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Gravity-wave detection via an optical parametric oscillator

N. C. Wong

Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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An active gravity-wave detector based on a two-arm optical parametric oscillator (OPO) is proposed and analyzed. The signal and idler waves of a nearly degenerate parametric oscillator are internally separated such that the signal beam propagates along one arm of the OPO and the idler beam along the orthogonal arm. By monitoring the signal-idler beat frequency, a sensitive measure is obtained of the differential path displacement in the two arms of this OPO interferometer. The phase noise of the output beat frequency is calculated and is shown to exhibit time-dependent squeezing. The detection sensitivity is limited by the signal-idler phase diffusion and photon shot noise, and is the same as that of the usual passive Michelson interferometer or active two-laser interferometer for a given apparatus size and detected optical power. Squeezed vacuum can, in principle, be used to improve the detection sensitivity.

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I. INTRODUCTION

The detection of gravitational waves remains one of the most challenging tasks in physics [1]. The minute strain that is expected from a periodic gravitational source is on the order of 10^{-21}, making its detection very difficult and highly sensitive to environmental perturbations. One of the promising methods is laser interferometry using a passive Michelson interferometer [2]. The path-length difference between test masses that are suspended in free space is measured via the interference fringes of the interferometer. Another laser-based method is the active two-laser detector [3] in which the heterodyne beat frequency of two lasers is monitored to measure the gravitationally induced phase shift of the beat frequency. The active detector has the advantage of working in a spectral window that can be chosen to minimize the effects of technical noise. Both the passive and active laser detectors have the same sensitivity for a given size of the apparatus and a given amount of optical power. The sensitivity of the passive interferometer is limited by the photon shot noise [3, 4] and can be improved with the injection of squeezed vacuum [4, 5]. For the active interferometer, spontaneous emission noise of the two lasers limits its detection sensitivity. The use of a correlated-emission laser [3, 6, 7] may significantly reduce the relative phase noise of the beat frequency in the active laser interferometer.

Here I describe a new type of gravity-wave detector that comprises a two-arm optical parametric oscillator (OPO) [8] whose output frequencies are nearly degenerate. The two subharmonic output waves, signal and idler, of the two-arm OPO are internally separated such that the signal beam travels along one arm of the OPO and the idler beam along the orthogonal arm. By measuring the phase shift of the signal-idler beat frequency it yields the relative displacement of the two orthogonal arms. The signal-idler beat frequency can be chosen to lie in a low-noise spectral region to minimize the effects of technical noise. Furthermore, the nonresonant nature of the parametric interaction in a highly transparent nonlinear optical crystal implies the absence of spontaneous emission noise and low internal losses such that the photon cavity lifetime can be made quite long. When the total losses of the signal and idler waves are well matched, the signal-idler beat frequency is insensitive to the pump-laser frequency fluctuation, thus relaxing the need for an extremely stable laser pump source. It should be pointed out that this proposed orthogonal-arm OPO gravity-wave detector is different from a suggestion by Björk and Yamamoto [9] in which a parallel-arm OPO is used and the signal-idler beat frequency is not utilized.

In Sec. II the basic OPO is briefly reviewed. In Sec. III the two-arm OPO is analyzed classically, and the gravitationally induced phase shift of the signal-idler beat frequency is obtained. In Sec. IV the internal phase noise of the signal-idler beat is calculated using the linearized quantum Langevin equations of an above-threshold OPO. A model beat-detection apparatus is used for calculating the external (measured) phase noise in Sec. V. It is shown that the external phase noise becomes squeezed for a measurement time comparable to the cavity lifetime. The detection sensitivity is calculated and is found to be the same as a passive Michelson interferometer. In Sec. VI the use of squeezed light to improve the detection sensitivity is discussed.

II. OPTICAL PARAMETRIC OSCILLATOR

An OPO converts a pump wave, of frequency ω_p, into two intense subharmonic waves, signal (ω_1) and idler (ω_2), whose frequencies are tunable and have narrow linewidths [8, 10, 11]. Energy conservation requires that

$$\omega_1 + \omega_2 = \omega_p \,. \tag{1}$$

The phase-matching condition of the parametric process determines the actual output frequencies of the signal-idler pair.

