Theory of the high-gain optical klystron

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We present an analytic expression of the harmonic bunching produced by the dispersive section of a free-electron laser (FEL) optical klystron operating in the high-gain exponential regime. This model allows the evaluation of the operating constraints and limits, the possible optimizations and advantages of such a device with respect to the conventional FEL configuration, i.e., shortening of the overall length and reduction of the induced energy spread, provided that the initial energy spread is smaller than the one required for the normal high-gain FEL operator. The analytical expressions derived here are confirmed by the numerical solution of the full nonlinear set of equations for the FEL.

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I. INTRODUCTION

An optical klystron [1] is composed of two wigglers, separated by a dispersive section, that can either be a free-space section or a proper magnetic device. In the first wiggler (the modulator) an electron beam, interacting with a radiation field (that can be provided either by an external laser or by the spontaneous radiation emitted by the electron beam itself), receive an energy modulation that is then transformed into a spatial modulation (bunching) in the dispersive section. The bunching provided by the dispersive section enhances the coherent emission of radiation in the second wiggler.

This scheme has, in principle, two distinct advantages over the bunching produced by the conventional high-gain free-electron-laser (FEL) amplifier scheme [2]: first, the total length of the device can be considerably reduced, second, the electron bunching in the dispersive section is gained with no additional growth of the FEL-induced energy spread. However, as we will show, in an optical klystron the bunching efficiency is very sensitive to the initial energy spread of the electron beam, which needs to be sensibly smaller than the one required for the normal high-gain FEL operation [3].

For small initial field intensities and short modulators the interaction between the electrons and the radiation field in the first wiggler can be described (as in the small-gain theory of a FEL) assuming a constant field and an induced sinusoidal modulation of the electron beam [4,5]. This is the typical situation for operation of an optical klystron in a storage ring [6]. It can be shown that there exists an optimum value both for the modulator length and the dispersive section strength, given the initial field amplitude and beam-energy spread.

The inclusion of the energy spread in such an analysis introduces a strong damping term for the bunching obtained in the dispersive section [5]; such energy-spread effects can be partially reduced by increasing the electron modulation amplitude, i.e., the initial field intensity or the length of the first wiggler. This can lead to a situation where the constant-field approximation no longer holds, and the modulator operates in the high-gain regime. This kind of operation has been recently proposed by several authors [7,8].

In the present paper we derive a complete analytical model, useful in describing the bunching process in both the small- and high-gain regimes and incorporating in a natural way energy-spread effects. We show that strong energy-spread limitations still hold in the high-gain regime and, furthermore, we describe the optimization and give the evaluation of the bunching on the fundamental and nth harmonic for both regimes. We stress that our model can describe the bunching process also in a conventional FEL, and even in this case its validity holds far into the exponential regime.

II. SOLUTION OF THE LINEAR EQUATIONS

To describe the field and the electron dynamics in the modulator, we consider the one-dimensional model of Ref. [3] for a FEL, written in the form

$$\frac{d\theta(\theta_0,p_0,\bar{z})}{d\bar{z}} = p(\theta_0,p_0,\bar{z}), \tag{1}$$

$$\frac{dp(\theta_0,p_0,\bar{z})}{d\bar{z}} = -\left(Ae^{i\theta(\theta_0,p_0,z)} + \text{c.c.}\right), \tag{2}$$

$$\frac{dA(\bar{z})}{d\bar{z}} = \langle e^{-i\theta}\rangle = \int_0^{2\pi}\int_{-\infty}^{+\infty} d\theta_0 dp_0 f(\theta_0,p_0)e^{-i\theta}, \tag{3}$$

where we have used the dimensionless scaling (the so-called "universal scaling") introduced in Ref. [3], namely,

$$\theta = (k+k_w)z - ckt,$$
$$p = \frac{\gamma-\gamma_r}{\rho\gamma_r},$$
$$A = \frac{\omega}{\omega_p\sqrt{\rho\gamma_r}}a,$$
$$\bar{z} = \frac{4\pi\rho}{\lambda_w}z \equiv \frac{z}{\ell_g},$$

