Spontaneous emission in a Fabry-Pérot cavity: The effects of atomic motion

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We show that the spontaneous emission of a moving atom inside a cavity is modified by two different mechanisms. The first one is a Doppler-induced detuning of the cavity line that leads to observable changes in the spontaneous emission rate for atomic velocities in excess of a characteristic velocity $v_{\text{cav}} = c/Q$, Q being the quality factor of the cavity. In addition, spatial mode effects can lead to a temporal modulation of the exponential decay rate. This modulation is observable for velocities that do not exceed another characteristic velocity v_{mod} which is typically several orders of magnitude smaller than v_{cav}.

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I. INTRODUCTION

Spontaneous emission from an excited electronic state probes the properties of the surrounding vacuum-field fluctuations. By placing an atom near a metallic or dielectric surface, the strength of the vacuum fluctuations is modified, leading to an inhibition of spontaneous emission [1]. This effect first became evident in fluorescence rate measurements performed on complex dye molecules radiating near a dielectric-metal interface [2]. Subsequent experiments have studied the modification of spontaneous emission in a resonator structure [3] which affects both the spectral density and the strength of the vacuum fluctuations. Enhanced spontaneous emission of a Rydberg atom in a microwave cavity [4] as well as inhibited spontaneous emission of a Rydberg atom moving between two parallel conducting plates [5] have been demonstrated. Recently, the suppression of spontaneous decay at optical frequencies has been measured for atoms moving between two closely spaced conducting plates [6]. Moreover, enhanced and inhibited spontaneous emission at visible wavelengths have been observed in a confocal resonator [7].

Along with this position dependence [8], the rate of spontaneous emission in a resonator may also depend on the velocity of the atom [9]. For example, an atom whose transition frequency is below the cavity cutoff, and would undergo inhibited spontaneous emission when at rest, may be brought into resonance by a suitable choice of velocity (Doppler effect), thereby experiencing a greatly enhanced spontaneous-emission rate. In another situation, the atom at rest may find itself either in a node or in an antinode of the cavity mode, with correspondingly drastically different spontaneous-emission rates. In contrast, a moving atom alternatively experiences both these situations, the resultant effective rate of spontaneous emission generally depending on its velocity.

These examples emphasize two quite distinct mechanisms leading to velocity-induced modifications of spontaneous emission. In the first example, it is the Doppler shift which tunes the atom in and out of resonance with the cavity field, thereby probing the "spectral density" of the vacuum fluctuations. This mechanism has been considered in a previous paper [9]. In the second example, it is the spatial variation of the strength of the field fluctuations which leads to a position dependence of the spontaneous emission, and—as we shall see—ultimately to a velocity dependence which is quite distinct in nature from the Doppler-caused velocity dependence.

In this paper we study the dynamics of the spontaneous emission of an atom traversing a lossy cavity, taking into account the interaction of the atom with the spatial part of the quantized vacuum field. In Sec. II, the model of a two-level atom traversing a small and lossy cavity is presented. In Sec. III, the dynamics of the spontaneous emission is studied in some detail with emphasis on the Markovian limit. Section IV finally contains the summary and discusses prospects for possible experiments.

II. TWO-LEVEL ATOM IN A LOSSY CAVITY

We consider a situation in which a two-level atom of mass M traverses a small cavity designed so as to significantly alter the vacuum mode structure near the atomic transition frequency. The atom enters the cavity in its excited electronic state $|e\rangle$ of energy $\hbar\omega_0$ which is radiatively coupled to a long-lived lower electronic state $|g\rangle$ of zero energy. While inside the cavity, the atom exchanges both energy and momentum with the cavity field. Since the cavity is not ideal, the emitted photon may escape from the cavity, and the atom eventually ends up in its ground state.

Taking into account the atomic translation along the cavity axis (the x axis), the Hamiltonian of the combined atom/field system reads

$$H = H_A + H_F + H_{AF} \ , \tag{1}$$

where

$$H_A = \hat{T} + \hbar\omega_0 |e\rangle\langle e| \tag{2}$$

is the Hamiltonian of the free atom, and

$$\hat{T} = \frac{\hat{p}^2}{2M} \tag{3}$$

is the kinetic-energy operator of its center-of-mass motion along the optical axis.

