflectivities. We derive an expression for the linewidth of such a laser operated well above threshold, when the inversion saturates and the carrier-density distribution becomes nonuniform. Results are given for the frequency and carrier-density dependence of the linewidth-enhancement factor of $Al_xGa_{1-x}As$ lasers. We find that saturation results in substantial nonuniformity of the distributions of the spontaneous-emission factor and the linewidth-enhancement factor. Although these factors codetermine the laser linewidth, their combined effect on the linewidth-power product is found to be negligible in most cases of practical importance. However, a small power-independent contribution to the laser linewidth is predicted for large mirror asymmetries.

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I. INTRODUCTION

The fundamental linewidth of semiconductor lasers is a topic that has received much attention in the past decade [1–3]. This attention has technical motivation originating from possible applications of narrow-linewidth lasers in coherent optical communications. However, this research is also driven by basic interest in laser linewidths. The high gain available in a semiconductor laser allows small cavity dimensions leading to a large and easily measurable fundamental linewidth, so that current theories for this linewidth may be verified. Also, high gain allows low mirror reflectivities, so that these lasers are suitable candidates for studying the effect of output coupling on the fundamental linewidth [2–10]. For large output coupling the intensity distribution inside the cavity becomes nonuniform, which has been shown to lead to an enhancement of the laser linewidth.

The fundamental lower limit on the laser linewidth is due to the disturbing influence of spontaneous emission on the phase of the intracavity optical field. For an ideal four-level laser with small output-coupling losses, tuned to the maximum of a symmetric gain profile, the full width at half maximum (FWHM) of the laser line, $\Delta\nu$, is given by the standard Schawlow-Townes expression [11],

$$\Delta\nu=\frac{\Gamma_c}{4\pi n}=\frac{h\nu\Gamma_c^2}{4\pi P_{\text{out}}} , \tag{1}$$

where Γ_c is the cavity-loss rate, n the average number of photons in the lasing mode, $h\nu$ the energy per photon, and P_{out} the combined output power from both laser mirrors.

As semiconductor lasers are not ideal four-level lasers and their gain profile is not symmetric, their linewidth is given by a more complicated expression. For a semiconductor laser with spatially uniform gain, but arbitrarily large output coupling, the laser linewidth is given by [5,9]

$$\Delta\nu=\Delta\nu_0+\frac{h\nu\Gamma_c^2}{4\pi P_{\text{out}}}\eta_{\text{opt}}(1+\alpha^2)n_{\text{sp}}K . \tag{2}$$

In Eq. (2), $\Delta\nu_0$ is the experimentally observed power-independent contribution to the linewidth, for which many explanations have been given, such as the competition of the lasing mode with nonlasing side modes [12] or the presence of spatial hole burning [13]; the issue seems to be undecided and will not be considered in this paper. In Eq. (2), $\eta_{\text{opt}}=P_{\text{out}}/P_{\text{st.emiss.}}$ is the optical power extraction efficiency, i.e., the fraction of the power produced by stimulated emission that couples out of the cavity, the rest being dissipated through internal losses; note that $\eta_{\text{opt}}=1$ if internal losses can be neglected. Finally, α is the linewidth-enhancement factor [1], $n_{\text{sp}}>1$ is the spontaneous-emission factor [14], and $K\geq 1$ is the excess-noise factor [5,8,10,14], describing the influence of large output coupling on the laser linewidth. The linewidth-enhancement factor is defined as

$$\alpha\equiv-\frac{\dfrac{\partial\chi_r}{\partial N}}{\dfrac{\partial\chi_i}{\partial N}} , \tag{3}$$

where χ_r and χ_i are the real and the imaginary parts of the complex susceptibility, related to the refractive index and the gain, respectively, and N is the carrier density. In semiconductor lasers the α factor gives the most drastic correction to the original equation for the laser linewidth [Eq. (1)], α being typically 3–7 [14].

In earlier papers [3,8] we addressed the effect of output coupling on laser linewidth. Comparing the linewidths of two sets of lasers with the same round-trip cavity losses, but with a different internal intensity profile, we confirmed the existence of the excess-noise factor K in Eq. (2); the lasers with the most asymmetric mirrors and thus the most nonuniform internal intensity profile had the largest linewidth. In these earlier papers we assumed that the gain was uniform over the length of the cavity, which is valid if the laser is close to threshold. In the present paper we drop that restriction and consider the fundamental linewidth of a high-gain–large-output-coupling semiconductor laser that is driven far above threshold. In this situation the large intracavity intensity will saturate the inversion and hence result in a spatially nonuniform inversion distribution and an intensity distribution that is changed as compared to the unsaturated case. The nonuniform carrier density leads to a nonuniform distribution of important parameters such as α and n_{sp}, which, apart from the output power P_{out} and the cavity-loss rate Γ_c, codetermine the linewidth. In this paper we present a method of calculating the abovementioned distributions. We also show how the fundamental linewidth can be calculated using the distributions of α and n_{sp}, i.e., we show how these distributions have to be averaged along the laser axis. We restrict the discussion to Fabry-Pérot-type $Al_xGa_{1-x}As$ lasers.