where $k_w = 2\pi/\lambda_w$ is the wiggler wave number, $\omega = ck = 2\pi c/\lambda$ is the radiation angular frequency, $\omega_p = \sqrt{4\pi e^2 n/m}$ is the plasma frequency, $\gamma_r = \sqrt{\lambda_w(1+a_w{}^2)/2\lambda}$ is the resonant energy (units of mc^2), $a_w = e\lambda_w B_w/\sqrt{2}\,2\pi mc^2$ is the rms wiggler parameter, and $a = e\lambda E/2\pi mc^2$ is the complex amplitude of the dimensionless radiation vector potential. The scaling of A has been choosen so that $|A|^2 = (1/\rho)(|E|^2/4\pi n_e \gamma_r mc^2)$ is the FEL power-conversion efficiency divided by ρ, $\ell_g = \lambda_w/4\pi\rho$ is the FEL gain length, i.e., the e-folding length of the high-gain exponential instability, and $\rho = \gamma_r^{-1}(a_w\omega_p/ck_w)^{2/3}$ is the fundamental FEL parameter.

In Eq. (3) $\langle\rangle$ represents an average over the initial distribution function $f(\theta_0, p_0)$ of the electrons.

Let us study the solution of the system (1)–(3) around the initial condition for the electrons, θ_0, p_0, and a small input field A_0. Defining $\bar\theta$, $\bar p$ as

$$\theta(\theta_0, p_0, \bar z) \equiv \theta_0 + p_0 \bar z + \bar\theta, \tag{4}$$

$$p(\theta_0, p_0, \bar z) \equiv p_0 + \bar p, \tag{5}$$

and linearizing Eqs. (1)–(3) for small values of $\bar\theta$, $\bar p$, and A, we obtain the following system:

$$\frac{d\bar\theta}{d\bar z} = \bar p, \tag{6}$$

$$\frac{d\bar p}{d\bar z} = -\left(Ae^{i(\theta_0 + p_0\bar z)} + \text{c.c.}\right), \tag{7}$$

$$\frac{dA}{d\bar z} = -i\langle\bar\theta e^{-i(\theta_0 + p_0 \bar z)}\rangle. \tag{8}$$

Solving the system (6)–(8) with Laplace transform techniques, we have the exact form of the self-consistent field evolution in the modulator, $A_s(\bar z)$:

$$A_s(\bar z) = \sum_{j=1}^{3} \tilde c_j e^{i\lambda_j \bar z}, \tag{9}$$

where the coefficients $\tilde c_j$ $(j = 1, \ldots, 3)$ satisfy the initial condition at $\bar z = 0$, and λ_j $(j = 1, \ldots, 3)$ are the solutions of the FEL dispersion relation

$$\lambda + \int_0^{2\pi}\int_{-\infty}^{\infty} d\theta_0 dp_0 \frac{f(\theta_0, p_0)}{(\lambda + p_0)^2} = 0. \tag{10}$$

This expression reduces to the usual cold-beam cubic relation of Ref. [3] in the case $f(\theta_0, p_0) = (1/2\pi)\delta(p_0 - \delta)$, where δ is the electron-energy detuning. Here the distribution function $f(\theta_0, p_0)$ takes into account the initial energy spread of the electron beam. As other authors have already reported [10], the electron-beam-emittance effects can be described by an equivalent energy spread. In the one-dimensional model used here, the electron-beam emittance can be included as a spread of the resonant energy of the electrons, and the overall FEL gain can be optimized by a suitable detuning of the initial average electron energy [11].

We can now use the self-consistent-field solution (9) and integrate analytically the linearized equations of motion for the electrons (6) and (7). Defining

$$A_{\rm I}(\bar z, p_0) \equiv \int_0^{\bar z} d\xi A_s(\xi) e^{ip_0\xi}, \tag{11}$$

$$A_{\rm II}(\bar z, p_0) \equiv \int_0^{\bar z} d\xi A_{\rm I}(\xi, p_0) = \int_0^{\bar z} d\xi (\bar z - \xi) A_s(\xi) e^{ip_0\xi}. \tag{12}$$

We can now solve exactly Eqs. (6) and (7) for the electrons, obtaining

$$\bar\theta(\bar z, \theta_0, p_0) = -[A_{\rm II}(\bar z, p_0) e^{i\theta_0} + \text{c.c.}], \tag{13}$$

$$\bar p(\bar z, \theta_0, p_0) = -[A_{\rm I}(\bar z, p_0) e^{i\theta_0} + \text{c.c.}]. \tag{14}$$

These equations give the small signal particle phase-space orbits under the action of the self-consistent field $A_s(\bar z)$, both in the small- and the high-gain regimes.