In Eq. (1), the forms of the free-field Hamiltonian H_F and of the atom-field interaction H_{AF} depend on the type of cavity the atom is placed in. In our recently published discussion of the Doppler-induced velocity dependence of spontaneous emission we utilized a three-mirror ring cavity [9]. For a given frequency ω and a given polarization, this kind of cavity supports two counterpropagating running modes, the momentum exchange between the atom and one such mode being restricted to one unit $\hbar\omega/c$. The relative smallness of this quantity compared to typical atomic momenta allows one to neglect the atomic recoil due to its interaction with the electric field. Another distinctive feature of running modes is that their spatial intensity is uniform. Consequently, there are no local modifications of the atom-field coupling which could lead to a nontrivial velocity dependence in the rate of spontaneous emission.

Here we consider instead a linear cavity of the Fabry-Pérot type. This kind of cavity supports one standing wave per frequency and poses no limits on the momentum exchange between atom and field: the mirrors serve as an unlimited sink or source of momentum [10]. Also, a standing mode gives rise to a spatial variation of the atom-field coupling which leads to a nontrivial velocity dependence of the rate of spontaneous emission.

To simplify matters we assume a quasi-one-dimensional cavity of length l, bound by a perfect mirror and a partially transparent input mirror which accounts for the cavity losses. Following the procedure in Ref. [11], the field quantization is performed with continuous-mode functions reflecting the fact that the resonator is embedded in a one-dimensional infinite half-space. If the transmittivity of the input mirror is sufficiently weak, the positive frequency part of the electric field inside the cavity for frequencies in the vicinity of the atomic transition frequency is

$$E^{(+)}(x) = \mathcal{E}\cos(qx)\int_0^\infty d\omega\,\gamma(\omega)a(\omega)\ , \tag{4}$$

where

$$\mathcal{E} = \sqrt{\frac{\hbar\omega_c}{V\epsilon_0}}\ , \tag{5}$$

V being the volume of the cavity, ω_c the resonance frequency, and $q \equiv \omega_c/c$ the corresponding wave number. In Eq. (4), $\gamma(\omega)$ is the spectral response function of the cavity

$$\gamma(\omega) = \sqrt{\frac{\Gamma}{2\pi}}\frac{1}{\Gamma/2 - i(\omega-\omega_c)}\ , \tag{6}$$

where Γ^{-1} is the inverse ringing time which is related to the cavity Q factor by $Q \equiv \omega_c/\Gamma$.

The photon annihilation and creation operators $a(\omega)$ and $a^\dagger(\omega)$ introduced in Eq. (4) obey the continuous quantization commutation relations

$$[a(\omega), a^\dagger(\omega')] = \delta(\omega-\omega')\ . \tag{7}$$

The vacuum state of the cavity is denoted $|\{0\}\rangle$. Application of a photon creation operator $a^\dagger(\omega)$ generates the one-photon-of-frequency-ω state in the sense

$$a^\dagger(\omega')a(\omega')[a^\dagger(\omega)|\{0\}\rangle] = \delta(\omega-\omega')[a^\dagger(\omega)|\{0\}\rangle]\ . \tag{8}$$

Higher-number states may be constructed in a similar manner.

In the canonical quantization scheme, the free-field Hamiltonian reads

$$H_F = \int_0^\infty d\omega\hbar\omega a^\dagger(\omega)a(\omega)\ , \tag{9}$$

and the atom-field interaction is, in the dipole and rotating-wave approximation,

$$H_{AF} = -\hbar g\cos(q\hat{x})\int_0^\infty d\omega\,\gamma(\omega)a(\omega)\sigma_+ + \text{H.c.} \tag{10}$$

Here $\sigma_+ \equiv |e\rangle\langle g|$ is the negative-frequency part of the atomic polarization operator, and $\hat{x}$ denotes the position operator of the atom measured along the optical axis, with $[\hat{x},\hat{p}] = i\hbar$. In Eq. (10) the vacuum Rabi frequency g is given by

$$\hbar g \equiv \wp\mathcal{E} = \wp\sqrt{\frac{\hbar\omega_c}{V\epsilon_0}}\ , \tag{11}$$

where $\wp \equiv \langle e|d|g\rangle$ is the dipole moment of the transition.