Several methods are available to evaluate the linewidth of a laser with nonuniform intracavity intensity and inversion. Eventually, all methods are based on an expansion of the intracavity optical field in the cavity eigenmodes, followed by an evaluation of the disturbing effect of spontaneous emission on the (complex) amplitude of the lasing mode. In this paper we use the nonorthogonal-mode formalism introduced in Refs. [8] and [15] to calculate the linewidth of a Fabry-Pérot-type semiconductor laser with arbitrary output coupling. An alternative approach that makes use of the Green's-function approach [5,16] has recently been applied by Tromborg, Olesen, and Pan [16] to calculate the linewidth of a distributed-feedback semiconductor laser. Unfortunately that derivation is not very transparent and we find the final expression difficult to implement in practical cases. Using the nonorthogonal-mode formalism, we will derive a much simpler expression for the linewidth of a laser with nonuniform population inversion. This new expression will be shown to agree with the expression derived using the Green's-function formalism.

Two basic approximations are made in the calculation. First, one might expect the distribution of carrier density over the cavity to exhibit a spatial profile similar to that of the standing-wave intensity (spatial hole burning with a period $\lambda_s/2$, where λ_s is the optical wavelength in the semiconductor medium) and thus provide a coupling of the intracavity traveling waves by the saturation-induced gain and refractive-index grating. We neglect this effect because, in semiconductors, carrier diffusion is so rapid (as compared to the spontaneous-decay rate) that the inversion grating is effectively washed out [18].

As a second approximation we use the rate-equation approximation, in which the active medium is treated as an ensemble of an identical two-level system. In fact, the carriers are distributed over conduction and valence energy bands. The seemingly crude discard of the effects of carrier redistribution over the bands can be justified by noting that the intraband relaxation process is much faster than any of the other processes involved. As a consequence, the carrier distribution in each of the bands remains practically in equilibrium and is well characterized by the integrated population. Therefore, we can neglect the so-called "nonlinear gain" in the calculations, an issue that is discussed further in Sec. II.

The paper is organized as follows. In Sec. II we describe the calculation of the intensity and carrier-density distributions and the derivation of the corresponding distributions of the linewidth-enhancement factor α and the spontaneous-emission factor n_{sp}. Section III describes how the fundamental linewidth can be found once the latter distributions are known. In Sec. IV we present several numerical examples, while conclusions are given in Sec. V. In the Appendix, an expression for the linewidth-enhancement factor of GaAs is derived.

II. CALCULATION OF INTRACAVITY DISTRIBUTIONS

In this section we calculate the stationary-carrier density and intensity distributions inside a semiconductor laser. This means that we neglect fluctuations. The fluctuations will be considered in Sec. III, where we calculate the linewidth. The carrier and intensity distributions were calculated using the standard propagation equations for the left- and right-traveling intensities

$$\frac{dI^{\pm}(z)}{dz}=\pm g(z)I^{\pm}(z) , \tag{4}$$

where $I^{\pm}(z)$ represents the intensity traveling to the right $(+)$ and to the left $(-)$, respectively, $g(z)=(\Gamma/v_g)G$ is the modal intensity-gain coefficient, Γ is the confinement factor, v_g is the optical group velocity, and G is the bulk intensity gain [14]. In a semiconductor laser the gain G is a function of carrier density N, for which a separate equation has to be solved at each position z in the laser. This equation is the standard rate equation for the carrier density [14],

$$\frac{d}{dt}N(z,t)=\frac{J}{ed}-\frac{N(z,t)}{\tau_{sp}}-G(N(z,t))\mathcal{S}(z) , \tag{5}$$

where J is the injected current density, e is the electron charge, d is the thickness of the active layer, τ_{sp} is the carrier lifetime, and $\mathcal{S}(z)$ is the volume photon density. We take the photon density $\mathcal{S}(z)$ appearing in Eq. (5) as proportional to the *envelope* of the intensity profile along the z direction because, as discussed in the Introduction, diffusion effectively washes out the carrier grating burned by the standing-wave optical intensity.