The behavior of the dispersive section in an optical-klystron configuration can then be approximated by a simple pointlike region that imposes the following transformation on the electron phase and energy variables:

$$\theta(\theta_0, p_0, \bar z_{\rm out}) = \theta(\theta_0, p_0, \bar z_{\rm in}) + Dp(\theta_0, p_0, \bar z_{\rm in}), \tag{15}$$

$$p(\theta_0, p_0, \bar z_{\rm out}) = p(\theta_0, p_0, \bar z_{\rm in}), \tag{16}$$

where D is the dispersive section strength, that, in the case of a magnetic device, can be evaluated, given a sufficiently large magnetic field $B(\bar z)$, by the formula

$$D = \rho k \left(\frac{e}{\gamma mc^2}\right)^2 \int_{\bar z_{\rm in}}^{\bar z_{\rm out}} d\xi \left(\int_{\bar z_{\rm in}}^{\xi} d\eta B(\eta)\right)^2. \tag{17}$$

For instance, a dispersive section made out of three dipoles of length $s/4$, $s/2$, and $s/4$ and field values $-B_0$, B_0, and $-B_0$, respectively, has a strength $D = \frac{1}{48}\rho k(eB_0/mc^2\gamma)^2 s^3$ [7].

In the case in which the dispersive section is a free-space region of length L, we have

$$D = \frac{4\pi\rho L}{\lambda_w(1 + a_w^2)} = \frac{L}{\ell_g(1 + a_w^2)} \tag{18}$$

as can be easily shown in the calculation

$$\theta_{\rm out} - \theta_{\rm in} = (k + k_w)z - ckt = \left(k + k_w - \frac{k}{\beta_\parallel}\right) z$$

$$\sim k_w\left(1 - \frac{\bar\gamma^2}{\gamma^2}\right) z \sim \frac{k_w}{1 + a_w^2}\frac{\gamma^2 - \gamma_r^2}{\gamma_r^2} z + \frac{k_w a_w^2}{1 + a_w^2} z,$$

where $\beta_{\|} = z/ct$, $\bar{\gamma}^2 \equiv \lambda_w/2\lambda$, and $\gamma^2 \gg 1$. The first term, in the Compton limit $(\gamma-\gamma_r)/\gamma_r \ll 1$ can be written as $[2k_w\rho/(1+a_w^2)]zp = Dp$, where D is given by expression (18). The other term is simply a phase shift due to electron detuning with respect to the ponderomotive FEL potential. Hence, being equal for all the electrons, it does not contribute to the bunching or debunching of the system.

III. CALCULATION OF THE BUNCHING PARAMETER

From the expression of the particle phase-space orbits in the modulator and from Eqs. (15) and (16), we can evaluate the bunching out of a dispersive section, of strength D, after a first wiggler section of length $\bar{z}$, given by the following formula:

$$b(\bar{z}) = \langle e^{-i\theta(\bar{z}_{\rm out})}\rangle = \langle e^{-i[\theta(\bar{z}_{\rm in})+Dp(\bar{z}_{\rm in})]}\rangle = \langle e^{-i[\theta_0+(D+\bar{z})p_0+\bar{\theta}+D\bar{p}]}\rangle$$
$$= \int_0^{2\pi} d\theta_0 \int_{-\infty}^{+\infty} dp_0 f(\theta_0,p_0)\exp\{-i[\theta_0+(D+\bar{z})p_0+\bar{\theta}(\bar{z},\theta_0,p_0)+D\bar{p}(\bar{z},\theta_0,p_0)]\}. \qquad (19)$$

From Eqs. (13) and (14) we can evaluate the term $\bar{\theta}+D\bar{p}$ in the previous expression, defining

$$A_{\rm III}(\bar{z},p_0,D) = A_{\rm II}(\bar{z},p_0) + DA_{\rm I}(\bar{z},p_0) = -\int_0^{\bar{z}} d\xi A_s(\xi)(D+\bar{z}-\xi)e^{ip_0\xi} \qquad (20)$$

we have

$$\bar{\theta}+D\bar{p} = -[A_{\rm III}(\bar{z},p_0,D)e^{i\theta_0} + \text{c.c.}]. \qquad (21)$$