Expressions (5) and (11) for the cavity electric field per photon and the vacuum Rabi frequency, respectively, are the same as those obtained in the discrete quantization scheme of an ideal cavity. Despite this similarity, however, the ideal cavity limit $\Gamma \to 0$ has to be performed with care since the spectral response function $\gamma(\omega)$ becomes singular at $\omega_c = \omega_0$ for $\Gamma \to 0$. The reason behind this is that in the continuous quantization scheme adopted here, the "photon"—i.e., the excitation of mode ω—is an entity which cannot be ascribed a position "inside" or "outside" the cavity. The photon is, in fact, everywhere; only its detection probability depends upon whether it is measured inside or outside the cavity. Note in this context that the electric field given in Eq. (4) is the in-cavity part of the total electric field. The out-of-cavity part is not given explicitly since it is not needed here.

As it stands, the model developed so far may be characterized by the following five parameters, all of which have the dimension of a frequency. The atomic characteristics enter the description via the electronic transition frequency ω_0 and the recoil frequency $\omega_r \equiv \hbar q^2/2M$. The properties of the cavity enter the description via the resonance frequency ω_c and the decay rate Γ. Finally, the properties of both the atom and the cavity determine the value of the vacuum Rabi frequency g.

It is worthwhile to recall the orders of magnitude these parameters may assume under experimental conditions. In the optical regime, atomic transition frequencies are of the order of 10^{15} Hz and the dipole moments are of the order of $\wp/ea_0 \approx 1$, where a_0 is the Bohr radius and e is the elementary charge. In microwave experiments, typical transition frequencies are of the order of 10^{10} Hz, and dipole moments are of the order of $\wp/ea_0 \approx 10^3$.

A typical candidate for an optical experiment is the D_2 line of sodium with a transition frequency $\omega_0/2\pi \approx 5 \times 10^{15}$ Hz and recoil frequency $\omega_r/2\pi \approx 2.5 \times 10^4$ Hz. For a cavity of volume 1 mm^3 we then find for the vacuum Rabi frequency $g/2\pi \approx 5 \times 10^5$ Hz. For cavities with $Q \approx 5 \times 10^5 \cdots 5 \times 10^{10}$, we have $\Gamma/2\pi \approx 10^{10} \cdots 10^5$ Hz, which is smaller than the frequency spacing of the corresponding ideal cavity $\Delta\omega/2\pi \equiv c/2l \approx 1.5 \times 10^{11}$ Hz. Hence typical situations are such that (i) the vacuum Rabi frequency and recoil frequency are of the same order, and (ii) the photon escape rate Γ is generally larger than all of the other characteristic frequencies. This situation is sometimes called the weak-coupling regime.

A possible candidate for microwave experiments is the $63p_{3/2} \leftrightarrow 61d_{3/2}$ transition in rubidium, with a transition frequency $\omega_0/2\pi \approx 2.2 \times 10^{10}$ Hz and a recoil frequency $\omega_r/2\pi \approx 1.2 \times 10^{-5}$ Hz. For a cavity of volume 1 cm^3 we find $g/2\pi \approx 3.6 \times 10^4$ Hz (we assume $\wp/ea_0 = 3600$). For the superconducting Nb cavities of the Garching micromaser experiments [16] with $Q \approx 2.2 \times 10^{10}$, we find $\Gamma/2\pi \approx 1$ Hz, but of course faster decay rates can easily be obtained by increasing the temperature of the resonator. Perhaps the most distinguishing feature of the microwave regime, as compared to the optical regime is (i) the smallness of the recoil frequency, and (ii) the dominance of the coherence-preserving Rabi frequency g over the incoherence-inducing photon leakage rate Γ in extremely high-Q cavities, a situation sometimes called the strong-coupling regime.