Note that in Eqs. (3) and (4) all variables are real valued. The *intensity* gain G is twice the real part of the complex *amplitude* gain sometimes used by other authors (see, e.g., [14]). The imaginary part of the complex amplitude gain, describing the dependence of the refractive index on the population inversion, does not enter Eqs. (3) and (4). However, it does affect the laser linewidth, as will be shown further on.

Also note that we assume that the gain is only a function of the population inversion and shows no direct intensity dependence. For an ensemble of real two-level systems, e.g., an atomic or molecular gas, this is obviously true. However, for a semiconductor gain medium, residual carrier redistribution over the conduction, and valence bands at large intensity, for example, spectral hole burning and/or carrier heating lead to a (small) dependence of the gain directly on intensity. Recently, numerical work has shown that this so-called nonlinear gain also leads to an intensity dependence of the linewidth-enhancement factor [2] and thus of the laser linewidth. For $In_{1-x}Ga_xAs_yP_{1-y}$ lasers this influence was calculated to be small [2]. As the nonlinear gain is a factor of 5 smaller in $Al_xGa_{1-x}As$ than in $In_{1-x}Ga_xAs_yP_{1-y}$ [19], we expect this effect to be negligible in $Al_xGa_{1-x}As$ lasers and feel that the simple two-level rate-equation approximation is valid in our case.

If the relation between the gain G and the carrier density N is known (see below), the stationary-carrier density and intensity distribution in the laser can be found by simultaneously solving Eqs. (4) and (5), setting the round-trip gain, including internal and mirror losses, to unity and setting $dN(z,t)/dt=0$. At the intensities that we use, a numerical solution for these distributions is most easily found by iteration. To first order, both carrier density and gain are uniform and the intracavity field is the sum of two counterpropagating, exponentially growing traveling waves. To second order the carrier density becomes inhomogeneous, as in the high-field regions in the cavity stimulated emission will reduce the carrier density more than in the low-field regions. Therefore, the gain also becomes inhomogeneous and the intensity profile is deformed compared to the sum of two simple exponentials. For practical intensities this iteration procedure converges rapidly to a consistent solution for the stationary-carrier density and the intensity distribution.

Knowledge of the relation between the gain G and the carrier density N is essential to our calculations. While in a true two-level system the gain is obviously proportional to the population inversion, the situation is more complicated in a semiconductor, where the population is distributed over energy bands and the relation $G(N)$ depends on the electronic band structure of the semiconductor. Furthermore, the dependence of the refractive index on carrier density, as characterized by α [see Eq. (3)], has potentially severe consequences for the laser linewidth. As all this information is not available in the literature we spent some effort to determine the functional dependence of both the gain and the linewidth-enhancement factor on the carrier density.

First we note that the approximation of a linear dependence of gain on carrier density, which is often used in

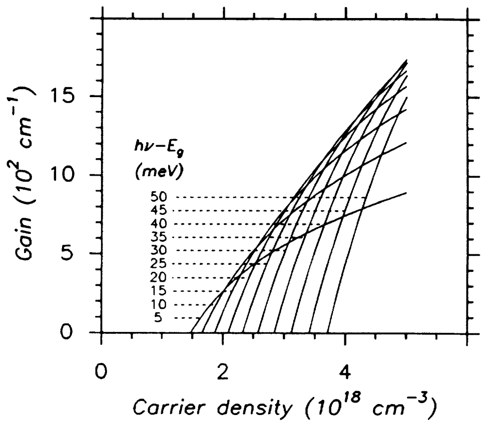

FIG. 1. Calculated gain coefficient g of $Al_xGa_{1-x}As$ as a function of carrier density for different photon energies $h\nu$ relative to the $Al_xGa_{1-x}As$ band gap E_g. The laser parameters used for the calculation of all figures are given in Table I.

the literature [20,21], applies for the maximum gain at each carrier density with the photon energy as a free parameter. However, in a semiconductor laser with a position-dependent saturation, the carrier density varies spatially, but the photon energy (i.e., laser oscillation frequency) is fixed. So we need a relation between gain and carrier density at fixed photon energy. We have used the standard strict k-selection model [20,21] to calculate such a relation for the case of GaAs. Typical results are shown in Fig. 1. The values of the device and material parameters used in the gain calculation are given in Table I.

