Triggered superradiance

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By applying microwave pulses to the two lower levels of three-level atoms in the V configuration, the collective state vector can be switched from forbidden to allowed transitions. This means that we can, in effect, switch the superradiant and subradiant processes from off to on and vice versa.

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I. INTRODUCTION

Superradiance and subradiance are well-known effects caused by constructive and destructive interatomic interference, respectively. Both concepts were developed by Dicke in his classic 1954 paper [1]. In this paper, he introduced superradiance by means of the simple system of two neutrons in a uniform magnetic field, one being in the excited spin state and the other in the ground state. He pointed out that the singlet or antisymmetrical state does not decay and is therefore subradiant. On the other hand, the triplet state of one excited and one unexcited state has the double radiation rate of a single excited neutron. This is a clear sign of superradiance.

Let us now consider two excited neutrons that decay only by spontaneous emission. The corresponding state is not superradiant but decays to the superradiant triplet state. The whole process, the evolution to the superradiant state, the superradiance, and the evolution to the ground state is called superfluorescence. In the Bloch space that means that the Bloch vector turns from the $+z$ direction in the beginning to the superradiant x-y plane and then to the $-z$ axis, indicating that all neutrons are in the ground state.

The reason for these effects and difference to fluorescence is the interaction of both neutrons with their common radiation field, which prevents them from being treated independently. Dicke, however, considered only two-level systems.

Since Dicke's paper, there has been a considerable amount of work done on superradiance [2,3], including subjects like multilevel systems [4,5], sudden symmetry breaking [6], degeneracy [7,8], and statistical treatment [9]. At this point we especially would like to emphasize the important contributions of Crubellier, Liberman, Pavolini, and Pillet on superradiance and in particular subradiance of three-level systems. They for the first time observed the subradiant phenomenon [10] and developed an admirable formalism, in which collective states are described in a very advantageous way by Young tableaus [5,8].

In the present work on three-level superradiance of the V configuration, we show how microwave pulses resonant with one transition can produce and destroy superradiance of the other optical transition. The spontaneous decay rate for this optical transition is hereby altered by means of changing the interference among the atoms instead of the number of excited atoms.

In preliminary considerations in Sec. II we introduce our system and the corresponding interaction Hamiltonian. This is followed in Sec. III by triggered two-atom superradiance, where we introduce an appropriate two-atom basis and explain qualitatively the triggering for this simple case. Beginning with more general arguments on the extension to multiatom systems, we will discuss the N-atom case for the symmetrical and so-called most antisymmetrical state in Sec. IV. In order to keep the analysis as simple as possible, we decided not to use the group-theoretical language of Young tableaus in the main part of the paper. We rather generalized Dicke's basis states to three-level atoms and presented the advantageous description of the N-atom basis in terms of Young tableaus in the Appendix as far as it is interesting for our problem.

II. PRELIMINARY CONSIDERATIONS

Our interest lies in superradiance of atoms in the quasidegenerate V configuration with a dipole-forbidden transition $|3\rangle$ to $|2\rangle$ (see Fig. 1). This can be achieved,

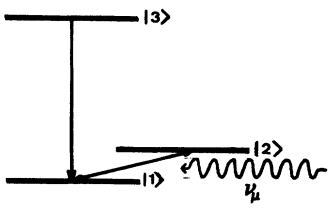

FIG. 1. The investigated three-level system. By changing the relative population of levels 1 and 2, the interatomic interference and therefore the spontaneous decay rate for the transition from 3 to 1 can be driven.

for example, by $|3\rangle$ and $|2\rangle$ being p states and $|1\rangle$ being an s state. With resonant microwave pulses ν_μ the relative population of atoms in the states $|1\rangle$ and $|2\rangle$ can be changed. This influences the decay rate from $|3\rangle$ to $|1\rangle$ as well. We here evaluate the corresponding spontaneous emission rates and shall see that microwave pulses can switch $|3\rangle$ to $|1\rangle$ transitions from not superradiant to highly superradiant and in some special cases even from forbidden to superradiant. The validity of these calculations is certainly restricted to Dicke's assumptions on the density of the atomic gas, i.e., the mean distance among the atoms should be small compared with the radiation wavelength but large compared with the particle wavelength.

We now wish to discuss the decay of atoms from the $|3\rangle$ to the $|1\rangle$ level, which leads us to the interaction Hamiltonian of the atoms in the states $|3\rangle$ and $|1\rangle$ with the corresponding field of frequency c/λ. We neglect the interaction of $|1\rangle$ and $|2\rangle$ because the coupling constant for microwave transitions is much smaller than for optical ones. In the dipole approximation the interaction Hamiltonian is

$$\mathcal{V}=-e\left[\sum_{i=1}^{N_3+N_1} r_i\right]E\ , \tag{2.1}$$

where N_3 and N_1 are the numbers of atoms in the states $|3\rangle$ and $|1\rangle$ and the r_i's are the atomic r vectors. The atoms are assumed to be close together, compared to the wavelength λ of the field, allowing us to consider the electric field E to be equal for all atoms.