Hence Eq. (19) can be rewritten as

$$b(\bar{z}) = \int_0^{2\pi} d\theta_0 \int_{-\infty}^{+\infty} dp_0 f(\theta_0,p_0)\exp\big(-i\{\theta_0+(D+\bar{z})p_0-[A_{\rm III}(\bar{z},p_0,D)e^{i\theta_0}+\text{c.c.}]\}\big). \qquad (22)$$

Again, given the initial electron distribution function $f(\theta_0,p_0)$, and hence the field $A_s(\bar{z})$ from Eq. (9), the above expression is the analytical form of the bunching in the linear regime of the exponential instability, including energy-spread and dispersive section effects. It should be noted that in the case $D=0$ this expression describes the bunching in a conventional FEL.

IV. ESTIMATES OF THE BUNCHING PARAMETER, LIMITS, AND OPTIMIZATIONS OF THE DISPERSIVE SECTION

In order to evaluate expression (22) we use an initial electron distribution of the form $f(\theta_0,p_0) = (1/2\pi)(1/\sigma\sqrt{2\pi})\exp[-(p_0-\delta)^2/2\sigma^2]$, corresponding to a beam uniformly spread in phase and Gaussian distributed in energy around the value of γ_0 [$\sigma \equiv \Delta\gamma/\rho\gamma$ and $\delta \equiv (\gamma_0-\gamma_r)/\rho\gamma_r$]. In the limit $\sigma\bar{z} \ll 1$, the term $e^{ip_0\xi}$ appearing in the expression (20) and the term $e^{-ip_0\bar{z}}$ in (22) can be approximated, respectively, with $e^{i\delta\xi}$ and $e^{-i\delta\bar{z}}$. Thus the two integrals can be performed independently, yielding the result

$$b(\bar{z}) = ie^{i[\varphi_{\rm III}(\bar{z})-\delta\bar{z}]}e^{-D^2\sigma^2/2}J_1(2|A_{\rm III}|(\bar{z},\delta,D)), \qquad (23)$$

where $\varphi_{\rm III}$ is the phase of the complex function defined in (20) and J_1 is the Bessel function of the first kind of order one. Note that the condition $\sigma\bar{z} \ll 1$ can be rewritten in terms of the relative energy spread as $\Delta\gamma/\gamma \ll 1/(4\pi N_w) \approx \Delta\gamma/\gamma|_{\rm nat}$, where N_w is the number of wiggler periods and $\Delta\gamma/\gamma|_{\rm nat}$ is the *natural* energy spread of the small-gain FEL process.

The above expression leads to the inequality $|b(\bar{z})|^2 < \exp(-D^2\sigma^2)$, that shows the strong effect of the energy spread, and that the requirement $D\sigma \simeq 1$, already referenced in the low-gain regime [5], holds also when the first wiggler operates in the high-gain exponential regime. The above requirement, in the interesting case $D \gg 1$, leads to $\sigma \lesssim 1/D$, which is much more restrictive than the condition $\sigma < 1$, necessary for the high-gain operation of a FEL amplifier.

Expression (23) suggests immediately the optimizing criteria for the dispersive section parameter D, given the length of the buncher $\bar{z}$ and the initial energy spread σ. Provided that the constraint on the energy spread is satisfied ($D\sigma \lesssim 1$), in order to reach the maximum bunching after the dispersive section, one has to maximize the Bessel function factor in Eq. (23), i.e., to choose the value of D for which $|A_{\rm III}| \approx 1$. An equivalent recipe has to be followed to find the optimum modulator length $\bar{z}$, given the dispersive section strength D.