For the calculation of the linewidth-enhancement factor α, the Kramers-Kronig relations were used to calculate $\partial\chi_r/\partial N$ from $\partial\chi_i/\partial N$, where $\chi_i(\omega)$ directly follows from the spectral dependence of the calculated gain. Details of the calculation are given in the Appendix. The main results are given in Figs. 2, 3, and 4. In Fig. 2 we show α as a function of carrier density. For each carrier density, α was calculated at the photon energy at which the gain is maximum. However, as stated above, we need a relation for α at a fixed photon energy. Figure 3 shows

TABLE I. Device and material parameters used in the calculation of intensity and carrier-density distributions.

Symbol	Description	Value	Units
L	laser length	250	μm
w	transverse width of intensity distribution	5	μm
d	active-layer thickness	0.075	μm
λ	emission wavelength	790	nm
τ_{sp}	spontaneous lifetime	1	ns
Γ	confinement factor	0.15	
m_c	effective mass in conduction band	0.067	m_0
m_v	effective mass in valence band	0.55	m_0
m_0	free-electron mass	9.11×10^{-31}	kg
$\|M\|$	momentum matrix element	1×10^{-24}	kg m s^{-1}

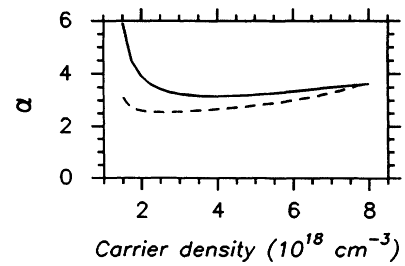

FIG. 2. The solid curve shows the numerically calculated value of α as a function of carrier density at the photon energy for which the gain is maximum. The dashed curve gives the (approximate) analytical results of Westbrook and Adams.

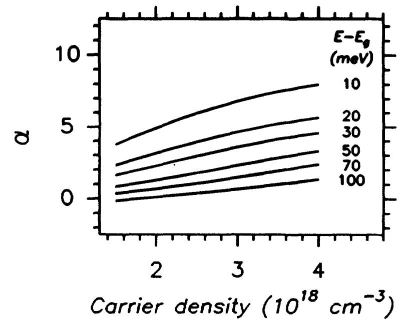

FIG. 4. Calculated value of α as a function of carrier density N for different photon energies E relative to the band gap E_g.

such a relation at a photon energy of 30 meV above the band gap. Clearly, the behavior is totally different from that in Fig. 2. Finally, Fig. 4 gives the result of α at several different photon energies in the region of carrier densities that is of interest to us.

Using the above results we have numerically calculated the steady-state intensity and carrier-density distributions in a semiconductor laser as a function of the injection current. The results for a laser with intensity-reflection coefficients of the facets equal to $R_1=10\%$ and $R_2=90\%$ (in the following called a 10-90 laser) are given in Fig. 5 for two values of the injection current, namely $I=I_{\mathrm{th}}$ and $I=1.5I_{\mathrm{th}}$. In the model the latter value corresponds to a total output power, i.e., from both facets, of $\simeq 15$ mW. Also shown in Fig. 5 are the distributions of the spontaneous-emission factor n_{sp} and the linewidth-enhancement factor α. The spontaneous-emission factor n_{sp} was calculated using known relations for the carrier-density dependence of the quasi-Fermi-levels [22].

III. FUNDAMENTAL LINEWIDTH

Evidently, the nonuniformity of the distributions of α and n_{sp} has to be properly averaged in any theory of laser linewidth. In order to see how this should be done we have performed an analysis based on the same principles used to derive the standard rate equations (see, e.g., [23,24]), modified to include the fact that mirror losses are localized and not homogeneously distributed over the length of the cavity. One approach to treating the localized nature of the mirror losses and possible inhomogeneities of the gain medium is to use a Green's-function approach as has been done in, e.g., [5,16]. Another approach, and the one we use, is based on direct mode expansion [8]. It starts from the fact that due to the localized nature of the mirror losses the longitudinal eigenmodes $\{u_q\}$ are nonorthogonal. It is then convenient to introduce the set of adjoint modes $\{w_q\}$ (i.e., the eigenmodes of the time-reversed laser [8]), which is biorthogonal to the set $\{u_q\}$ [17]. We write the eigenmodes and adjoint modes of the laser as

$$u_q(z)=C\{[I^+(z)]^{1/2}e^{-ikz}+[I^-(z)]^{1/2}e^{ikz}\}\ , \tag{6a}$$

$$w_q(z)=\frac{1}{C2L(I^+I^-)^{1/2}}\{[I^-(z)]^{1/2}e^{-ikz}+[I^+(z)]^{1/2}e^{ikz}\}\ , \tag{6b}$$

$$C=\left(\int_0^L dz[I^-(z)+I^+(z)]\right)^{1/2}\ , \tag{6c}$$

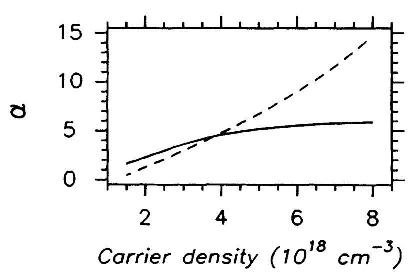

FIG. 3. The solid curve shows the calculated value of α as a function of carrier density at fixed photon energy $E=E_g+30$ meV. The dashed curve shows the (approximate) analytical expressions of Westbrook and Adams.