III. DYNAMICS OF SPONTANEOUS EMISSION

The spontaneous emission of an atom moving inside a lossy cavity is given by the solution of the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = H|\Psi\rangle \ , \tag{12}$$

subject to the initial condition

$$|\Psi(t=0)\rangle = |\phi(t=0)\rangle|e\rangle|\{0\}\rangle \ , \tag{13}$$

which describes an excited atom in the translational state $\phi(x,t=0) \equiv \langle x|\phi(t=0)\rangle$ entering the cavity whose field is in the vacuum state. With this initial condition, the Schrödinger dynamics (12) takes place exclusively in the one-excitation subspace of the entire Hilbert space. The general ket in such a subspace may be written

$$|\Psi(t)\rangle = e^{-i\omega_0 t}|\phi(t)\rangle|e\rangle|\{0\}\rangle + e^{-i\omega_0 t}\int_0^\infty d\omega\, \gamma^*(\omega)|\phi(\omega,t)\rangle|g\rangle[a^\dagger(\omega)|\{0\}\rangle] \tag{14}$$

where the exponential $e^{-i\omega_0 t}$ and the integration measure $\gamma^*(\omega)d\omega$ have been introduced for future convenience. Here $|\phi(t)\rangle$ is the partial amplitude of the center-of-mass motion of the atom in the excited state, and $|\phi(\omega,t)\rangle$ is the corresponding partial amplitude for the atom in its ground state with one photon of frequency ω present.

In terms of the partial amplitudes $|\phi(t)\rangle$ and $|\phi(\omega,t)\rangle$, the Schrödinger equation (12) takes the form

$$i\hbar|\dot\phi(t)\rangle = \hat T|\phi(t)\rangle - \hbar g\cos(q\hat x)\int_0^\infty d\omega|\gamma(\omega)|^2|\phi(\omega,t)\rangle \ , \tag{15}$$

$$i\hbar|\dot\phi(\omega,t)\rangle = [\hat T + \hbar(\omega-\omega_0)]|\phi(\omega,t)\rangle - \hbar g^*\cos(q\hat x)|\phi(t)\rangle \ , \tag{16}$$

where the kinetic-energy operator $\hat T$ and the spectral function $\gamma(\omega)$ have been introduced in Eqs. (3) and (6), respectively. We note that in Eq. (15) $|\phi(\omega,t)\rangle$ enters only via the integral quantity

$$|\Phi(t)\rangle \equiv \int_0^\infty d\omega|\gamma(\omega)|^2|\phi(\omega,t)\rangle \ . \tag{17}$$

To derive the equation of motion for this quantity, we formally integrate Eq. (16),

$$|\phi(\omega,t)\rangle = ig^*\int_0^t dt'\exp\left(-\frac{i}{\hbar}[\hat T + \hbar(\omega-\omega_0)](t-t')\right) \times\cos(q\hat x)|\phi(t')\rangle \tag{18}$$

with $|\phi(\omega,t=0)\rangle = 0$ in accordance with the initial condition (13). To proceed, we multiply this expression by the Lorentzian $|\gamma(\omega)|^2$ and integrate the resulting expression over ω, see Eq. (17). Extending the integration to $-\infty$ and closing the contour in the lower complex ω plane, one finds

$$|\Phi(t)\rangle = ig^*\int_0^t dt'\exp\left\{-\frac{i}{\hbar}\left[\hat T + \hbar\left(\omega_c-\omega_0-i\frac{\Gamma}{2}\right)\right](t-t')\right\}\cos(q\hat x)|\phi(t')\rangle \ , \tag{19}$$

up to corrections of order Γ/ω_c which stem from the extension of the ω integration in Eq. (17) towards $-\infty$. Using Eq. (17) in Eq. (15) and differentiating Eq. (19) with respect to t, we arrive at

$$i\hbar|\dot\phi(t)\rangle = \hat T|\phi(t)\rangle - \hbar g\cos(q\hat x)|\Phi(t)\rangle \ , \tag{20}$$

$$i\hbar|\dot\Phi(t)\rangle = \left[\hat T + \hbar\left(\delta - i\frac{\Gamma}{2}\right)\right]|\Phi(t)\rangle - \hbar g^*\cos(q\hat x)|\phi(t)\rangle \ , \tag{21}$$

where $\delta \equiv \omega_c - \omega_0$ is the detuning between the cavity resonance frequency and the atomic transition frequency.