In the limit $\bar{z} \ll 1$ the expression (23) reduces to the usual expression of the bunching in a small-gain optical klystron [4,5]. In this case $|A_{\rm III}(\bar{z},\delta,D)| \approx A_0D\bar{z}$ and we obtain

$$|b(\bar{z})| = e^{-D^2\sigma^2/2}|J_1(2A_0D\bar{z})|, \qquad (24)$$

where A_0 is the initial field amplitude. In this regime, the optimal modulator length for a given D is

$$\bar{z}_{\rm opt} \approx \frac{1}{A_0 D} \gtrsim \frac{\sigma}{A_0}. \qquad (25)$$

In the high-gain regime, however, we can use the solution of systems (6)–(8) in order to provide an explicit form of expression (23). Considering a case of an initial uniform energy spread of full width Δ around the average value of δ, the field can be explicitly evaluated as

$$A_s(\bar{z}) = A_0 e^{-i\delta\bar{z}} \sum_{j=1}^{3} c_j e^{i\lambda_j \bar{z}} = A_0 e^{-i\delta\bar{z}} \sum_{j=1}^{3} \frac{\lambda_j^2 - \Delta^2/4}{(\lambda_j - \lambda_h)(\lambda_j - \lambda_k)} e^{i\lambda_j \bar{z}}, \quad j \neq k \neq h \qquad (26)$$

where the λ_j are the solutions of the FEL dispersion relation, namely,

$$(\lambda - \delta)\left(\lambda^2 - \frac{\Delta^2}{4}\right) + 1 = 0. \qquad (27)$$

In order to derive the analytical solution of the radiation field we have used for the Gaussian energy distribution a uniform distribution of full width $\Delta = 2\sqrt{3}\sigma$, corresponding to the same rms value. Substitution of the self-consistent-field evolution of (26) yields the following expression for the electron bunching parameter after the dispersive section:

$$|b(\bar{z})| = e^{-D^2\sigma^2/2} \left| J_1 \left\{ \left| 2A_0 \sum_{j=1}^{3} \frac{c_j}{i\lambda_j} \left[\left(D + \frac{1}{i\lambda_j} \right) (e^{i\lambda_j \bar{z}} - 1) - \bar{z} \right] \right| \right\} \right|. \qquad (28)$$

In the limit $\bar{z} \ll 1$, expanding the exponential term and keeping in mind that, from Eq. (26), $\sum_{j=1}^{3} c_j = 1$, this expression reduces to the small-gain expression of Eq. (24). When $\sigma < 1$, the roots of Eq. (27) are not significantly different from the cold-beam values, hence we can write the previous expression taking into account only the exponentially growing solution of the resonant cold-beam case:

$$|b(\bar{z})| = e^{-D^2\sigma^2/2} \left| J_1\left(\tfrac{2}{3} A_0 \left(D^2 + \sqrt{3}D + 1 \right)^{1/2} e^{\sqrt{3}\bar{z}/2} \right) \right|. \qquad (29)$$

Again, if the condition $D\sigma \lesssim 1$ is satisfied, the optimum value for the modulator length $\bar{z}$ corresponding to a given D is obtained maximizing the Bessel function J_1, choosing its argument to be approximately 2, namely,

$$\bar{z}_{\rm opt} \approx \frac{2}{\sqrt{3}} \ln 3 \frac{1}{A_0 (D^2 + \sqrt{3}D + 1)^{1/2}}. \qquad (30)$$

When the energy-spread condition is badly violated, or to perform a simultaneous optimization of D and $\bar{z}$, it is useful to look at the level curves of $|b(\bar{z})|$, such as those obtained by solving numerically expression (28) and reported in Figs. 1 and 2, respectively, in the case of $\sigma = 0.01$ and $\sigma = 0.1$. From these plots it can be seen easily that, increasing σ, the advantage of the optical-klystron scheme in the overall length reduction becomes irrelevant. In fact, as we can see in Figs. 1 and 2, for $\sigma = 0.01$ the value $|b| = 0.5$ is reached at $\bar{z} = 1.5$ (and $D \approx 50$), for $\sigma = 0.1$ the modulator has to be increased to $\bar{z} \approx 4.5$ (and $D \approx 5$), whereas the normal FEL instability ($D = 0$) induces the same amount of bunching at $\bar{z} \approx 6$. There is clearly an advantage in the decrease of the modulator length using a dispersive section in the

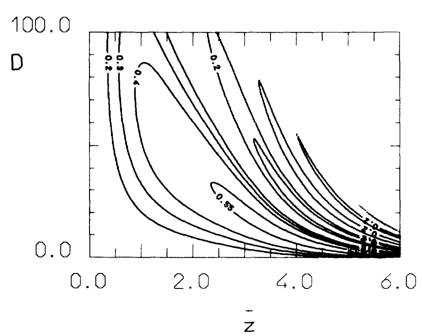

FIG. 1. Level curve of the bunching factor given by expression (28) as a function of the dimensionless length of the modulator $\bar{z}$ (horizontal axis) and the dimensionless dispersive section strength parameter D (vertical axis). The parameters used here are $\sigma = 0.01$ and $A_0 = 0.01$.

