Phase fluctuations in the Jaynes-Cummings model with and without the rotating-wave approximation

Jin-sheng Peng*
Chinese Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China

Gao-xiang Li
Department of Physics, Huazhong Normal University, Wuhan 430070, People's Republic of China

(Received 19 July 1991)

In this paper, we have investigated the time evolution of the phase operator of the radiation field in the Jaynes-Cummings model with and without the rotating-wave approximation, making use of the phase formalism of Barnett and Pegg [J. Mod. Opt. **36**, 7 (1989); Phys. Rev. A **39**, 1665 (1989)]. We verify by an analytical method that the atomic Rabi oscillation leads to phase dissipation of the radiation field, and also that the effect of the virtual-photon field can exhibit quantum fluctuations in the atom-field-coupling system and lead to a frequency shift of the radiation field.

PACS number(s): 42.50.Dv, 42.50.Md

I. INTRODUCTION

One of the simplest nontrivial systems in quantum optics is the Jaynes-Cummings (JC) model, where a two-level atom is coupled to a light mode [1]. Starting with a radiation field in the coherent state and with the atom in different initial states, this simple model already shows such interesting features as periodic revival and collapse of the inversion of the atom [2–4], and the squeezing of the light field [5–8]. The predicted collapse and revival of the inversion are in agreement with experiment [9].

As we know, the phase property of the light field is very important [10]. Recently, Barnett and Pegg [11,12] introduced a new formalism based on a unitary phase operator that has properties coincident with those normally associated with the phase. Recently, Pegg and Barnett [13] rectified three minor errors in their previous paper [12] following a suggestion by Ma and Rhodes [14]. Some authors investigated the phase properties of squeezed light [15], an anharmonic oscillator [16], a one-photon laser [17], correlated-emission laser [18,19], and a laser with atomic-memory effects [20]. However, less attention has been paid to the time evolution of the phase operator [21] and the phase fluctuations in the JC model. In particular, no one has studied the influence of phase fluctuations due to the effect of the virtual-photon field [22–27] (counterrotating terms) in the JC model.

In this paper, we have investigated the time evolution of the phase operator of the radiation field in the JC model both with and without the rotating-wave approximation (RWA) according to the phase theory introduced by Barnett and Pegg. That the atomic Rabi oscillation leads to phase dissipation of the radiation field that is initially in a coherent state is verified. We also show that the influence of the virtual-photon field leads not only to quantum fluctuations in the atom-field-coupling system, but also to a frequency shift of the radiation field.

II. TIME EVOLUTION OF THE PHASE OPERATOR IN THE JC MODEL WITH THE RWA

We consider a system of a two-level atom and a mode of the radiation field. These two are coupled by the dipole interaction within the RWA, and the system is described by the Hamiltonian

$$H=H_0+V \;, \tag{1}$$

where

$$H_0=\omega a^\dagger a+\omega_0 s_z \;, \tag{2}$$

$$V=g(a^\dagger s_-+as_+) \;. \tag{3}$$

Here $a^\dagger,a$ are the Bose creation and annihilation operators for the photons at frequency ω. The two-level atom is described by the usual spin-flip operators and the inversion operator s_z, and g is the coupling constant. For simplicity, we take the exact-resonance case $\omega_0=\omega$. To study the phase properties of the field, we need to know the state evolution of the system. In the interaction picture, the interacted Hamiltonian can be described by

$$V^I(t)=g(a^\dagger s_-+as_+) \;. \tag{4}$$

If at time $t=0$ the state vector of the atom-field-coupling system is

$$|\psi_{\rm Af}(0)\rangle=\sum_{n=0}^{\infty}F_n(0)|a,n\rangle \;, \tag{5}$$

it means that the atom is in the excited state $|a\rangle$, and the radiation field is in the arbitrary state $\sum_n F_n(0)|n\rangle$. With time development, the state vector is

$$|\psi_{\rm Af}^I(t)\rangle=\sum_n a_n(t)|a,n\rangle+b_n(t)|b,n\rangle \;, \tag{6}$$

which obeys the Schrödinger equation in the interaction

 ©1992 The American Physical Society

picture,

$$i\frac{d}{dt}|\psi_{Af}^I(t)\rangle = V^I(t)|\psi_{Af}^I(t)\rangle \ . \quad (7)$$