Equations (20) and (21) are very similar to the frequently studied pair of Schrödinger equations which describe the interaction of an atom with a classical standing-light mode [12]. The only difference is the appearance of Γ in the equation for Φ which accounts for photon leakage through the partially transparent input mirror. In fact, these equations are quite universal, and have been studied to some extent in Ref. [13] with the roles of $|\phi\rangle$ and $|\Phi\rangle$ interchanged to account for a decay of the upper level to levels other than the ground state.

The effective Hamiltonian pertaining to the pair of Schrödinger Eqs. (20) and (21) may be diagonalized in the band-theoretic framework developed in Ref. [14]. Alternatively, a multiple time-scale analysis similar to the one developped in Ref. [15] may be used to integrate Eqs. (20) and (21).

In this paper we concentrate on the weak-coupling limit in which the photon leakage is the fastest of all processes. In this case a Markov approximation may be employed, and the results are easily compared to those obtained in the standard Weisskopf-Wigner theory where the effects of the atomic motion and the spatial mode variation are neglected.

A. Spontaneous emission in the Markovian limit

To derive an equation for the excited-state amplitude $|\phi(t)\rangle$ which is valid in the limit of large Γ, we temporarily transform to an interaction picture via the unitary transformation

$$|\tilde{\phi}(t)\rangle \equiv \exp\left(\frac{i}{\hbar}\hat{T}t\right)|\phi(t)\rangle \ , \tag{22}$$

$$|\tilde{\Phi}(t)\rangle \equiv \exp\left(\frac{i}{\hbar}\hat{T}t\right)|\Phi(t)\rangle \ , \tag{23}$$

in terms of which the Schrödinger equations (20) and (21) become

$$i\hbar|\dot{\tilde{\phi}}(t)\rangle = -\hbar g \cos\left[q\left(\hat{x}+\frac{\hat{p}}{M}t\right)\right]|\tilde{\Phi}(t)\rangle \ , \tag{24}$$

$$i\hbar|\dot{\tilde{\Phi}}(t)\rangle = \hbar\left(\delta - i\frac{\Gamma}{2}\right)|\tilde{\Phi}(t)\rangle - \hbar g^* \cos\left[q\left(\hat{x}+\frac{\hat{p}}{M}t\right)\right]|\tilde{\phi}(t)\rangle \ . \tag{25}$$

To proceed, we formally integrate Eq. (25) to obtain

$$|\tilde{\Phi}(t)\rangle = ig^*\int_0^t dt \exp\left[-i\left(\delta - i\frac{\Gamma}{2}\right)(t-t')\right] \times \cos\left[q\left(\hat{x}+\frac{\hat{p}}{M}t'\right)\right]|\tilde{\phi}(t')\rangle \ . \tag{26}$$

Equation (24) shows that $|\tilde{\phi}\rangle$ is a slow variable if g is small compared with Γ. This allows us to perform the Markov approximation in Eq. (26). To be specific, we replace $|\tilde{\phi}(t')\rangle$ by $|\tilde{\phi}(t)\rangle$ in Eq. (26), but leave the cosine function intact. Using the Baker-Campbell-Haussdorff formula to disentangle the operator-valued argument of the cosine, $\hat{x}+\hat{p}t'/M$, the integration over t' is then easily carried out with the result

$$|\tilde{\Phi}(t)\rangle = \frac{g^*}{2}\left(e^{iq(\hat{x}+\hat{p}t/M)}\frac{1}{\delta' + q\hat{p}/M - i\Gamma/2} + e^{-iq(\hat{x}+\hat{p}t/M)}\frac{1}{\delta' - q\hat{p}/M - i\Gamma/2}\right)|\tilde{\phi}(t)\rangle \ , \tag{27}$$

where

$$\delta' = \omega_c - \omega_0 + \omega_r \tag{28}$$

is an effective atomic detuning. Upon inserting Eq. (27) into Eq. (24) and using Eq. (22) to transform back to the Schrödinger picture, we finally obtain