According to Ref. [3] the quantum-mechanical form corresponding to $\mathcal{V}$ in Eq. (2.1) is

$$\mathcal{V}=g_\lambda\left[\sum_{i=1}^{N}(\sigma_i+\sigma_i^\dagger)(a_\lambda+a_\lambda^\dagger)\right]\ , \tag{2.2}$$

where a_λ is the annihilation operator of photons of wavelength λ, g_λ is the coupling constant, and σ_i the operator interchanging the $|3\rangle$ level of the ith atom into the $|1\rangle$ level defined by

$$\sigma_i|l_1l_2\ldots l_i\ldots l_N\rangle=\begin{cases}|l_1l_2\ldots 1\ldots l_N\rangle & \text{if } l_i=3\\ 0 & \text{otherwise}\ .\end{cases} \tag{2.3}$$

On making the rotating-wave approximation, going into the interaction picture, and expanding the evolution operator, we get [3]

$$U(t,0)=1-\frac{i}{\hbar}tg_\lambda\left[\left[\sum_{i=1}^{N}\sigma_i\right]a_\lambda^\dagger+A\right]+O(t^2)\ , \tag{2.4}$$

where A represents the adjoint.

We thus realize that the change of the system by spontaneous emission of one photon of wavelength λ is governed by

$$|\psi\rangle|n_\lambda\rangle\rightarrow\left[\sum_{i=1}^{N}\sigma_i|\psi\rangle\right]|(n+1)_\lambda\rangle\ . \tag{2.5}$$

By switching on the microwave field we find by an analogous calculation that the absorption of one microwave photon by the atomic system goes along with

$$|\psi\rangle|n_{\nu_\mu}\rangle\rightarrow\left[\sum_{i=1}^{N}\tau_i^\dagger|\psi\rangle\right]|(n-1)_{\nu_\mu}\rangle\ , \tag{2.6}$$

where τ_i and $\tau_i^\dagger$ are the operators flipping levels $|2\rangle$ into $|1\rangle$ and $|1\rangle$ into $|2\rangle$, respectively. In this case it was possible to neglect the $|3\rangle$ to $|1\rangle$ interaction because we assume the microwave field to be strong.

We now turn to a discussion of the triggering effect of the microwave field on the $|3\rangle$ to $|1\rangle$ transition in two-atom and multiatom systems.

III. TRIGGERED TWO-ATOM SUPERRADIANCE

Analogous to the construction of the two-level Dicke states we first find the basis of $3^2=9$ mutually orthonormal states for two three-level atoms. Obviously the basis is not unique, giving us the possibility to impose additional conditions that turn out to be useful later on. With each state of the basis we want it, if existing, to contain also its decay product and the state that decays into this state. Moreover, following Dicke, we want the basis to include all symmetric states. And naturally we only allow linear combinations of states that are distinguished from each other by their properties under permutation, which are the numberings of the atoms or of the eigenstates each atom occupies. This makes the orthonormal basis for two atoms unique (see Fig. 2).

For example, beginning with $|33\rangle$, necessarily a basis state because of its symmetry, we obtain its decay product and therefore new basis state by applying the atomic flip operator ($\sum_{i=1}^{2}\sigma_i$):

$$\sum_{i=1}^{2}\sigma_i|33\rangle=|31\rangle+|13\rangle\ . \tag{3.1}$$

The subspace corresponding to one atom being in the 3 level and the other one in the 1 level is two-dimensional and the orthonormal supplement to $(1/\sqrt{2})(|31\rangle+|13\rangle)$ is $(1/\sqrt{2})(|31\rangle-|13\rangle)$.

Looking at the decay rates of the states in Fig. 2, one can already recognize the enormous effect of microwave pulses switching atoms from state $|1\rangle$ to state $|2\rangle$. The photon trapped by the state $(1/\sqrt{2})(|31\rangle-|13\rangle)$ because of destructive atomic interference can be liberated by a microwave pulse transforming the atomic state into

both atoms in upper level	$\|33\rangle$				
	$\updownarrow \Gamma = 2$				
one atom in upper level	$\frac{1}{\sqrt{2}}(\|31\rangle + \|13\rangle)$	$\frac{1}{\sqrt{2}}(\|31\rangle - \|13\rangle)$	$\frac{1}{\sqrt{2}}(\|32\rangle + \|23\rangle)$	$\frac{1}{\sqrt{2}}(\|32\rangle - \|23\rangle)$	
	$\updownarrow \Gamma = 2$		$\updownarrow \Gamma = 1$	$\updownarrow \Gamma = 1$	
both atoms in lower levels	$\|11\rangle$		$\frac{1}{\sqrt{2}}(\|12\rangle + \|21\rangle)$	$\frac{1}{\sqrt{2}}(\|12\rangle - \|21\rangle)$	$\|22\rangle$