Substituting Eqs. (4)–(6) into Eq. (7), we obtain

$$a_n(t)=F_n(0)[\exp(-i\gamma_n t)+\exp(i\gamma_n t)]/2 \ , \quad (8)$$

$$b_{n+1}(t)=F_n(0)[\exp(-i\gamma_n t)-\exp(i\gamma_n t)]/2 \ , \quad (9)$$

where

$$\gamma_n = g\sqrt{(n+1)}$$

is associated with the frequency of atomic Rabi oscillation. Substituting Eqs. (7) and (8) into Eq. (6), the state vector of the system at time t is decided in the interaction picture. We change it into the Schrödinger picture

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} [a_n(t)|a,n\rangle + b_{n+1}(t)|b,n+1\rangle] \times \exp[-i(n+1/2)\omega t] \ . \quad (10)$$

According to Barnett and Pegg's approach [11,12], the phase operator operates on an $(s+1)$-dimensional subspace Ψ spanned by the number state $|0\rangle, |1\rangle, \ldots, |s\rangle$. The value of s can be made arbitrarily large. A complete orthonormal basis of the $(s+1)$ phase state is defined on Ψ as

$$|\theta_m\rangle = (s+1)^{-1/2} \sum_{n=0}^{s} \exp(in\theta_m)|n\rangle \quad (11)$$

with

$$\theta_m = \theta_0 + 2m\pi/(s+1), \quad m=0,1,2,\ldots,s \ .$$

The value of θ_0 is arbitrary. These states are eigenstates of the Hermitian phase operator

$$\hat{\Phi}_\theta = \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m| \ . \quad (12)$$

So the state vector $|\psi(t)\rangle$ of the atom-field-coupling system is spanned by the phase eigenkets as

$$|\psi(t)\rangle = \sum_{m=0}^{s} \langle a,\theta_m|\psi(t)\rangle |a,\theta_m\rangle + \langle b,\theta_m|\psi(t)\rangle |b,\theta_m\rangle \ . \quad (13)$$

Here $|\langle a,\theta_m|\psi(t)\rangle|^2 + |\langle b,\theta_m|\psi(t)\rangle|^2$ represents the phase-probability distribution. Thus the expectation value of the phase operator is

$$\langle\hat{\Phi}_\theta\rangle = \sum_m \theta_m [|\langle a,\theta_m|\psi(t)\rangle|^2 + |\langle b,\theta_m|\psi(t)\rangle|^2] \ , \quad (14)$$

$$\langle\hat{\Phi}_\theta^2\rangle = \sum_m \theta_m^2 [|\langle a,\theta_m|\psi(t)\rangle|^2 + |b,\theta_m|\psi(t)\rangle|^2] \ , \quad (15)$$

if the radiation field is initially in a coherent state, i.e.,

$$F_n(0) = \exp(-\bar{n}/2)\alpha^n/\sqrt{n!} \ , \quad (16)$$

where

$$\alpha = \bar{n}^{1/2}\exp(i\zeta) \ .$$

Here $\bar{n}$ is the mean photon number, and ζ is the phase angle of α. If $n \gg 1$, then the photon field is in the Gaussian distribution [11,18]

$$F_n(0) \approx (2\pi\bar{n})^{-1/4}\exp[-(n-\bar{n})^2/4\bar{n}]\exp(in\zeta) \ . \quad (17)$$

Recalling the saddle-approximation method [2], i.e.,

$$g\sqrt{(n+1)} \approx g\sqrt{(\bar{n}+1)} + g(n-\bar{n})/2\sqrt{(\bar{n}+1)} \approx g\sqrt{\bar{n}} + g(n-\bar{n})/2\sqrt{\bar{n}} \ , \quad (18)$$

the phase-probability distribution can be approximated as

$$P(\theta_m,t) = |\langle a,\theta_m|\psi(t)\rangle|^2 + |\langle b,\theta_m|\psi(t)\rangle|^2$$
$$= [2\pi/(s+1)](4\bar{n}/2\pi)^{1/2}\{\exp[-2\bar{n}(\zeta-\theta_m-\omega t+gt/2\sqrt{\bar{n}})^2] + \exp[-2\bar{n}(\zeta-\theta_m-\omega t-gt/2\sqrt{\bar{n}})^2]\}/2 \ . \quad (19)$$

In the continued limit, i.e., $s \to \infty$, θ_m is a continued variation. The phase-probability density is normalized according to Refs. [11,16,18,19]

$$\int P(\theta,t)(s+1)\Big/2\pi\, d\theta = 1 \ ,$$

where $(s+1)/2\pi$ is the density of the states.