$$i\hbar|\dot{\phi}(t)\rangle = (\mathcal{T} + \mathcal{V})|\phi(t)\rangle \ , \tag{29}$$

where

$$\mathcal{T} = \frac{\hat{p}^2}{2M} - \hbar\frac{|g|^2}{4}\left(\frac{1}{\delta' + q\hat{p}/M - i\Gamma/2} + \frac{1}{\delta' - q\hat{p}/M - i\Gamma/2}\right) \tag{30}$$

is a (non-Hermitian) generalized kinetic-energy operator and

$$\mathcal{V} = -\hbar\frac{|g|^2}{4}\left(e^{2iq\hat{x}}\frac{1}{\delta' + q\hat{p}/M - i\Gamma/2} + e^{-2iq\hat{x}}\frac{1}{\delta' - q\hat{p}/M - i\Gamma/2}\right) \tag{31}$$

is a momentum-operator-dependent "optical" potential.

The Schrödinger equation (29) describes the dynamics of the upper-state probability amplitude in the Markovian limit $\Gamma \gg g$. The separation of the effective Hamiltonian in Eq. (29) into a kinetic and a potential part, respectively, reveals the two different roles they play for the dynamics of the spontaneous emission, i.e., for the time dependence of the overall upper-state probability

$$P(t) \equiv \langle\phi(t)|\phi(t)\rangle = \int_{-\infty}^{\infty} dx|\phi(x,t)|^2 \ . \tag{32}$$

The imaginary part of $\mathcal{T}$, which is spatially uniform, implies different loss rates for different velocities resulting from a simple Doppler effect. In contrast, the imaginary part of the optical potential $\mathcal{V}$ is spatially modulated, and accounts for different loss rates at different positions within the cavity. In addition, the optical potential leads to the scattering of atoms into different velocity states, which in turn have different Doppler-induced decay rates.

B. Spatial modulation neglected

Let us first concentrate on the kinetic part alone. Setting $\mathcal{V} = 0$ in Eq. (29), the resulting equation is equal in form to the one derived in our previous work describing the spontaneous decay of an atom traversing a ring cavity [9]. In this limit an atom which enters the interaction region with velocity component v along the optical axis undergoes a purely exponential decay $P(t) = \exp(-\gamma_v t)$ where the decay rate is given by

$$\gamma_v = \frac{|g|^2}{\Gamma}\left(\frac{1}{1+4(\delta'+qv)^2/\Gamma^2} + \frac{1}{1+4(\delta'-qv)^2/\Gamma^2}\right) . \tag{33}$$

Setting $v = 0$, $\omega_c = \omega_0$, and $\omega_r = 0$ (i.e., $\delta' = 0$) in the above formula, and using Eq. (11) to express the vacuum Rabi frequency in terms of the cavity volume V and the atomic dipole moment $\wp$, we readily find the well-known relation [1]

$$\gamma_{v=0,\delta'=0} = \frac{3Q}{4\pi^2}\frac{\lambda^3}{V}\gamma_f , \tag{34}$$

where $\gamma_f = (1/4\pi\epsilon_0)(4\omega_0^3\wp^2/3\hbar c^3)$ is the free-space value of the spontaneous decay rate and $\lambda \equiv 2\pi/q$ is the wavelength of the transition.

The decay rate γ_v is a superposition of two Lorentzians centered around $v = \pm\delta'/q$. These two Lorentzians account for the interaction of the atom with two symmetrically Doppler-shifted counterpropagating cavity field modes. The full width at half maximum of these Lorentzians is given by $v_{\rm cav} = c/Q$. If the atom at rest is in resonance with the cavity field, $\delta' = 0$, an increase in atomic velocity leads to a noticeable decrease in the spontaneous decay rate for velocities which are close to or exceed the "characteristic cavity velocity" $v_{\rm cav}$. For $v = v_{\rm cav}$ the decay is half as fast as given by the standard expression (34). On the other hand, if the cavity is strongly detuned, $\delta' > qv_{\rm cav}$, and the atom is at rest, it will rarely decay since it is detuned with respect to both counterpropagating waves. An increase of the atomic velocity, however, will lead to an increase of the decay rate. This increase is a result of a pure Doppler shift which tunes the atom in resonance with one component of the two counterpropagating waves, while it gets even more detuned with respect to the other component.