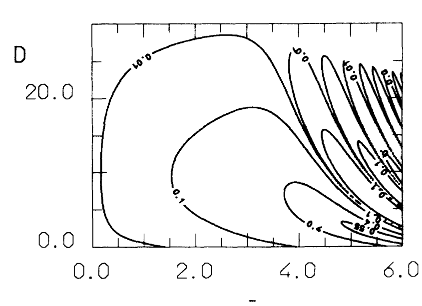

FIG. 2. As Fig. 1, for the parameters $\sigma = 0.1$ and $A_0 = 0.01$.

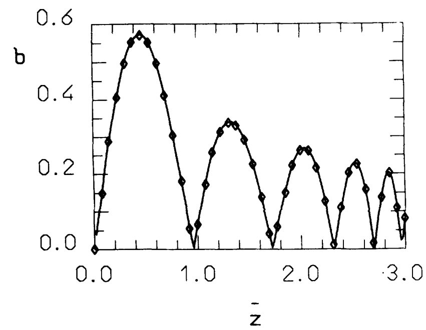

FIG. 3. Bunching at the end of the dispersive section as a function of the dimensionless modulator length. The solid line was produced integrating the system of electron-field equation of Ref. [3]; the symbols are evaluated from the expression (28). The parameters used in this simulation are $D = 2000$, $A_0 = 10^{-3}$, and $\sigma = 10^{-4}$.

first case, where the energy spread is small, but this advantage is nearly vanished in the second case, where the energy spread is larger. In Fig. 3 we plot the bunching as a solution of the full electron-radiation nonlinear system of Ref. [3] (solid line) and expression (28) (symbols) for a case of a small initial energy spread, $\sigma = 10^{-4}$ and $A_0 = 10^{-3}$. Here, for these values of energy spread and input field, the normal FEL saturation occurs around $\bar{z} = 10$ at $|b|_{\rm sat} \approx 0.75$ and a value of $|b| \approx 0.6$ is reached at $\bar{z} \approx 9$, whereas, as we can see from the figure, the insertion of a suitable dispersive section ($D = 2000$) allows us to reduce the modulator length by a factor 20, approximately. We stress that this shortening of the first wiggler implies a dramatic reduction of the intrinsic FEL energy spread, as can be easily evaluated by Fig. 4, which depicts the behavior of the induced spread, $\sigma \equiv \langle (p - \langle p \rangle)^2 \rangle^{1/2}$, as a function of the distance along the wiggler. Moreover, from Fig. 3, we can see that expression (28) turns out to be very accurate.

In Fig. 5 we show another comparison between expression (28) (symbols) and the numerical solution of the particle-field equations (solid line), for $D = 5$, $A_0 = 10^{-2}$, and $\sigma = 0.2$. We can clearly see that the analytical expression fits the numerical result well even when the condition $\sigma \bar{z} \ll 1$ is not strictly satisfied, given the initial value for the energy spread used in the simulation. Here

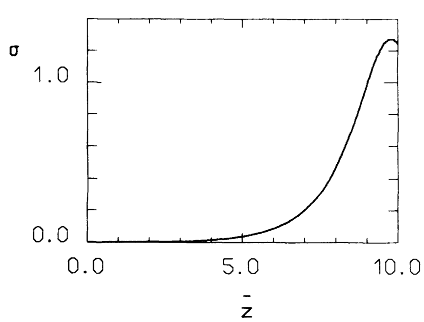

FIG. 4. FEL-induced energy spread, σ as a function of the dimensionless modulator length $\bar{z}$ for $A_0 = 10^{-3}$.