If considering $g/2\sqrt{\bar{n}} \ll \omega$ in Eq. (19) when $\bar{n} \gg 1$, we can neglect the term $g/2\sqrt{\bar{n}}$ associated with the atomic Rabi oscillation. Using Eqs. (15) and (16), we obtain the time evolution of the phase operator

$$\langle\hat{\Phi}\rangle = \int \theta P(\theta,t)(s+1)\Big/2\pi\, d\theta = \zeta - \omega t \ , \quad (20)$$

$$\langle\hat{\Phi}^2\rangle = \int \theta^2 P(\theta,t)(s+1)\Big/2\pi\, d\theta = (\zeta-\omega t)^2 + 1/4\bar{n} \ . \quad (21)$$

So the phase fluctuations can be expressed as

$$\langle\Delta\hat{\Phi}\rangle^2 = \langle\hat{\Phi}^2\rangle - \langle\hat{\Phi}\rangle^2 = 1/4\bar{n} \ . \quad (22)$$

Since the photon number distribution is still Poissonian, we have [18]

$$\langle\Delta\hat{N}\rangle^2 = \bar{n} \ ,$$

so that the number-phase-uncertainty product is

$$\langle \Delta\hat{\Phi} \rangle^2 \langle \Delta\hat{N} \rangle^2 = \tfrac{1}{4} \; . \tag{23}$$

We see that the field, which is initially in a coherent state, retains coherence with time development if one neglects the influence of the atomic Rabi oscillation on the phase-probability distribution.

However, if we do not neglect $g/(2\sqrt{\bar{n}})$ in Eq. (19), then

$$\langle \hat{\Phi}^2 \rangle = (\zeta - \omega t)^2 + 1/4\bar{n} + (gt)^2/4\bar{n} \; , \tag{24}$$

and $\langle \hat{\Phi} \rangle$ is satisfied by Eq. (20). So the phase fluctuations can be shown

$$\langle \Delta\hat{\Phi} \rangle^2 = 1/4\bar{n} + (gt)^2/4\bar{n} \; . \tag{25}$$

Also, the number-phase-uncertainty product changes:

$$\langle \Delta\hat{\Phi} \rangle^2 \langle \Delta\hat{N} \rangle^2 = \tfrac{1}{4} + (gt)^2/4 \; . \tag{26}$$

This means that the phase fluctuations are enhanced, and the field does not keep its phase-number-minimum-uncertainty product [28]. From Eqs. (23) and (25), we can see that the cause leading to the enhancement of the phase fluctuations is the atomic Rabi oscillation due to the atomic-field coupling.

For a chaotic field, the phase fluctuations are maximum [11,12]

$$\langle \Delta\hat{\Phi} \rangle^2_{\max} = \pi^2/3 \; , \tag{27}$$

and we find, when

$$t = 2\pi\bar{n}^{1/2}/g\sqrt{3} = T_R/\sqrt{3} < T_R \; , \tag{28}$$

that the phase fluctuations in the JC model reach the maximum value due to the atomic Rabi oscillation, and that the field is in a chaotic state. Here T_R is the revival time of the atomic inversion that exhibits the repeated collapse and revival [2–4].

It is evident that our result is not in agreement with Ref. [21], which studied the phase property in the JC model. Comparing our result with that of Ref. [21], we find that the difference between Eq. (20) here and Eq. (20) in Ref. [21] is ωt. And from Eq. (23) in Ref. [21], we can see that the maximum phase fluctuations are larger than $\pi^2/3$. Among the causes for these differences is that the state vector described by Eq. (12) in Ref. [21] is in the interaction picture, but the phase operator in Eqs. (21) and (23) is in the Schödinger picture. Because our phase operators are all in the Schrödinger picture, our result regarding phase fluctuations [Eqs. (25) and (28)] is not only exact, but is also in agreement with Refs. [11,12].