FIG. 2. The nine basis states of the two-atom three-level system.

$$\sum_{i=1}^{2} \tau_i^{\dagger} \frac{1}{\sqrt{2}}(|31\rangle - |13\rangle) = \frac{1}{\sqrt{2}}(|32\rangle - |23\rangle) \,. \tag{3.2}$$

The new state $(1/\sqrt{2})(|32\rangle - |23\rangle)$ decays into $(1/\sqrt{2})(|12\rangle - |21\rangle)$ with the decay rate of uncorrelated atoms. As yet another effect, we see that a switching from $(1/\sqrt{2})(|32\rangle + |23\rangle)$ to $(1/\sqrt{2})(|31\rangle + |13\rangle)$ increases the decay rate by a factor of 2, converting a nonsuperradiant to a superradiant state. Microwave pulses here switch $|2\rangle$ to $|1\rangle$ by stimulated emission. The inverse effects of decreasing radiation is also possible. At this point we would also like to mention that the decay rate of one isolated excited atom is set equal to 1 in the whole paper.

IV. TRIGGERED N-ATOM SUPERRADIANCE

In the case of more than two atoms, there are states that are neither symmetrical nor antisymmetrical. In order to determine these so-called mixed symmetry states, it is useful to divide the whole 3^N-dimensional space into the $N!/(N_3!N_1!N_2!)$ dimensional subspaces, characterizing states with N_3, N_1, N_2 atoms in the levels 3, 1, 2, respectively. All subspaces together form the whole configuration space yielding for the dimensions of space and subspaces

$$3^N = \sum_{\substack{N_3,N_1,N_2 \\ \text{with } N_3+N_1+N_2=N}} \frac{N!}{N_3!N_1!N_2!} \,. \tag{4.1}$$

The construction of a subspace (N_3, N_1, N_2) can be carried out by calculating the decay products of the subspace (N_3+1, N_1-1, N_2), where the first, second, and third entry give the number of atoms in the levels 3, 1, and 2, respectively. When this subspace is empty or does not determine the whole subspace under consideration, the remaining states can be constructed as an orthonormal supplement. It may be noted that in case of a more than one-dimensional supplement the choice of basis states is not unique.

However, this way of constructing basis states becomes awkward for higher numbers of atoms. In the Appendix we introduce a technique that allows us to represent the basis states more efficiently and is suitable to describe the extension to N atoms. For simplicity, we restrict ourselves in the following N-atom treatment to the important symmetrical and so-called most antisymmetrical states.

A. N-atom symmetrical states

The symmetrical states have the form

$$|\psi_s(N_3,N_1,N_2)\rangle \equiv \left[\frac{N!}{N_3!N_1!N_2!}\right]^{-1/2} \sum_{\mathcal{P}} |[3\ldots3]_{N_3}[1\ldots1]_{N_1}[2\ldots2]_{N_2}\rangle \,, \tag{4.2}$$

where $\sum_{\mathcal{P}}$ means that all $N!/(N_3!N_1!N_2!)$ different permutations of $[3\ldots3]_{N_3}\,[1\ldots1]_{N_1}\,[2\ldots2]_{N_2}\rangle$ are included in the sum, and x in $[\,]_x$ denotes the amount of numbers embraced.

Such symmetrical states are very interesting because they show the most constructive atomic interference and therefore highest superradiance [1]. They are, moreover, easy to prepare since a maximal excitement of all atoms gives always a symmetrical state.

A straightforward analysis leads to the spontaneous emission rate

$$\Gamma = \left| \left\langle \psi_s(N_3-1,N_1+1,N_2) \middle| \sum_{i=1}^{N} \sigma_i \middle| \psi_s(N_3,N_1,N_2) \right\rangle \right|^2 = N_3(N_1+1) \tag{4.3}$$

of the state $|\psi_s(N_3,N_1,N_2)\rangle$ into $|\psi_s(N_3-1,N_1+1,N_2)\rangle$. As a consequence the decay rate Γ can be changed by a factor of the order of the number of unexcited atoms, because (N_1+1) can be varied between 1 and $(N-N_3+1)$ by microwave pulses.

Thus, such pulses can switch states with decay rate of N uncorrelated atoms N_3 to highly superradiant states. The highest decay rate can be achieved by selecting $N_3=N_1=N/2$, which gives $\Gamma=(N/2)[(N/2)+1]$. This is the typical quadratical dependence of Γ on N for superradiance.