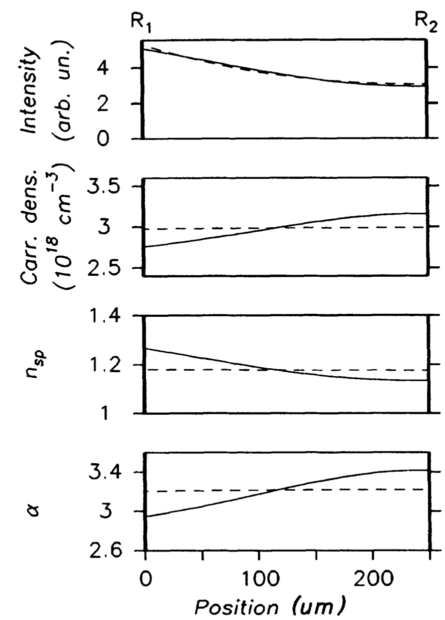

FIG. 5. Calculated intensity, carrier density, spontaneous-emission factor n_{sp}, and linewidth-enhancement factor α as a function of the axial coordinate inside the laser. $R_1=10\%$, $R_2=90\%$. The dashed curve corresponds to $I=I_{\mathrm{th}}$ (no saturation) and the solid curve to $I=1.5I_{\mathrm{th}}$.

where C is introduced for normalization and L is the cavity length. Note that the complex optical wave vector k used in Eqs. (6a) and (6b) depends on the position z in the cavity because in general neither the gain nor the refractive index is uniform. Both $I^{\pm}(z)$ and $k(z)$ are taken from the numerically calculated stationary distributions of $I(z)$ and $N(z)$. Notice that the product $I^{+}(z)I^{-}(z)$ is independent of the position z in the cavity; the reason for this is that the left- and rightward gains are identical since a possible carrier grating is washed out by diffusion [25–27].

We have used the formalism of biorthogonal eigenmodes to derive the laser linewidth. The derivation of the equations for the stationary and fluctuating quantities is completely analogous to that given in other papers [24], except for the fact that almost all quantities become z dependent now. Writing

$$E(t)=E_0[1+\rho(t)]e^{i\phi(t)}, \tag{7a}$$

$$N(z,t)=N_0+n(z,t), \tag{7b}$$

where $E(t)$ is the slowly varying complex field amplitude, we find the following linearized (i.e., products of small quantities are neglected) equations for the fluctuating quantities

$$\frac{d}{dt}[\rho(t)+i\phi(t)]=\frac{1}{2L}\int_0^L dz\,\xi(z)[1+i\alpha(z)]n(z,t)+\frac{F_{\rm sp}(t)}{E_0}e^{-i\phi(t)}, \tag{8a}$$

$$\frac{\partial}{\partial t}n(z,t)=-\frac{1}{\tau_R(z)}n(z,t)-2\frac{\omega_R^2(z)}{\xi(z)}\rho(t), \tag{8b}$$

where $F_{\rm sp}$ is the Langevin force corresponding to spontaneous emission, $\xi=\Gamma\partial G/\partial N$ is the effective differential gain and

$$\frac{1}{\tau_R(z)}\equiv\frac{1}{\tau_{\rm sp}}+\frac{\xi(z)\mathcal{S}_0(z)}{\Gamma}, \tag{9a}$$

$$\omega_R^2(z)\equiv\xi(z)\mathcal{S}_0(z)G(z). \tag{9b}$$

It should be realized that τ_R and ω_R are now functions of z and have thus lost their simple physical meaning of a relaxation-oscillation damping time and frequency. The spontaneous-emission term $F_{\rm sp}(t)$ is the randomly varying optical field produced by spontaneous emission in the laser mode, integrated over the cavity. It differs from the one normally used in the laser rate equations [24] as it results from the projection of the usual δ-function correlated-noise amplitude on the *adjoint* mode $w_q(z)$ instead of on the eigenmode $u_q(z)$. This difference has been shown to result in the so-called excess-noise factor K [8,15]. We repeat here the essence of the argument, extending it in such a way that the effect of saturation is incorporated. Equations (8a) and (8b) describe the dynamics of the system and are the starting point for the calculation of the linewidth. We neglect fast variations in $n(z,t)$, such as those related to the relaxation oscillation, and set the left-hand side of Eq. (8b) to zero. This gives

