Interfaces driven by quenched random fields

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(Received 7 November 1991)

The growth of interfaces in a discrete solid-on-solid model driven by quenched random fields have been simulated in (1+1) dimensions. It is found that the interface width initially grows with time t as $\ln^2(t)$, and with a power law $t^{1/2}$ for long times. The crossover time increases sharply with increasing randomness.

PACS number(s): 05.40.+j, 05.50.+q, 68.10.Gw

The scaling behavior of growing interfaces has recently attracted extensive theoretical and numerical explorations [1–4]. Few of these investigations consider the effect of quenched randomness on the kinetics of interfacial growth. The available experimental data [5] are, however, normally obtained in porous media where quenched randomness plays an important role. Similar problems arise in the context of pinning and roughening of domain walls by impurities [6–11], which is important because below the critical temperature many static and dynamic properties of systems with broken Ising symmetry are dominated by the behavior of the domain walls. The interfacial fluctuations are also important in determining the equilibrium and non-equilibrium properties of wetting and wetting transitions [12]. In this paper, therefore, we report results of Monte Carlo simulations of interfacial growth in a discrete solid-on-solid (SOS) model driven by quenched random fields in (1+1) dimensions. We have found that the interface initially behaves as one-dimensional random walker driven by quenched random forces, with the interfacial width w growing in time t as $\ln^2(t)$; for longer times, the growth law crosses over to $w \sim t^{1/2}$ consistent with a Kolmogorov-type argument [4]. The crossover time t_c increases sharply with the degree of randomness.

The model used in our simulation is described by a Hamiltonian

$$H = \sum_{\langle ij \rangle} J|h_i - h_j| - \sum_i \sum_{h=0}^{h_i} [v(h,i)+\mu] , \tag{1}$$

where h_i is the height at site i of an array of columns describing the position of the interface of a $(d-1)$-dimensional lattice of size L^{d-1}. $\langle i,j \rangle$ denotes a sum over all nearest-neighbor pairs; $v(h,i)$ is a random function of h and i uniformly distributed on the interval $[-\Delta,\Delta]$; and μ is a constant independent of i and h. Equation (1) is the strong-anisotropic limit of the random-field Ising model [13]. Similar Hamiltonians without quenched randomness have been used to study equilibrium and nonequilibrium interfaces in pure systems [14], crystal growth [15], as well as wetting film growth [16].

The standard Metropolis Monte Carlo method is used in our simulations. The simulation process consists of the following steps: (a) randomly pick a site i; (b) decide with equal probability to try an increase or decrease of h_i by 1; (c) calculate the change of energy ΔH due to the change of h_i, accept the change of h_i with probability P, where $P=\exp(-\Delta H/T)$ for $\Delta H > 0$, and $P=1$ for $\Delta H \le 0$. T is the temperature. The time t is measured as the number of Monte Carlo trials per site (MCS). Periodic boundary conditions are employed in directions parallel to the interface. The heights are initially set to $h_i=0$ for all sites. The time-dependent interface width $w(t)$ is calculated as

$$w(t) = \left[L^{-(d-1)} \sum_i [h_i(t) - \bar{h}(t)]^2 \right]^{1/2} , \tag{2}$$

where

$$\bar{h}(t) = L^{-(d-1)} \sum_i h_i(t) \tag{3}$$

is the mean position of the interface at t. The time is typically run up to 10^5 MCS. Finally, for given L, μ, Δ, and T, an average is performed over several different realizations of the random-field configurations. The typical system size L used in our simulations is 4000. Some runs on larger L up to 10 000 have been done. For all the results shown in this paper, we do not see any size dependence.