However, a standing wave is not to be considered as two counterpropagating running waves [10], a fact which is manifest in the optical potential which we have neglected so far, and whose influence we shall discuss now.

C. Standing-wave effects

To identify the role played by the spatial variations of the cavity mode function, it is convenient to transform Eq. (29) to a moving frame in which the atomic motion becomes slow. This Galilei transformation is achieved by means of the unitary operator

$$G_v = \exp\left[-\frac{i}{\hbar}Mv\left(\hat{x} - \frac{\hat{p}}{M}t\right)\right] , \tag{35}$$

where v is the initial atomic velocity. In terms of the transformed ket $|\phi'(t)\rangle \equiv G_v|\phi(t)\rangle$, the Schrödinger equation is formally identical to Eq. (29) with the transformed quantities $\mathcal{T}'$ and $\mathcal{V}'$ obtained from $\mathcal{T}$ and $\mathcal{V}$ by the substitution $\hat{p} \to \hat{p} + Mv$ in the Lorentzian denominators and $\hat{x} \to \hat{x} + vt$ in the exponents of $\mathcal{V}$. Under the assumption that the momentum spread in the atomic rest frame remains small, the Schrödinger equation for $|\phi'\rangle$ may be further simplified by setting $\hat{p} = 0$ (Raman-Nath approximation), so that eventually, with $\hat{x}_t \equiv \hat{x} + vt$,

$$i\hbar|\dot{\phi}'\rangle = -\frac{\hbar|g|^2}{4}\left(\frac{1+e^{2iq\hat{x}_t}}{\delta'+qv-i\Gamma/2} + \frac{1+e^{-2iq\hat{x}_t}}{\delta'-qv-i\Gamma/2}\right)|\phi'\rangle . \tag{36}$$

Being diagonal in the position representation, this equation is easily integrated to obtain the overall upper-state probability

$$P(t) = e^{-\gamma_v t}\int_{-\infty}^{\infty} dx\, e^{-\eta(x,t)}|\phi(x,0)|^2 , \tag{37}$$

where γ_v has been defined in Eq. (33), and

$$\eta(x,t) = \frac{\gamma_v \sin(qvt)}{qv}\left(\cos(2qx+qvt) + \frac{2qv}{\Gamma}\frac{1-4(\delta'^2-q^2v^2)/\Gamma^2}{1+4(\delta'^2+q^2v^2)/\Gamma^2}\sin(2qx+qvt)\right) \tag{38}$$

accounts for the local variation of the spontaneous decay due to the spatial modulation of the resonant mode function. Further simplification of this somewhat opaque expression is achieved by noticing that the amplitude of the spatial sine function never exceeds $2g^2/\Gamma^2$, a quantity which is assumed to be small. That is, we may neglect the spatial sine function in Eq. (38) so that

$$\eta(x,t) = \frac{\gamma_v \sin(qvt)}{qv}\cos(2qx+qvt) . \tag{39}$$

If the atom is initially well localized around some x_0, the excited-state probability $P(t) = \exp[-\gamma_v t - \eta(x_0,t)]$ displays an oscillatory behavior superimposed on the standard exponential decay. If the atom enters the cavity at an antinode of the cavity field, for example, $x_0 = 0$, its decay is initially twice as fast as in the case where the spatial mode function has been neglected. If the atom, however, enters the cavity in a region of a node of the cavity field ($x_0 = \lambda/4$, for example), the atom does not decay at all initially. Only after some time, when the velocity of the atom brings it into a region of nonzero field strength, does the decay start to take place.

In general, the effect of the spatial mode variation on the decay of the atom results is an oscillation with period λ/v. This period has to be comparable to the lifetime γ^{-1} in order to lead to observable effects, thereby setting the atomic velocity to $v \simeq v_{\rm mod} \equiv (c/Q)(g/\Gamma)^2$.