$$n(z,t)=-\frac{2\tau_R(z)\omega_R^2(z)}{\xi(z)}\rho(t), \tag{10}$$

with $\tau_R(z)$ and $\omega_r(z)$, as defined in Eqs. (9a) and (9b). Substitution of Eq. (10) in Eq. (8a) yields two coupled linear first-order differential equations for $\rho(t)$ and $\phi(t)$, driven by the real and imaginary parts of the spontaneous-emission force. The standard Fourier-transform technique can be applied to solve these equations [1,28], yielding for the mean-square phase fluctuations of the optical field

$$\langle\Delta\phi^2(\tau)\rangle=\frac{1}{4\omega^2E_0^2}[1+(\bar{\alpha})^2]\times\left[\frac{1}{L}\int_0^L dz\,2D_L(z)|w_q(z)|^2\right]\tau, \tag{11}$$

where $\bar{\alpha}$ is defined below as Eq. (13a) and $D_L(z)$ is the local Langevin-diffusion coefficient. Although this cannot be rigorously justified, we take the heuristic point of view that we can indeed define a local-diffusion coefficient $D_L(z)$, which is proportional to the local spontaneous-emission rate [24] and thus is position dependent when the carrier density is nonuniform. Only the component of $D_L(z)$ that varies slowly in space should be used in Eq. (11), because the spatially fast component, associated with the population grating, is washed out by carrier diffusion. According to Eq. (11) the mean-square phase fluctuations increased linearly in time, a signature of phase diffusion. The laser linewidth $\Delta\nu$ is given by the time derivative of $\langle\Delta\phi^2(\tau)\rangle$. After relating the intracavity intensity to the total output power $P_{\rm out}$, i.e., from both facets (see Ref. [28]), we obtain

$$\Delta\nu=\frac{\Gamma_c}{4\pi n}(1+\bar{\alpha}^2)K_{\rm sat}=\frac{h\nu\Gamma_c^2}{4\pi P_{\rm out}}\eta_{\rm opt}G_{\rm sat}(1+\bar{\alpha}^2)K_{\rm sat}. \tag{12}$$

Equation (12) is the fundamental result of this paper; it is of the same form as the standard expression for the semiconductor laser linewidth [Eq. (2)], where the effects of saturation and nonuniform population inversion were neglected, with the following generalizations:

$$\bar{\alpha}=\frac{\int_0^L dz\,\alpha(z)\tau_R(z)\omega_R^2(z)}{\int_0^L dz\,\tau_R(z)\omega_R^2(z)}, \tag{13a}$$

$$K_{\rm sat}=\frac{1}{2L}\int_0^L dz\frac{r_{\rm sp}(z)}{\Gamma_c}\left[\frac{1}{I^-(z)}+\frac{1}{I^+(z)}\right]\times\int_0^L dz\frac{I^-(z)+I^+(z)}{2L}, \tag{13b}$$

$$G_{\rm sat}=\frac{[1-(R_1)^{1/2}]^2I^-(0)+[1-(R_2)^{1/2}]^2I^+(L)}{-(1/2L)\ln R_1R_2\int_0^L dz[I^-(z)+I^+(z)]}. \tag{13c}$$

Note that the weight factor $\tau_R(z)\omega_R^2(z)$ used to derive $\bar{\alpha}$ is proportional to the local gain if the laser is driven far above threshold. Also note that $K_{\rm sat}$ is determined by averaging the ratio of the local excess spontaneous-emission rate $r_{sp}(z)/\Gamma_c$ over the local traveling-wave intensity. This is similar to the result found using the model for traveling-wave phase diffusion [9]. Equation (12) reduces to Eq. (2) when the spontaneous-emission rate and the linewidth-enhancement factor are taken as independent of z, as in the case of negligible saturation. In absence of saturation $K_{sat} = n_{sp} K$ with

$$K = \left| \frac{[(R_1)^{1/2}+(R_2)^{1/2}][1-(R_1R_2)^{1/2}]}{(R_1R_2)^{1/2}\ln R_1R_2} \right|^2 , \qquad (14)$$

as the excess-noise factor that describes the effect of output coupling on the laser linewidth in the absence of saturation [5,8,10].