At this point we would like to emphasize that microwave pulses converting 1 into 2 decrease the decay rate. The simple explanation is that there is a correlation among atoms in 3 and 1 levels as opposed to atoms in 3 and 2 levels. This correlation means constructive interference for symmetrical states, and it vanishes by switching the atoms in level 1 to level 2.

B. The most antisymmetrical state

We now present a subradiant N-atom state in the levels 3 and 1 that traps the highest possible number of photons. The maximum number of photons is trapped if we have the maximum number of nondecaying atoms in the 3 level.

We found out that such a state is given by

$$|\psi\rangle \equiv \sum_{k=0}^{N/2} (-1)^k k! \left[\frac{N}{2}-k\right]! \psi_{k,k} \tag{4.4}$$

with

$$\psi_{k,l} \equiv \sum_{\mathcal{P}_a} |[1\ldots1]_k[3\ldots3]_{N/2-k}[1\ldots1]_{N/2-l}[3\ldots3]_l\rangle \ , \tag{4.5}$$

where the summation $\mathcal{P}_a$ is only over those permutations which have exactly k atoms of the first $N/2$ atoms and $N/2-l$ of the second $N/2$ atoms unexcited. Such a state $|\psi\rangle$ is called the most antisymmetrical state.

From the Appendix it is obvious that the $N/2$ photon-trapping N-atom state is described by the Young tableau with $N/2$ 3's in the first row and $N/2$ 1's in the second row. It becomes also clear why this state is called the most antisymmetrical state. However, the formalism given in the Appendix is quite complicated to explicitly construct $|\psi\rangle$ in Eq. (4.4) out of the known Young tableau. Without using any group theory, we here directly show that the operator $\sum_{i=1}^{N}\sigma_i$ applied to $|\psi\rangle$ gives zero, meaning that $|\psi\rangle$ does not decay by a 3 to 1 transition. To begin with we find

$$\sum_{i=1}^{N/2} \sigma_i \psi_{k,k} = (k+1)\psi_{k+1,k} \ . \tag{4.6}$$

In order to understand the factor $k+1$ we consider one arbitrary summand of $\psi_{k+1,k}$ which has by definition $k+1$ 1's on certain positions of the first $N/2$ atoms. Whenever we have a summand of $\psi_{k,k}$ that has its k 1's of the first $N/2$ atoms on any k of the $k+1$ positions of our arbitrary summand of $\psi_{k+1,k}$ and a 3 on the remaining position j, σ_j applied on the first gives our summand of $\psi_{k+1,k}$. This always happens exactly $k+1$ times in the sum on the left side of Eq. (4.6). Similarly we get

$$\sum_{i=N/2+1}^{N} \sigma_i \psi_{k,k} = \left[\frac{N}{2}-k+1\right] \psi_{k,k-1} \ . \tag{4.7}$$

Returning to the object of main interest, we have with Eq. (4.4)

$$\sum_{i=1}^{N} \sigma_i |\psi\rangle = \sum_{k=0}^{N/2-1} (-1)^k k! \left[\frac{N}{2}-k\right]! (k+1)\psi_{k+1,k} + \sum_{k=1}^{N/2} (-1)^k k! \left[\frac{N}{2}-k\right]! \left[\frac{N}{2}-k+1\right] \psi_{k,k-1} = 0 \ , \tag{4.8}$$

where we have split $\sum_{i=1}^{N}\sigma_i$ in two parts and dropped the terms with $k=N/2$ and $k=0$ since their contributions vanish. Having then applied Eqs. (4.7) and (4.8) and having carried out an index transformation has thus shown that $|\psi\rangle$ does not decay. And since at least one excited atom is needed to prevent one unexcited atom from decaying via antisymmetration, $|\psi\rangle$ traps the most possible number of photons.

C. Switching from subradiant to superradiant transition

By applying resonant pulses for the $|1\rangle$ to $|2\rangle$ transition as represented by the operator $(\sum_{i=1}^{N}\tau_i^{\dagger})^{N/2}$ on the state $|\psi\rangle$ of $N/2$ trapped photons, we get apart from a constant factor

$$|\bar{\psi}\rangle \equiv \sum_{k=0}^{N/2} (-1)^k k! \left[\frac{N}{2}-k\right]! \sum_{\mathcal{P}_a} |[2\ldots2]_k[3\ldots3]_{N/2-k}[2\ldots2]_{N/2-k}[3\ldots3]_k\rangle \ , \tag{4.9}$$

which is in the Young tableau notation, the tableau with $N/2$ 3's in the first row and $N/2$ 2's in the second (see Appendix).

Thus, for the sake of simplifying the analysis, we consider the case of full inversion of $|1\rangle$ and $|2\rangle$. This cannot be realized with the three-level system introduced in Sec. II, since the microwave pulses can achieve only equal population of $|1\rangle$ and $|2\rangle$. However, adding a fourth level $|0\rangle$, lower than the other three and only coupled to level $|2\rangle$, accomplishes our wish to empty level $|1\rangle$ and to shift its population to other levels that do not influence the spontaneous emission from $|3\rangle$ to $|1\rangle$. Including the population of level $|0\rangle$ into that of $|2\rangle$ allows us to keep our notation.