Another point we would like to mention is that from Ref. [21] the phase-probability distribution, which obeys $P_-(\theta,t) = P_+(\theta,t)$, is the cause of the revival effect of atomic inversion. However, this result is only based on the interaction picture. But if we consider the phase-probability distribution in the Schrödinger picture, Eq. (22) in Ref. [21] changes,

$$p^s_\pm(\theta,t) = \left\{ 1 + 2\sum_{n,k} b_n b_k \cos[(n-k)(\theta+\omega t) - (\sqrt{n}-\sqrt{k})gt] \right\} \Big/ 2\pi \; .$$

It is easy to see, not only when $t = mT_R$ but also when $\omega t = m\pi/(n-k) - \theta$ ($m = 0, 1, 2, \ldots$), that the phase-probability distribution satisfies $P_-(\theta,t) = P_+(\theta,t)$. As we know, the revivals of atomic inversion happen only at $t = mT_R$ and not at $\omega t = m\pi/(n-k) - \theta$. This means that the proof deduces the result, that the revivals of the atomic inversion appear when $P_-(\theta,t) = P_+(\theta,t)$ is not sufficient.

III. TIME DEVELOPMENT OF THE PHASE OPERATOR WITHOUT THE RWA

As we know, the virtual-photon processes are represented by the counterrotating terms in the JC model [4,23–27]. In order to investigate the role of virtual-photon processes in the time development of the phase operator in the JC model, we cannot neglect the counterrotating terms in the Hamiltonian. The Hamiltonian for a system of a two-level atom interacting with a single-mode radiation field in the interaction picture is

$$H = H_0 + V^I(t) \; , \tag{29}$$

where

$$H_0 = \omega a^\dagger a + \omega_0 s_z \; , \tag{30}$$

$$V^I(t) = g[a^\dagger s_- + a s_- + a^+ s_+ \exp(i2\omega t) + a s_- \exp(-i2\omega t)] \; . \tag{31}$$

For simplicity, we also take the field to be resonant with the atomic-transition frequency.

Substituting eqs. (6) and (31) into the Schrödinger equation, we obtain

$$i a_n = g[\sqrt{n+1}\, b_{n+1} + \sqrt{n}\, b_{n-1} \exp(i2\omega t)] \; , \tag{32}$$

$$i b_{n+1} = g[\sqrt{n+1}\, a_n + \sqrt{n+2}\, a_{n+2} \exp(-i2\omega t)] \; . \tag{33}$$

It is easy to see that the last terms in Eqs. (32) and (33) represent the influence of the virtual-photon processes on $a_n(t)$ and $b_{n+1}(t)$. If we regard the last terms in Eqs. (32) and (33) as perturbation terms, and define a perturbation parameter that is the ratio of Rabi frequency to the field frequency, i.e., $g\sqrt{\bar{n}}/2\omega$, the $a_n(t)$ and $b_{n+1}(t)$ are then in the form of a perturbation series. The zeroth-order terms in $a_n(t)$ and $b_{n+1}(t)$ correspond to the solution with the RWA, and the terms including $g\sqrt{\bar{n}}/2\omega$ are the first-order correction due to energy-nonconserving terms. We only retain terms up to first order in $g/2\omega$, in $a_n(t)$, and in $b_{n+1}(t)$. Then the solutions of $a_n(t)$, $b_{n+1}(t)$ are

$$a_n(t)=F_n(0)[\exp(-i\gamma_n t)+\exp(i\gamma_n t)]/2 +g\sqrt{n}F_{n-2}(0)(\{\exp[i(2\omega+g\sqrt{n-1})t]-1\}/(2\omega+g\sqrt{n-1}) -\{\exp[i(2\omega-g\sqrt{n-1})t]-1\}/(2\omega-g\sqrt{n-1}))/2\ , \tag{34}$$

$$b_{n+1}(t)=F_n(0)[\exp(-i\gamma_n t)-\exp(i\gamma_n t)]/2 +g\sqrt{n+2}F_{n+2}(0)(\{\exp[-i(2\omega+g\sqrt{n+3})t]-1\}/(2\omega+g\sqrt{n+3}) +\{\exp[-i(2\omega-g\sqrt{n+3})t]-1\}/(2\omega-g\sqrt{n+3}))/2\ . \tag{35}$$

From Eqs. (34) and (35), we can see that $g\sqrt{n}/(2\omega\pm g\sqrt{n-1})$ and $g\sqrt{n+2}/(2\omega\pm g\sqrt{n+3})$ remain infinitesimal only when the radiation field is not intensive: otherwise, the RWA in the JC model is destroyed [29]. For simplicity, we suppose that the terms $g\sqrt{n}/(2\omega\pm g\sqrt{n-1})$ and $g\sqrt{n+2}/(2\omega\pm g\sqrt{n+3})$ are not large, so we can use perturbation theory. Substituting Eqs. (34) and (35) into Eq. (10), the state vector of the system in the Schrödinger picture is determined.