IV. DISCUSSION

In this paper, we have shown that the spontaneous emission of a moving atom inside a cavity is modified by two different mechanisms, each of which is observable on a different velocity scale. The modification of the spontaneous emission due to a Doppler-induced detuning of the cavity line is observable for velocities which exceed the characteristic velocity $v_{\text{cav}} = c/Q$, while the modulation of the exponential decay rate due to spatial mode effects is observable for velocities which do not exceed the characteristic velocity v_{mod}, which is smaller by a factor $(g/\Gamma)^2$ than v_{cav}. This means that the decay of fast atoms is primarily governed by the spectral cavity response function (sometimes called "mode density"), while the decay of slow atoms is primarily influenced by the spatial structure of the cavity modes.

Experimentally, the inhibition of spontaneous emission of fast atoms is most easily confirmed in the optical regime. For a cavity of 1-mm linear dimension and quality factor $Q = 10^8$, we have $\Gamma/2\pi = 5 \times 10^7$ Hz. The lifetime of an atom at rest inside the cavity is then $\Gamma/(2|g|^2) = 1/2\pi \times 10^{-4}$ sec and $v = c/Q = 3$ m/sec. At this velocity, the time of flight of the atom through the cavity is by 0.3×10^{-3} sec, which is long enough to ensure decay. With the weak-coupling parameter $g/\Gamma = 10^{-2}$, on the other hand, we have $v_{\text{mod}} = 0.3$ mm/sec, so that spatial mode effects require exceedingly slow atomic beams.

In fact, the influence of the spatial mode structure should be most easily observed in microwave experiments. For a cavity of 1-cm linear dimension and quality factor $Q = 0.5 \times 10^6$, we have $\Gamma/2\pi = 4.4 \times 10^4$ Hz and a lifetime at rest 0.3×10^{-5} sec. In this case, the weak-coupling parameter is $g/\Gamma = 0.8$. Accordingly, the spatial mode structure influences the dynamics of spontaneous decay most dramatically for atoms of velocity slower than 3.6×10^2 m/sec.

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[1] E. M. Purcell, Phys. Rev. **69**, 681 (1946).

[2] K. H. Drexhage, in *Progress in Optics XII*, edited by E. Wolf (North-Holland, New York, 1974).

[3] D. Kleppner, Phys. Rev. Lett. **47**, 233 (1981).

[4] P. Goy, J. M. Raimond, M. Gross, and S. Haroche, Phys. Rev. Lett. **50**, 1903 (1983).

[5] R. G. Hulet, E. S. Hilfer, and D. Kleppner, Phys. Rev. Lett. **55**, 2137 (1985).

[6] W. Jhe *et al.*, Phys. Rev. Lett. **58**, 666 (1987).

[7] D. J. Heinzen, J. J. Childs, J. E. Thomas, and M. S. Feld, Phys. Rev. Lett. **58**, 1320 (1987).

[8] F. De Martini *et al.*, Phys. Rev. A **43**, 2480 (1991).

[9] Z. Bialynicka-Birula, P. Meystre, E. Schumacher, and M. Wilkens, Opt. Commun. **85**, 315 (1991).

[10] B. W. Shore, P. Meystre, and S. Stenholm, J. Opt. Soc. Am. B **8**, 903 (1991).

[11] L. Knöll, W. Vogel, and D.-G. Welsch, Phys. Rev. A **43**, 543 (1991).

[12] A. P. Kazantsev, G. J. Surdovich, and V. P. Yakovlev, *Mechanical Action of Light on Atoms* (World Scientific, Singapore, 1990).

[13] B. Ya. Dubetskii, A. P. Kazantsev, V. P. Chebotaev, and V. P. Yakovlev, Zh. Eksp. Teor. Fiz. **89**, 1190 (1985) [Sov. Phys.—JETP **62**, 685 (1985)].

[14] M. Wilkens, E. Schumacher, and P. Meystre, Phys. Rev. A **44**, 3130 (1991).

[15] J. Mizerski, W. Miklaszewski, and S. Stenholm, J. Opt. Soc. Am. B **8**, 509 (1991).

[16] G. Rempe, F. Schmidt-Kaler, and H. Walther, Phys. Rev. Lett. **64**, 2783 (1990).