Since, as mentioned, atoms in level $|2\rangle$ (and $|0\rangle$) do not influence the spontaneous decay from $|3\rangle$ to $|1\rangle$, $|\bar{\psi}\rangle$ should radiate like $N/2$ atoms in state $|3\rangle$. However, because of its importance, we will turn to the detailed calculation of the

decay of $|\bar{\psi}\rangle$, with the additional wish to give a relatively simple example for this kind of decay-rate determination.

We first obtain a general formula for calculating the decay rate of unnormalized states of $k|\phi\rangle$, where $|\phi\rangle$ is a normalized state. We then calculate the decay rate of $|\bar{\psi}\rangle$ of Eq. (4.9) and show that it radiates like $N/2$ atoms in level 3.

The state $k|\phi\rangle$ decays to the state $\sum_{i=1}^{N}\sigma_i k|\phi\rangle$, which we may write in the form $\tilde{k}|\tilde{\phi}\rangle$, where $|\tilde{\phi}\rangle$ is a normalized state. The decay rate then is

$$\Gamma=\left|\left\langle\tilde{\phi}\left|\sum_{i=1}^{N}\sigma_i\right|\phi\right\rangle\right|^2=\left|\frac{1}{k}\left\langle\tilde{\phi}\left|\sum_{i=1}^{N}\sigma_i k\right|\phi\right\rangle\right|^2=\left|\frac{1}{k}\langle\tilde{\phi}|\tilde{k}|\tilde{\phi}\rangle\right|^2=\left|\frac{\tilde{k}}{k}\right|^2 . \tag{4.10}$$

Thus we see that the decay rate is just the square of the ratio of the lengths of the final state to that of the initial state.

Applying the operator $\sum_{i=1}^{N}\sigma_i$ exactly N_1 times on $|\bar{\psi}\rangle$, we get, apart from a factor,

$$\sum_{k=0}^{N/2}(-1)^k k!\left[\frac{N}{2}-k\right]!\sum_{\mathcal{P}_s}\sum_{\mathcal{P}_a}|[2\ldots2]_k[1\ldots1\ 3\ldots3]_{N/2-k}[2\ldots2]_{N/2-k}[1\ldots1\ 3\ldots3]_k\rangle , \tag{4.11}$$

which decays by applying $\sum_{i=1}^{N}\sigma_i$ once again into the state

$$(N_1+1)\sum_{k=0}^{N/2}(-1)^k k!\left[\frac{N}{2}-k\right]!\sum_{\mathcal{P}_s}\sum_{\mathcal{P}_a}|[2\ldots2]_k[1\ldots1\ 3\ldots3]_{N/2-k}[2\ldots2]_{N/2-k}[1\ldots1\ 3\ldots3]_k\rangle , \tag{4.12}$$

where in carrying out the sum over $\mathcal{P}_a$ the 1's have to be treated like 3's, and $\mathcal{P}_s$ now stands for all distinct permutations among all the 1's and all the 3's. The number of atoms in level 1 of the states in Eqs. (4.11) and (4.12) is N_1 and N_1+1, respectively. Using Eq. (4.10) we obtain the expected spontaneous decay rate of $N/2$ superradiant atoms:

$$\begin{aligned}\Gamma&=\frac{\sum_{k=0}^{N/2}(k!)^2[(N/2-k)!]^2\begin{bmatrix}N/2\\k\end{bmatrix}^2\begin{bmatrix}N/2\\N_1+1\end{bmatrix}(N_1+1)^2}{\sum_{k=0}^{N/2}(k!)^2[(N/2-k)!]^2\begin{bmatrix}N/2\\k\end{bmatrix}^2\begin{bmatrix}N/2\\N_1\end{bmatrix}}\\&=(N_1+1)^2\frac{\begin{bmatrix}N/2\\N_1+1\end{bmatrix}}{\begin{bmatrix}N/2\\N_1\end{bmatrix}}=(N_1+1)(N/2-N_1)=N_3(N_1+1) ,\end{aligned} \tag{4.13}$$

where, for example, the binominal coefficients

$$\begin{bmatrix}N/2\\k\end{bmatrix}^2\quad\begin{bmatrix}N/2\\N_1\end{bmatrix}$$

in the denominator, respectively, are the numbers of elements of $\mathcal{P}_a$ and $\mathcal{P}_s$ for the initial state. This then shows that a nondecaying state can be switched to a superfluorescent state, which becomes superradiant after the first emission of a photon. If we switched half of the atoms in $|1\rangle$ to $|2\rangle$, what is possible by keeping the three-level system is an analogous but more complicated calculation, which would lead to a decay of $N/4$ symmetrically coupled atoms. For high enough N, this still means switching from a forbidden to a superradiant transition, though the radiation is substantially smaller than that of $N/2$ symmetrically coupled atoms.