The growth kinetics described by the Monte Carlo simulations is equivalent to a stochastic process [17] which, in the continuous limit, is described by a Langevin equation

$$\frac{\partial h}{\partial t} = \sigma \nabla^2 h + v(h,\mathbf{x}) + \mu + \xi(\mathbf{x},t) , \tag{4}$$

where σ is the interfacial stiffness, which is generally proportional to the coupling J; while $v(h,\mathbf{x})$ and $\xi(\mathbf{x},t)$ are the quenched random fields and thermal noises, respectively, with correlations

$$\langle v^{\alpha}(\mathbf{x},h) v^{\beta}(\mathbf{x}',h') \rangle = \Delta' \delta^{\alpha\beta} \delta(\mathbf{x}-\mathbf{x}') \delta(h-h') , \tag{5}$$

$$\langle \xi^{\alpha}(\mathbf{x},t) \xi^{\beta}(\mathbf{x}',t') \rangle = D \delta^{\alpha\beta} \delta(\mathbf{x}-\mathbf{x}') \delta(t-t') , \tag{6}$$

and

$$\langle v \rangle = \langle \xi \rangle = 0 , \tag{7}$$

where the superscripts α and β represent different replicas of the random-field configurations.

Maintain σ and Δ' invariant after rescaling $\mathbf{x} \to \mathbf{x}b$,

$t \to tb^z$, and $(h-\mu)t \to (h-\mu t)b^\alpha$, one obtains that $z=2$, $\alpha=(5-d)/3$, and $D \to Db^{-(d+1)/3} \to 0$. Therefore, we expect that, at long times, the interface fluctuations will be dominated by the random fields, and the interfacial width scales as

$$w \sim L^{(5-d)/3} , \tag{8}$$

and

$$w \sim t^{\alpha/z} \sim t^{(5-d)/6} . \tag{9}$$

This is consistent with both the Imry-Ma argument [18] and the Kolmogorov argument [4]. In particular, for a $(1+1)$-dimensional interface simulated in this work, we expect $w \sim t^{1/2}$ growth law for long times. Equation (8) has been numerically verified for $d=2$ and 3 by the transfer matrix method [10,11], yet no effort has been made to check Eq. (9).

For $\sigma=0$ and $\mu=0$, Eq. (4) describes a one-dimensional random walker driven by random forces. The mean-square displacement is expected [19] to increase with time as $\ln^2(t)$. Heuristically, $t \sim \exp(E/T)$, where $E \sim \Delta w^{1/2}$ is the energy gain in the random fields, thus

$$w \sim \left[\frac{T}{\Delta}\right]^2 \ln^2(t) . \tag{10}$$

Now we turn to present our simulation results in $(1+1)$ dimensions. Without losing generality, we always take $J=1$. Figure 1 is a plot of $\log_{10}(w)$ versus $\log_{10}(t)$ for $L=4000$, $T=1.0$, $\Delta=1.0$. The different curves are for $\mu=0.06$, 0.04, 0.02, and 0.00, respectively. We observe that the curves for larger values of μ reach a slope of $\frac{1}{2}$ for long times. This confirms the prediction of the random-field-dominated growth law $w \sim t^{1/2}$. We have done some longer runs for smaller systems to check that the width goes to saturation after the $t^{1/2}$ regime. It is important not to confuse this $t^{1/2}$ behavior with normal random-walk behavior in pure systems. Here the $t^{1/2}$ law is due to the dynamics of searching through the random-field configurations to minimize the energy given by Eq. (2), while the $t^{1/2}$ behavior in normal random walk is purely an entropy effect. For general dimensions d, the interface growth law given by Eq. (9) is not the same as normal random-walk behavior.

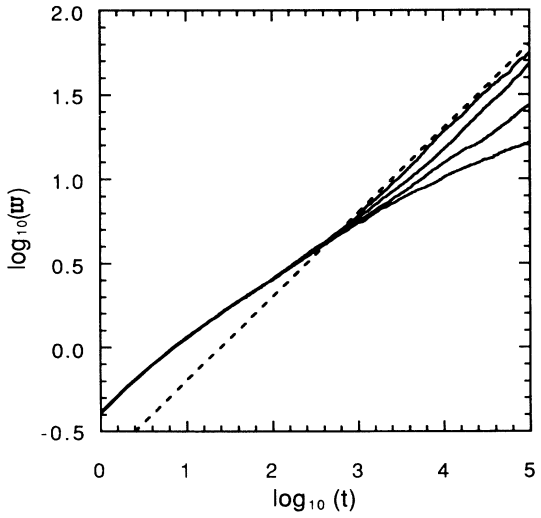

FIG. 1. For $T=1.0$, $\Delta=1.0$, and $L=4000$, $\log_{10}(w)$ is plotted against $\log_{10}(t)$ for different μ. The solid curves from top to bottom correspond to $\mu=0.06$, 0.04, 0.02, and 0.00, respectively. The data for larger μ reach a slope of $\frac{1}{2}$ at long times. The data in early times fit $w \sim \ln^2(t)$ well, cf., Fig. 2. The crossover time increases with decreasing μ. A dashed straight line of slope $\frac{1}{2}$ is included to guide the eyes.