OPO's have been widely used as tunable sources of radiation, from the ultraviolet to the infrared [8]. Recently, OPO's have been utilized with much success to generate strongly correlated twin beams whose intensity correlation falls below the usual shot-noise level [12–15]. Its use as an ultrasensitive intracavity absorption spectrometer [16, 17] has been suggested to take advantage of the strong quantum correlation between the signal and idler photon numbers. Optical parametric amplifiers (OPA's), or below-threshold OPO's, have been extensively used for the generation of squeezed states of light [18–20].

The availability of high-quality nonlinear crystals and the advances in highly stable laser pump sources and frequency stabilization techniques [21] have led to new uses of OPO's that take advantage of their unique coherence properties [11]. Its possible use as a tunable optical frequency divider has been proposed [22] and demonstrated [23], which makes use of the energy conservation relation Eq. (1) and a simple beat-frequency measurement of the signal-idler difference frequency

$$\omega_{12} = \omega_1 - \omega_2 , \tag{2}$$

such that the subharmonic frequencies are precisely determined:

$$\omega_{1,2} = \tfrac{1}{2}(\omega_p \pm \omega_{12}) .$$

Here I propose a new ultrasensitive strain gauge based on a specially configured two-arm OPO and a measurement of the signal-idler beat frequency. When the OPO cavity is scaled to the size of the currently proposed long-baseline interferometric gravity-wave observatory [2], it should be capable of detecting a strain of 10^{-21} that is expected of the weak gravity-wave induced deformation of the space-time metric.

III. THEORETICAL FRAMEWORK

Consider a cw nearly degenerate doubly resonant OPO, in which both subharmonic waves are nearly resonant within the OPO cavity. The classical equations for the internal modes are

$$\begin{aligned} \dot{A}_p &= -\kappa_p A_p - \chi A_1 A_2 + \sqrt{2\kappa_p} E_p , \\ \dot{A}_1 &= -(\kappa_1 - i\Delta_1) A_1 + \chi A_p A_2^* , \\ \dot{A}_2 &= -(\kappa_2 - i\Delta_2) A_2 + \chi A_p A_1^* , \end{aligned} \tag{3}$$

where χ is the positive coupling constant, $E_p = e_p \exp(i\theta_p)$ is the input pump, A_i is the internal field mode, and κ_i is the field total-loss rate. The input power e_p^2 is in units of photons per second. Here the subscript i refers to the pump (p), signal (1), and idler (2). The cavity detunings for the signal and idler modes are defined by

$$\Delta_i = \omega_i - \omega_i^c , \quad i = 1, 2 , \tag{4}$$

where ω_i^c is the cavity resonance frequency for mode i. The cavity is assumed to be a low-finesse type for the pump, with $\Delta_p = 0$.

Writing $A_i = r_i \exp(i\phi_i)$ and setting $\dot{A}_i = 0$, we obtain in the steady state [10]:

$$\Delta_1/\kappa_1 = \Delta_2/\kappa_2 \equiv \tan\theta_\Delta , \quad \pi/2 > \theta_\Delta > -\pi/2 , \tag{5}$$

$$\phi_1 + \phi_2 - \phi_p = \theta_\Delta , \tag{6}$$

$$r_p = \sqrt{\kappa_1\kappa_2 + \Delta_1\Delta_2}\,/\,\chi , \tag{7}$$

$$r_i = C/\sqrt{\kappa_i} , \quad i = 1, 2 , \tag{8}$$

$$e_p \cos(\theta_\Delta + \phi_p - \theta_p) = \tilde{e}_p \left(1 + C^2/2\tilde{e}_p^2\right) , \tag{9}$$

$$e_p \sin(\theta_\Delta + \phi_p - \theta_p) = \tilde{e}_p(\Delta_1/\kappa_1) , \tag{10}$$

where the minimum zero-detuning pump threshold ($\Delta_i = 0$)

$$\tilde{e}_p = \frac{\sqrt{\kappa_1\kappa_2}}{\chi}\sqrt{\frac{\kappa_p}{2}} . \tag{11}$$

Using Eqs. (9) and (10), we obtain

$$C^2 = 2\,\tilde{e}_p^2 \left[\sqrt{F_p - (\Delta_1/\kappa_1)^2} - 1\right] , \tag{12}$$

where $F_p = (e_p/\tilde{e}_p)^2$ is the number of times above the zero-detuning pump threshold power. The output signal and idler powers, including losses, are given by

$$P_1 = 2\kappa_1 r_1^2 = P_2 = 2\kappa_2 r_2^2 = 2C^2 . \tag{13}$$

From the Δ_i definitions (4) and the constraints (1) and (5), we obtain for the beat frequency

$$\omega_{12} = \frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2}\,\omega_p + \frac{2(\kappa_2\omega_1^c - \kappa_1\omega_2^c)}{\kappa_1 + \kappa_2} . \tag{14}$$

The sensitivity of the beat frequency ω_{12} to the pump frequency ω_p is greatly reduced by matching the signal and idler loss rates, $\kappa_1 \approx \kappa_2$.