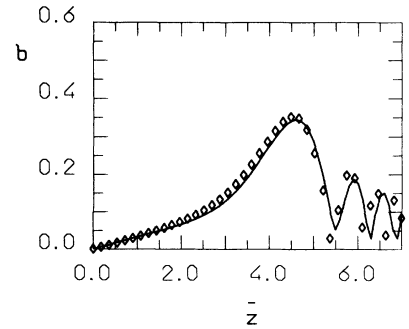

FIG. 5. As Fig. 3, but for the following set of parameters: $D = 5$, $A_0 = 10^{-2}$, and $\sigma = 0.2$.

the modulator clearly operates in the high-gain regime, and the saturation of the bunching occurs at $\bar{z} \approx 4.5$, at a value of $|b| \approx 0.4$, whereas normal FEL saturation, at a value $|b|_{\rm sat} \approx 0.75$, requires a gain of $\bar{z} \approx 7$, and the value $|b| = 0.4$ is reached for $\bar{z} \approx 5.5$. Note that expression (30) provides an accurate estimate of the optimum value for the modulator length, indeed $\bar{z}_{\rm opt} = 4.53$.

Clearly, for such a value of energy spread the advantage of the dispersive section over the normal FEL bunching process is considerably reduced with respect to the previous case, and its use is questionable. For greater values of the beam-energy spread, in order to satisfy the condition $D\sigma \lesssim 1$, smaller values of the dispersive section parameter D have to be choosen, so that the use of a dispersive section becomes a disadvantage, if not useless.

V. HARMONIC BUNCHING

The theory can be easily extended to the calculation of the bunching on the nth harmonic:

$$b_n(\bar{z}) = \langle e^{-in(\theta + Dp)} \rangle. \tag{31}$$

To be exact, we should study the field and electron-beam evolution in the modulator using the FEL equations that take into account the harmonic contents of the radiation field [9]. However, since it has been shown [2] that only the fundamental is relevant in driving the bunching process of the electron beam, we will use the previous expressions for the single electron trajectories, namely, Eqs. (13) and (14). Repeating the calculation we obtain

$$|b_n(\bar{z})| = e^{-n^2 D^2 \sigma^2/2} |J_n(2n|A_{\rm III}|(\bar{z}, \delta, D))| \tag{32}$$

which in the small-gain and high-gain regime become, respectively,

$$|b_n(\bar{z})| = e^{-n^2 D^2 \sigma^2/2} |J_n(2nA_0 D\bar{z})| \tag{33}$$

and

$$|b_n(\bar{z})| = e^{-n^2D^2\sigma^2/2}\left|J_n\left(\tfrac{2}{3}nA_0\left(D^2+\sqrt{3}D+1\right)^{1/2}e^{\sqrt{3}\bar{z}/2}\right)\right|. \tag{34}$$

Hence we can easily extend all our previous conclusions in this general case, in particular, the following.

(i) The energy-spread limitation is more restrictive for the harmonic bunching, since the condition $\sigma \leq 1/nD$ must be satisfied.

(ii) Since $J_n(x)$ goes, for small value of its argument, as x^n, taking $D = 0$ we recover the result that the growth rate of the nth harmonic in a conventional FEL is n times larger than the growth rate of the fundamental [2].

(iii) Again, when $D = 0$, since the maximum value of J_n decreases very slowly with n, we have the analytical evidence of the fact that the harmonic bunching near saturation is very large even if the harmonic field is very small [2].

VI. CONCLUSIONS

We have presented here an analytical model for the optical-klystron configuration of an FEL, valid both in the small- and the high-gain regime. An explicit expression, describing the two cases, has been derived and confirmed with numerical results. This expression suggests both the operating constraints of such a device and the optimization criteria. The effect of energy spread on the operation of an optical klystron turns out to be strong even in the high-gain regime, imposing an upper limit on the allowable values of the dispersive section parameter, D. In detail, the condition $D \lesssim 1/\sigma$ must be always satisfied, independently from the gain of the system. Hence, the maximum D allowable scales as $1/\sigma$. A decrease in D must be compensated, in order to retain a high value of the bunching, by a correspondent increase in $\bar{z}$, i.e., one is compelled to go to the high-gain regime in the modulator, where the bunching itself grows due to the FEL process. In conclusion the optical-klystron configuration is convenient only for very small values of σ, and practically ceases to be convenient as $\sigma \approx 0.1$ (see Fig. 2) whereas in the high-gain system high values of the bunching ($|b| \approx 0.8$) are reached until $\sigma \lesssim 1$. In the framework of the one-dimensional theory, the effect of the emittance of the electron beam can be modeled by a suitable spread of the resonant energy [11], and all the limits derived in this paper apply to this particular kind of spread as well.

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