For early times, the curves virtually overlap with each other. They fit to $w \sim \ln^2(t)$ nicely as displayed in Fig. 2, which is a plot of w versus $[\log_{10}(t)]^2$ for $\mu=0.06$ and the same L, T, and Δ as in Fig. 1. This logarithmic growth can be easily understood by the fact that, at the early times, the lateral correlation length is small, and therefore each column moves independently as a random walker driven by random forces in one dimension [19].

We also observe that, as shown in Fig. 1, the crossover time t_c increases rapidly as μ increases. Note that Δ/μ is a measure of the relative degree of randomness of the total driving force $\mu+v(h,x)$. The reason for this increase is that, for larger μ, the average position of the interface moves faster, thus increasing the chances of searching through the quenched random fields. The development of the random-field-dominated fluctuations is basically an optimization process in a multidimensional random surface. For given Δ and T, the speed of optimization increases with μ as $\exp(\mu/T)$. Therefore, we expect t_c to be given by

$$t_c = t_{c0}(J,\Delta,T)\exp(-\mu/T) . \tag{11}$$

$t_{c0}(J,\Delta,T)$ can be derived by the following argument: The logarithmic law, which is the result of independent growth, will cease to be valid when the energy due to the

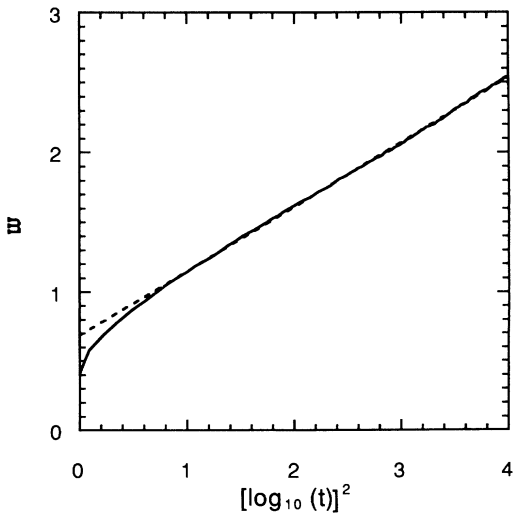

FIG. 2. The interface width w is plotted against $\log_{10}^2(t)$ for $\mu=0.06$, $T=1.0$, $\Delta=1.0$, and $L=4000$. The dashed straight line is a fit to the data.

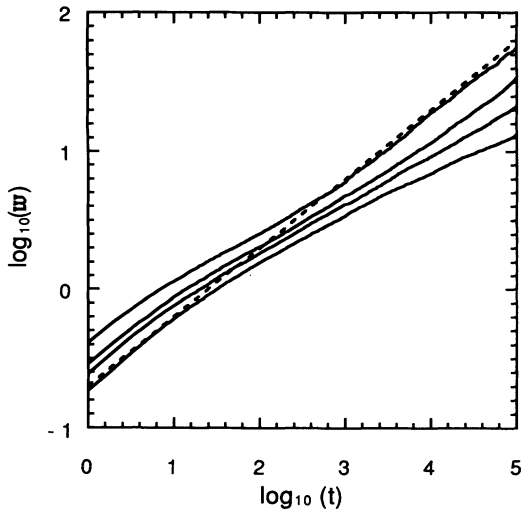

FIG. 3. A plot of $\log_{10}(w)$ vs $\log_{10}(t)$ for $\mu=0.06$, $\Delta=1.0$, and $N=4000$ at different temperatures. The solid curves from top to bottom correspond to $T=1.0$, 0.7, 0.6, and 0.5, respectively. The crossover time increases with decreasing temperature. A dashed line of slope $\frac{1}{2}$ is included to guide the eyes.