As before, the phase-probability distribution for which the field is initially in a coherent state for $\bar{n}\gg 1$ is

$$P(\theta,t)=A[2\pi/(s+1)](4\bar{n}/2\pi)^{1/2}\{\exp(-2\bar{n}x)+\exp(-2\bar{n}y)-\exp[-\bar{n}(x+z)]\cos(2\zeta+g\sqrt{\bar{n}}t)g\sqrt{\bar{n}}/\omega +\exp[-\bar{n}(y+z)]\cos(2\zeta-g\sqrt{\bar{n}}t)g\sqrt{\bar{n}}/\omega -2\exp[-\bar{n}(x+y)]\sin[2(\omega t-\zeta)]\sin(2g\sqrt{\bar{n}}t)g\sqrt{\bar{n}}/\omega\}/2\ , \tag{36}$$

where

$$x=(\zeta-\theta-\omega t+gt/2\sqrt{\bar{n}})^2\ , \tag{37}$$

$$y=(\zeta-\theta-\omega t-gt/2\sqrt{\bar{n}})^2\ , \tag{38}$$

$$z=(\zeta-\theta-\omega t)^2\ , \tag{39}$$

$$A=1-\sin(2\zeta)\sin(g\sqrt{\bar{n}}t)g\sqrt{\bar{n}}\exp(-g^2t^2/8)/\omega+\sin[2(\omega t-\zeta)]\sin(2g\sqrt{\bar{n}}t)g\sqrt{\bar{n}}\exp(-g^2t^2/2)/\omega\ . \tag{40}$$

Here we have neglected the second-order correction due to energy-nonconserving terms, and A is the normalized probability parameter.

Using Eqs. (15) and (16), and (36)–(40), we only retain terms up to first order in $g/2\omega$ and obtain the time evolution of the phase operator,

$$\langle\hat{\Phi}\rangle=\zeta-\omega t+g^2t\cos(2\zeta)\cos(g\sqrt{\bar{n}}t)\exp(-g^2t^2/8)/4\omega\ , \tag{41}$$

$$\langle\hat{\Phi}^2\rangle=(\zeta-\omega t)^2+1/4n+\frac{(8t)^2}{4\bar{n}}+(gt)^2/4n\{\sin[2(\omega t-\zeta)]\sin(2g\sqrt{\bar{n}}t)\exp(-g^2t^2/2) -\sin(2\zeta)\sin(g\sqrt{\bar{n}}t)\exp(-g^2t^2/8)\}3g\sqrt{\bar{n}}/4\omega +(\zeta-\omega t)g^2t\cos(2\zeta)\cos(g\sqrt{\bar{n}}t)\exp(-g^2t^2/8)/2\omega\ . \tag{42}$$

So the phase fluctuations are

$$\langle\Delta\hat{\Phi}\rangle^2=(gt)^2/4\bar{n}\{\sin[2(\omega t-\zeta)]\sin(2g\sqrt{\bar{n}}t)\exp(-g^2t^2/2)-\sin(2\zeta)\sin(g\sqrt{\bar{n}}t)\exp(-g^2t^2/8)\} \times 3g\sqrt{\bar{n}}/4\omega+(gt)^2/4\bar{n}+1/4\bar{n}\ . \tag{43}$$

From Eqs. (41)–(43), we can find that terms of the first order in $g/2\omega$ appear in the expectation value of the phase operator. The appearance of $g/2\omega$ occurs because of the role of virtual-photon processes in the state vector. These terms containing $g/2\omega$ arise from an interference between the counterrotating- (virtual-photon processes) and the rotating-wave (real-photon processes) contributions. The interference induces small-amplitude fluctuations in the expectation value of the phase operator, which represent the quantum fluctuations of the atom-field-coupling system. That is to say, the system without the RWA can explicitly exhibit the quantum fluctuations of the system, but these quantum fluctuations are not exhibited in the RWA. The amplitudes of quantum fluctuations are associated with the coupling g, the mean photon number $\bar{n}$, the resonant frequency ω, and the phase angle ζ. This means the quantum fluctuations are decided by the atom-field coupling. This result coincides with the result we obtained previously [3]. Comparing Eqs. (42) and (43) with (25), we find that the phase fluctuations are associated with not only the coupling constant g and the mean photon number $\bar{n}$, but also the frequency ω and the initial phase ζ. So the different frequency and the initial phase of the field induce different fluctuations due to the effect of the virtual field, even if the coupling constant and mean photon number are the same. In the RWA, the conclusion is opposite. That is to say, the phase fluctuations in the JC model without RWA are associated with not only the intensity properties of the field, but also the phase properties of the field.