The occurrence of the factor G_{sat} in Eq. (12) is due to the fact that saturation leads to a redistribution of the intracavity intensity and thus affects the loss rate through the mirrors [9,29]. Saturation leads to a reduced gain in the high-field region close to the output-coupling mirror(s) and tends to flatten out the intracavity intensity profile, thereby effectively reducing the cavity-loss rate $G_{sat} < 1$. In the absence of saturation, $G_{sat} = 1$.

Other procedures for the averaging of α have been reported in Refs. [2,16,30]. Some algebra shows that the results are similar and that the weight factor $\tau_R(z)\omega_R^2(z)$ in the α averaging [Eq. (13a)] that we find with the formalism of nonorthogonal eigenmodes is, e.g., identical to that which can be deduced from Eq. (57) in Ref. [16] if we set the nonlinear gain to zero [31]. The formalism presented here has the advantage that it allows a clear physical interpretation of the results and that the derived equations are easy to use, as will be demonstrated in the examples below.

IV. EXAMPLES

The values of $\bar{\alpha}$, K_{sat}, and G_{sat} as a function of injection current are given in Fig. 6 for the case of a 30-30, a 10-90, a 10-10, and a 1-100 laser. Clearly the axial variation of N has only a small influence on $\bar{\alpha}$, K_{sat}, and G_{sat}, except in the case of the extremely asymmetric 1-100 laser. The linewidth [Eq. (12)] has been calculated for several values of the inverse output power for the same lasers; the results are shown in Fig. 7. The main result is that for all four lasers the slope of the calculated linewidth versus inverse output power is within 0.2% of the value predicted by theory when saturation is neglected. Therefore we conclude that, although saturation causes important laser parameters such as α and n_{sp} to vary appreciably along the laser axis (see Fig. 5), the net effect on the fundamental linewidth is negligibly small, even when the asymmetry of the mirror reflectivities is very large.

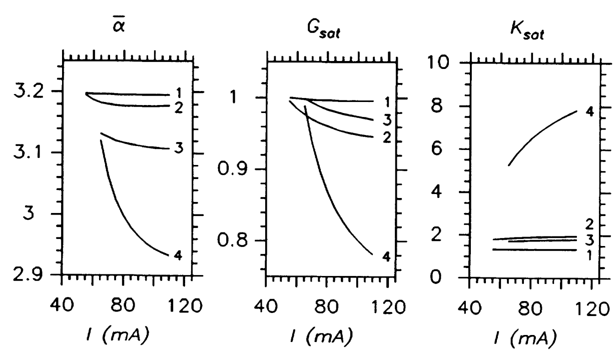

FIG. 6. Calculated values of $\bar{\alpha}$, G_{sat}, and K_{sat} as a function of injection current for four lasers with different facet reflectivities (in percent). 1: 30-30; 2: 10-90; 3: 10-10; and 4: 1-100. The threshold currents lie at the starting point of each curve. Except for the 1-100 laser the averaged quantities depend only slightly on injection current.

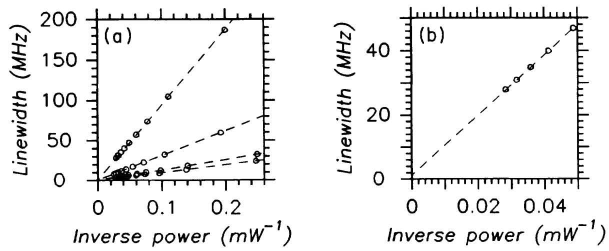

FIG. 7. (a) The circles show the calculated linewidth vs inverse output power for the same lasers as in Fig. 6. The dashed curves represent a linear fit. For all four lasers, the slope of these curves equals the slope predicted by the theory without saturation within 0.2%. (b) Enlarged section in the region of high output power for the 1-100 laser. The intercept shows a power-independent contribution of 1.1 ± 0.3 MHz, apparently due to the effect of saturation.

It should be noted that, for the 1-100 laser, Fig. 7(b) indicates the existence of a power-independent contribution to the laser linewidth due to the effect of saturation. A power-independent linewidth is actually observed in semiconductor lasers, but generally attributed to other causes such as mode-mode interactions [12,32–34].

It should also be noted that we have assumed that the carrier density and thus the quasi-Fermi-level separation, which is proportional to the voltage across the laser junction, are free to vary across the laser axis. This would indeed be the case if the laser were a purely current-driven device. However, a semiconductor laser is not perfectly current driven: the series resistance along the laser axis is finite and will tend to remove the axial voltage difference imposed by the variations of the carrier density. Thus the carrier-density variations calculated in this section present an upper limit to the actual variations.