Some time ago Crubellier, Liberman, and Pillet and Alicki, Rudnicki, and Sadowski pointed out the strong relation, almost equivalence, between the statistical mixture and the symmetry of a state for two- and N-level atoms, respectively [9]. This has been taken advantage of when Pavolini *et al.* observed the above discussed most antisymmetrical state $|\psi\rangle$ [10]. They prepared gallium atoms in a $j=\frac{3}{2}$ level in a full statistical mixture and thus most likely in an antisymmetrical combination [9] of all four Zeeman sublevels (as prepresented by a density matrix, which is a multiple of the unitary matrix) and observed that only half of the atoms decay to the $j'=\frac{1}{2}$ level. For an experimental implementation of our effect, atoms in levels 1 and 3 should be prepared in a full statistical mixture along the lines of Ref. [10] to obtain $|\psi\rangle$. The application of the microwave and the detection of the superradiant radiation should be less problematic. As a concrete situation we could think of the $(5S_{1/2},5P_{1/2},6P_{3/2})$ configuration of the very handy rubidium atoms, where driving on the infrared $5P_{1/2}$ to $5S_{1/2}$ transition could influence the interatomic interference on the ultraviolet $6P_{3/2}$ to $5S_{1/2}$ transition. According to Refs. [5] and [8] the atoms should be placed in a so-called pencil-shaped volume in order to secure invariance of the Hamiltonian under atomic permutation.

The most antisymmetrical state $|\psi\rangle$ has a simple interpretation that arises at a glance from the corresponding Young tableau (see Appendix) but also from the fact that just two two-level atoms can form a fully antisymmetrical state and therefore trap a photon. Similar to a system of

$N/2$ Cooper pairs, $|\psi\rangle$ represents a system of $N/2$ bosonlike coupled small systems of two fermionlike coupled atoms. The two-atom pairs consisting of one excited and one deexcited atom cannot decay because of their antisymmetry, and that is why the whole system does not decay either. The switching of unexcited atoms to a level not coupled to the excited state then leaves behind symmetrically coupled excited atoms thus developing constructive interference.

V. CONCLUSION

For the V configuration we have seen that microwave pulses switching atoms from 1 to 2 can change the decay rate of 3 to 1 drastically by means of altering the interatomic interference. Especially, we have shown that the state with $N/2$ trapped photons can be switched to a superradiant state, and the decay rate of symmetrical states can be changed by a factor of the order as high as N due to switching superradiance from off to on and vice versa. This can also be looked on as a device that triggers the release of very high energy by low-energy pulses.

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APPENDIX: A CONSIDERABLE HELP: YOUNG TABLEAUS

The following group-theoretical discussion is intended to explain how an N atom n-level system can be described in a complete and effective manner by standard Young tableaus. We consider n-level atoms in the first part since it does not require any extra effort. The connection of atomic states with well-defined symmetry properties and Young tableaus that characterize the irreducible representations of the $S_N \times SU(n)$ group was first formulated by Crubellier *et al.* Their justification is based on the fact that the N atoms can be considered as indistinguishable and that the atomic operators are infinitesimal operators of the $SU(n)$ group [5,8]. We here concentrate on showing that standard Young tableaus corresponding to the $S_N \times SU(n)$ group describe a basis of the n^N-dimensional configuration space and that they indeed are a convenient notation for our problem. They especially allow us to recognize the decay rate and the maximal number of photons that can be released at a glance. Thus it becomes easy to find states with many trapped photons and to determine the new Young tableau and its decay rate after the microwave pulses.

To make our discussion self-contained we introduce a few well-known facts about Young tableaus.

1. Mathematical background [11–13]

A partition $\lambda=[\lambda_1\lambda_2\ldots\lambda_h]$ of the integer N fulfills $\sum_{i=1}^{h}\lambda_i=N$ and $\lambda_i \geq \lambda_j$ for $i>j$. A set of N squares arranged in h lines placed one above the other, the ith having λ_i squares ($i=1,\ldots,h$) represents the Young diagram Y_λ corresponding to the above partition. The scheme of numbering the N squares of a given Young tableau from left to right and up to down in increasing order is not unique. Each configuration gives rise to a standard Young tableau θ_s^λ ($s\in\{1,\ldots,f^\lambda\}$), where f^λ is the number of those different configurations and therefore the number of standard Young tableaus belonging to the Young diagram Y_λ. The operators σ_{rs}^λ transform one standard Young tableau to another belonging to the same Young diagram:

$$\sigma_{rs}^\lambda\theta_s^\lambda=\theta_r^\lambda \quad (r,s\in\{1,\ldots,f^\lambda\}) . \tag{A1}$$