Comparing Eq. (41) with Eq. (20), we find $(d/dt)\langle\hat{\Phi}\rangle \neq -\omega$. This means that the frequency of the field is shifted. Its value is decided by the property of the atom-field-coupling system. The cause of the frequency shift of the field is the interference between the counterrotating- and the rotating-wave contributions. So the role of virtual field induces not only the atomic energy shift [22,25] but also the frequency shift of the field.

IV. CONCLUSIONS

In conclusion, we have studied the time evolution of the phase operator in the JC model with and without the RWA. That the atomic Rabi oscillation induces the initially coherent state of the field to lose its property of phase-number-minimum uncertainty product is shown. We have verified that the virtual-photon field is the cause for quantum fluctuations, and revealed that the frequency shift of the field is due to the effect of virtual-photon processes.

ACKNOWLEDGMENTS

One of the authors (J.S.P.) would like to thank Professor Abdus Salam for his hospitality at the International Center for Theoretical Physics, Trieste, and thank Professor F. Persico for helpful discussions.

---

*Permanent address: Department of Physics, Huazhong Normal University, Wuhan, People's Republic of China.

[1] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963).
[2] J. H. Eberly, N. B. Narozhy, and J. Sanchez-Mondragon, Phys. Rev. Lett. **40**, 1323 (1980).
[3] J. S. Peng, G. X. Li, and P. Zhou, Acta Phys. Sin. Abstr. **40**, 1042 (1991).
[4] K. Zaheer and M. S. Zubairy, Phys. Rev. A **37**, 1628 (1988).
[5] P. Meystre and M. S. Zubairy, Phys. Lett. A **89**, 390 (1982).
[6] G. Compagno, J. S. Peng, and F. Persico, Opt. Commun. **57**, 415 (1986).
[7] J. R. Kuklinski and J. L. Madajczyk, Phys. Rev. A **37**, 3175 (1988).
[8] P. Lais and T. Steimle, Opt. Commun. **78**, 346 (1990).
[9] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987).
[10] R. Loudon, *The Quantum Theory of Light* (Clarendon, Oxford, 1983), p. 140.
[11] S. M. Barnett and D. T. Pegg, J. Mod. Opt. **36**, 7 (1989).
[12] D. T. Pegg and S. M. Barnett, Phys. Rev. A **39**, 1665 (1989).
[13] D. T. Pegg and S. M. Barnett, Phys. Rev. A **43**, 2579 (1991).
[14] X. Ma and W. Rhodes, Phys. Rev. A **43**, 2576 (1991).
[15] J. A. Vaccaro and D. T. Pegg, Opt. Commun. **70**, 529 (1989).
[16] C. C. Gery, Opt. Commun. **75**, 168 (1990).
[17] S. M. Barnett, S. Stenholm, and D. T. Pegg, Opt. Commun. **73**, 314 (1989).
[18] P. Anantha Lakahmi and S. Swain, Phys. Rev. A **42**, 5632 (1990).
[19] M. Orszag and C. Saavedra, Phys. Rev. A **43**, 2557 (1991).
[20] M. Orszag and C. Saavedra, Phys. Rev. A **43**, 554 (1991).
[21] H. T. Dung, R. Tanas, and A. S. Shumovsky, Opt. Commun. **79**, 462 (1990).
[22] X. Y. Huang and J. S. Peng, Phys. Scr. **T21**, 100 (1988).
[23] G. Compagno, G. M. Palma, R. Passante, and F. Persico, Europhys. Lett. **9**, 215 (1989).
[24] A. K. Biswas, G. Compagno, G. M. Palma, R. Passante, and F. Persico, Phys. Rev. A **42**, 4291 (1990).
[25] G. Compagno, R. Passante, and F. Persico, J. Mod. Opt. **37**, 1337 (1990).
[26] M. D. Crisp, Phys. Rev. A **43**, 2430 (1991).
[27] S. J. D. Phoenix, J. Mod. Opt. **36**, 1163 (1989).
[28] F. W. Cummings, Phys. Rev. **140**, 1051 (1965).
[29] M. Tavis and F. W. Cummings, Phys. Rev. **170**, 379 (1968).