V. CONCLUSIONS

In conclusion, we have presented the results of calculations of the linewidth of a Fabry-Pérot-type $Al_xGa_{1-x}As$ laser with various facet reflectivities. The calculations include the saturation inside the laser. In cases of practical interest this results in appreciable (tens of percent) nonuniformity of the distributions of the linewidth-enhancement factor and the spontaneous-emission factor. However, we have found that after proper averaging of these distributions and taking into account the effect of saturation on the output-coupling efficiency, the resulting calculated linewidth as a function of the inverse output power is hardly affected by saturation. A small power-independent contribution, however, was noted for the case of very asymmetrically coated lasers.

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APPENDIX: CALCULATION OF THE LINEWIDTH-ENHANCEMENT FACTOR

In this Appendix we comment briefly on the calculation of the linewidth-enhancement factor α, the results of which have already been presented in Fig. 2–4. Generally, α depends upon the photon energy E and the carrier density N. In this appendix we derive a simple formula that permits a straightforward calculation of $\alpha(N,E)$ for $Al_xGa_{1-x}As$. Apart from the context of the present paper this formula might also be useful when modeling waveguides of semiconductor lasers [35,36].

Our calculation of the linewidth-enhancement factor α starts with the definition given as Eq. (3). To find α, $\partial\chi_i/\partial N$ is calculated from the optical-gain spectrum and $\partial\chi_r/\partial N$ is then deduced using the Kramers-Kronig transformation:

$$\frac{\partial\chi_r(E')}{\partial N}=\frac{2}{\pi}P\int_0^\infty \frac{E}{(E^2-E'^2)}\frac{\partial\chi_i(E)}{\partial N}dE\ , \tag{A1}$$

where $E=h\nu-E_g$ and P indicates the principal part. We have numerically performed this calculation using strict k selection in the calculation of the optical gain; relaxation of k selection has been shown to have a negligible effect upon α [37].

The numerical calculation of α was done for a range of values of the carrier density N and photon energy E. The energy at which the gain reaches its maximum value for a specific carrier density is denoted by E_{max}. The value of α at this energy is relevant, as laser oscillation will take place at or very close to this gain maximum. In Fig. 1 we have presented the results of our calculation for the gain $G(N,E)$. In Figs. 2 and 3 the calculated values of $\alpha(N,E_{max})$ and $\alpha(N,E)$ at a *fixed* photon energy ($E=E_g+30$ meV) have been shown, respectively. In both figures, also curves based on approximate analytical expressions of Westbrook and Adams [37] are given. For $In_{1-x}Ga_xAs_yP_{1-y}$ lasers the results of Westbrook and Adams have been found to yield excellent agreement with experiment [37].

Over a part of the operating range, reasonable agreement is obtained between the two methods of calculating α, but significant discrepancies appear, in particular, in the results of Fig. 3, i.e., for $\alpha(N,E_{fixed})$. The discrepancies appear to have their origins in certain approximations made in the Westbrook-Adams derivation that, although valid for $In_{1-x}Ga_xAs_yP_{1-y}$ lasers, give rise to quite large errors for $Al_xGa_{1-x}As$ lasers. Specifically, it seems that, due to the relatively large carrier density in $Al_xGa_{1-x}As$ lasers, an analytical approximation to the carrier-density derivative of the Fermi-Dirac distribution functions, as made by Westbrook and Adams, becomes rather inaccurate.

In Fig. 4 we have presented calculations of $\alpha(N)$ for a variety of E values. Calculations of the carrier-density dependence of α for 1.42-eV band-gap GaAs lasers (and a photon energy of 1.446 eV) have also been reported by Chow, Dente, and Depatie [36]. We find that our results (obtained for a 1.54-eV band gap) are within 5% of those given in Fig. 1 of Ref. [36]. From a linear fit to the computed results displayed in Fig. 4, we obtain

$$\alpha(N,E-E_g)\simeq[1.4-7.85\times10^{-3}(E-E_g)]N + [1.349-0.04(E-E_g)]\ , \tag{A2}$$

where N is in units of 10^{18} cm^{-3} and $E-E_g$ is in units of meV. This formula describes α, for the case of $Al_xGa_{1-x}As$ lasers, with less than a 10% error when compared with the numerically computed values displayed in Fig. 4, i.e., for carrier densities in the range $(1.5-4)\times10^{18}$ cm^{-3} and for photon energies in the range of 10–100 meV relative to the band gap.

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