The symmetrizer of the rows of θ_s^λ, P_s^λ, is defined by the sum over all $\lambda_1!\lambda_2!\cdots\lambda_h!$ permutations that leave the rows of θ_s^λ invariant. In analogy, the antisymmetrizer of the columns of $\theta_s^\lambda, Q_s^\lambda$, is the sum over all permutations that only permute inside the columns, times the sign of the respective permutation. Finally we need to define

$$E_{rs}^\lambda\equiv Q_r^\lambda P_r^\lambda\sigma_{rs}^\lambda \quad (r,s\in\{1,\ldots,f^\lambda\}) . \tag{A2}$$

The f^λ values for s lead to the f^λ permutations belonging to the standard Young tableau characterized by r and λ. According to Ref. [11] the system

$$\{E_{rs}^\lambda\}_{r,s\in\{1,\ldots,f^\lambda\},\lambda\in\{[N],\ldots,[1^N]\}} \tag{A3}$$

is a basis of the $N!$-dimensional vector space spanned by the elements of S_N and $E_{rs}^\lambda\perp E_{vw}^\mu$ for $\lambda\neq\mu$ and arbitrary $r,s\in\{1,\ldots,f^\lambda\}$ and $v,w\in\{1,\ldots,f^\mu\}$.

In order to construct quantum-mechanical states, these permutations must be applied on the ket $|l_1l_2\ldots l_N\rangle$ defined as usual as the state in which the ith atom occupies the level l_i ($i\in\{1,\ldots,N\}$). There are two possibilities in principle: permuting the numbering of atoms or of the states the atoms occupy. The example

$$(12)_{\text{atoms}}|l_1l_3l_2\rangle=|l_3l_1l_2\rangle , \tag{A4a}$$

$$(12)_{\text{states}}|l_1l_3l_2\rangle=|l_2l_3l_1\rangle \tag{A4b}$$

shows that different results are possible. In order to avoid ambiguity we decide to interpret the permutations in the following as indicated by (A4b), where the indices i of the levels l_i are the objects under permutation.

2. Application to the N n-level system

Analogous to Eq. (4.1) we divide the n^N-dimensional configuration space in $N!/(\prod_{j=1}^{n}N_j!)$ dimensional subspaces characterized by N_j atoms in the jth level ($j\in\{1,\ldots,n\}$, $\sum_{j=1}^{n}N_j=N$), and we have

$$\sum_{\{N_j\},\,\sum_{j=1}^{n}N_j=N} N!/\left(\prod_{j=1}^{n}N_j!\right)=n^N , \tag{A5}$$

i.e., the sum over the dimensions of all disjoint subspaces gives the dimension of the whole configuration space. For the sake of generality, all parameters N,n,N_j and l_i are arbitrary natural numbers apart from the above given

restrictions and $n \leq N$, what can be realized easily by omitting the levels that are not occupied by any atom.

We now describe how to obtain the basis states for these subspaces. In the case that all the N atoms occupy different levels ($N_j \in \{0,1\}$ for all $j \in \{1,\ldots,n\}$) designated by the indices from 1 to N, the system

$$\{E_{rs}^{\lambda}|l_1 l_2 \ldots l_N\rangle\}_{r,s\in\{1,\ldots,f^{\lambda}\},\lambda\in\{[N],\ldots,[1^N]\}}, \tag{A6}$$

consists of all $N!$ linearly independent states of the corresponding subspace [compare with the set of permutations (A3)].

The situation of different atoms in the same level ($N_j \geq 2$ for at least one $j \in \{1,\ldots,n\}$) leads to states in (A6) being equal or vanishing. Neglecting double and vanishing states, we find that the maximal number of $N!/(\prod_{j=1}^{n} N_j!)$ linearly independent states remains. It cannot be less because taking all subspaces into account, there must be altogether n^N linearly independent states. This can be understood considering that all basis states are obtained from the different possible Young diagrams. Those are characterized by the so-called outer product of N single squares, which is $n \times n \times \cdots n = n^N$ dimensional [12,14].

In the following, for convenience, we substitute the numbers i ($i \in \{1,\ldots,N\}$), with which Young tableaus are usually characterized, by the corresponding eigenstates l_i and may refer to these standard Young tableaus as collective atomic states. Then the most transparent description of N n-level atoms is in terms of these standard Young tableaus rather than in considering the explicit basis states. However, when the explicit f^{λ} states corresponding to a standard Young tableau are of interest, the E_{rs}^{λ}'s of (A3) have to be evaluated where r and λ are determined by the corresponding standard Young tableau. The f^{λ} operators then have to be applied on the ket $|l_1 l_2 \ldots l_N\rangle$ according to (A4b) after the l_i's in the Young tableau have been replaced by their indices i.