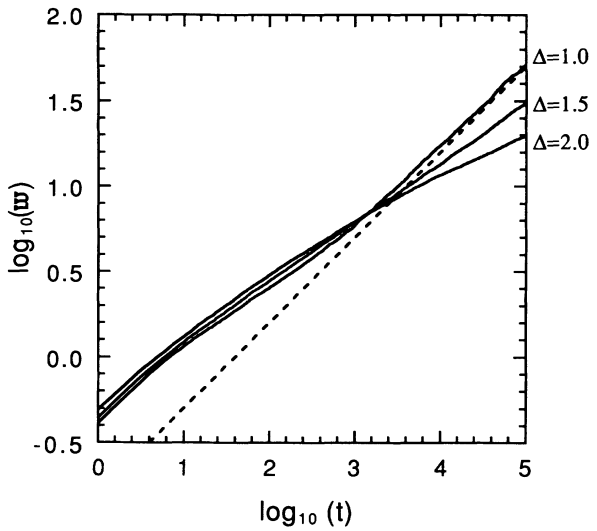

FIG. 4. A plot of $\log_{10}(w)$ vs $\log_{10}(t)$ for $\mu=0.05$, $T=1.0$, and $N=4000$ for $\Delta=1.0$, 1.5, and 2.0. The crossover time increases with increasing Δ. A dashed line of slope $\frac{1}{2}$ is included to guide the eyes.

coupling Jw is of the order of the energy gain in the random fields $\Delta w^{1/2}$. Setting $Jw \sim \Delta w^{1/2}$ and using Eq. (10) one obtains

$$t_{c0}(J,\Delta,T) \sim \exp\left[\frac{\Delta^2}{JT}\right] . \tag{12}$$

Combining Eqs. (11) and (12), we have a general expression for the crossover time

$$t_c \sim \exp\left[\frac{\Delta^2}{JT} - \frac{\mu}{T}\right] . \tag{13}$$

Equation (13) implies that the crossover time increases with decreasing temperatures for given μ and Δ. This is demonstrated in Fig. 3 which is a plot of $\log_{10}(w)$ versus $\log_{10}(t)$ for $\mu=0.06$ and $\Delta=1.0$ at four different temperatures $T=1.0$, 0.7, 0.6, and 0.5. The data for early times again fit the $\ln^2(t)$ behavior well for all temperatures. Decreasing the temperature makes it more difficult to overcome local minima in the random-field configuration space, thus increasing the crossover time. Comparing the data for early times in Figs. 1 and 3, we see that the interface width is sensitive to the temperature but insensitive to the average driving force μ. This is not surprising since the early growth is mainly a process of local thermal excitations.

The crossover time also increases with increasing Δ for given μ and T. This is shown in Fig. 4 where we plot $\log_{10}(w)$ versus $\log_{10}(t)$ for $\mu=0.05$, $T=1.0$, and $\Delta=1.0$, 1.5, and 2.0 respectively. This is again in qualitative agreement with Eq. (13). It is, however, difficult to check Eq. (13) quantitatively since precise values for t_c cannot be obtained from our simulations.

In conclusion, we have presented the results of Monte Carlo simulations of interface growth in a discrete solid-on-solid model with quenched random fields in $(1+1)$ dimensions. In the early times, the correlation length in the direction parallel to the interface is small; therefore each column of the interface moves independently like a random walker driven by quenched random forces, and the interface width grows with a logarithmic law $w \sim \ln^2(t)$. As the correlations among the columns increase with time, the growth behavior crosses over to a random-field-dominated growth regime in which the interface width grows with a power law $w \sim t^{1/2}$. The crossover time t_c increases exponentially with an increasing degree of randomness of the driving field. All these results are consistent with simple scaling arguments. It would be interesting to investigate these behaviors in higher dimensions, especially for $(2+1)$ dimensions.

The simulations were performed on the Emory UNIX system. We are grateful for funding support from the Department of Physics at Emory University.

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