The f^{λ} states corresponding to a standard Young tableau have the same symmetry and the same radiation rate and are thus physically identical. Therefore, the calculation of mostly not more than one state [$r=s$ is the easiest because of $\sigma_{ss}^{\lambda}=1$, see Eq. (A1)] is necessary. For qualitative considerations it is already enough to look at the standard Young tableau. Thus, back to our system of Fig. 1, a 3 and a 1 in one column is an indication of a trapped photon since two 1's in one column would give states equal to 0 because of the antisymmetration in the columns. Moreover, the radiation rate increases generally with the symmetry of the state. So the symmetrical state, where the Young tableau consists of just one row, has the highest decay rate, and the antisymmetrical state (if existent, then its Young tableau is just one column) does not decay at all.

The spontaneous decay rate can be evaluated from the Young tableau as follows. At first all squares denoted by levels being neither the upper nor the lower level of the considered transition can be omitted. Then all pairs of antisymmetrically coupled atoms can be omitted as well, so that one row of say k excited atoms and l unexcited ones (level 1) remains. The corresponding spontaneous decay rate as determined in Sec. IV B is $k(l+1)$ times the spontaneous decay rate of one independently radiating atom (set 1 in this paper).

In the final section of the Appendix, we present the simple basis for two three-level atoms in terms of Young tableaus and present a helpful way to construct the complete set of N three-level atoms.

3. The basis for the $n=3$ problem in terms of Young tableaus

The basis for two three-level atoms in terms of standard Young tableaus is given in Fig. 3. Again for convenience, we have substituted the numbers i of the squares, which were mentioned in Sec. III A, by the atomic states l_i, where $(l_1,l_2,l_3)=(3,1,2)$. We here selected level 2 to be last in the ordering so that it can easily be omitted when the 3 to 1 collective decay is of interest. For instance, the subspace with one atom in level 3 and the other one in level 1 delivers the Young tableaus:

$$\boxed{3}\boxed{1} \quad \text{and} \quad \begin{array}{|c|}\hline 3 \\ \hline 1 \\ \hline \end{array} ,$$

characterizing the states $(1/\sqrt{2})(|31\rangle+|13\rangle)$ and $(1/\sqrt{2})(|31\rangle-|13\rangle)$, respectively. The higher transparency of the Young tableau notation becomes more evident for multiatom systems.

We now display how to construct the whole set of standard Young tableaus of the subspaces mentioned above, which altogether form the whole configuration space.

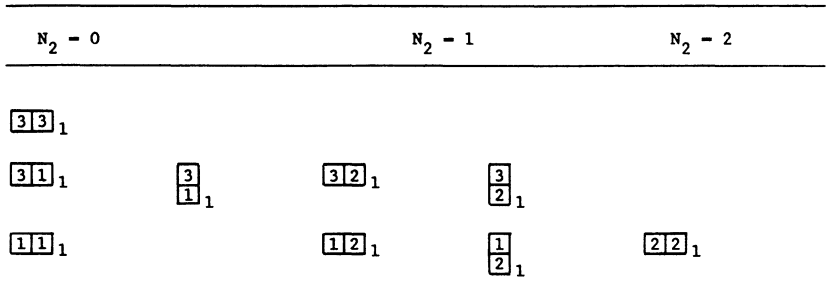
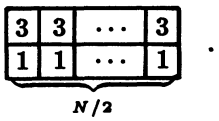


FIG. 3. The basis for two three-level atoms in terms of standard Young tableaus.

Thus imagine the number of atoms in the levels 3, 1, and 2 (N_3, N_1, and N_2, respectively) to be fixed after being chosen arbitrarily. The N_3 squares marked with a 3 have to appear in the first row on the left. The squares with 1's are to be placed right or under these 3's. The symmetrical combination with the highest decay rate is obtained when all 1's are situated in the first row. Taking these 1's successively and placing them under the 3's leads to more and more antisymmetrical Young tableaus. If there is a 1 under every 3 or if all 1's are in the second row, the most antisymmetrical combination has been obtained in the corresponding subspace. The squares with 2's have to be placed right or under the 3's and 1's in a similar manner, keeping in mind that not more than one 2 is supposed to appear in one column, and no row should be longer than the one above it.

As a general feature, each 3 that has a 1 in the same column stands for an excited atom that is prevented from decaying via destructive interatomic interference. We therefore, understand at this point that the most antisymmetrical state of Sec. IV C can be described and calculated via the Young tableau:



It stands for a system of $N/2$ symmetrically coupled small systems consisting of two antisymmetrically coupled atoms. The Young tableau reveals the whole physics of the state, whereas the explicit representation of Eq. (4.4) hides it.

Young tableaus allow us to recognize the decay rate, the number of trapped photons, and its decay product immediately and are, therefore, a helpful notation.

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