A schematic for an OPO gravity-wave detector is sketched in Fig. 1. The nondegenerate signal and idler waves are internally separated such that the signal beam propagates along one arm of the OPO cavity and the idler beam along the orthogonal arm. The separation of the signal and idler waves can be achieved with the use of a highly efficient dual-wavelength Mach-Zehnder interferometer [14]. For the case of a type-II phase-matched crystal, in which the signal and idler waves are orthogo-

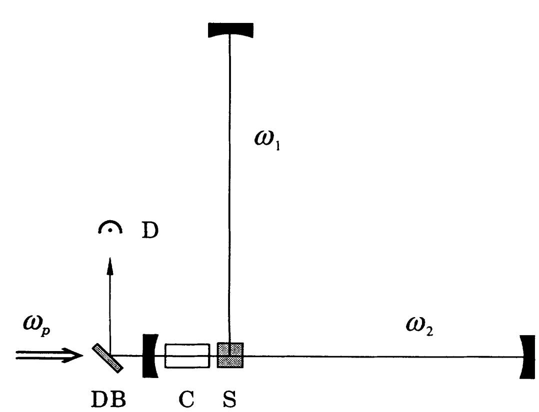

FIG. 1. Schematic of an OPO gravity-wave detector: signal (ω_1) and idler (ω_2) waves are separated (S) to propagate along orthogonal paths. ω_p, pump frequency; C, crystal; DB, dichroic beamsplitter; D, detector.

nally polarized, they can be efficiently separated either by a polarizing beam splitter or by spatial separation inside the birefringent crystal through walk-off of their Poynting vectors. The lengths of the two orthogonal arms are assumed to be much greater than the overlap region.

The three end mirrors in Fig. 1 can be considered as test masses suspended in free space. A gravitational wave propagating perpendicularly to the plane of the OPO induces a length contraction in one arm, say the signal arm, l_1, and a length expansion in the other arm, l_2. The cavity resonance frequency is defined as $\omega_i^c = \pi m_i c/l_i$, where m_i is the number of optical half wavelengths inside the cavity. Therefore a length change δl_i causes a cavity resonance-frequency change, for fixed m_i,

$$\delta\omega_i^c = -\omega_i^c(\delta l_i/l_i) \; . \tag{15}$$

It is customary to write the gravitationally induced strain as

$$\delta l_1/l_1 = -\tfrac{1}{2}h_g(t) \; , \quad \delta l_2/l_2 = \tfrac{1}{2}h_g(t) \; , \tag{16}$$

with

$$h_g(t) = h_0 \cos(\omega_g t) \tag{17}$$

for periodic gravity waves of strain strength h_0 and frequency ω_g.

Substituting Eqs. (15) and (16) into Eq. (14) yields a signal-idler beat-frequency deviation

$$\delta\omega_{12} = \frac{\kappa_1-\kappa_2}{\kappa_1+\kappa_2}\,\delta\omega_p + \frac{\kappa_2\omega_1^c+\kappa_1\omega_2^c}{\kappa_1+\kappa_2}\,h_g(t) \; . \tag{18}$$

If $\kappa_1 = \kappa_2$, the first term in Eq. (18) vanishes and the beat-frequency shift is independent of the pump-frequency jitter. Since the signal- and idler-loss rates (inverse of the photon cavity lifetimes) are functions of both the power losses per round-trip and the cavity round-trip times $2l_i/c$, the lengths of the two arms can be fine tuned to match κ_1 and κ_2 to a high degree. Moreover, by operating the OPO near frequency degeneracy, $\omega_1 \simeq \omega_2$, the signal and idler power losses per round-trip should be about equal. We shall therefore assume that the total-loss rates are well matched and that the pump laser is highly stabilized so that the first term of Eq. (18) can be neglected. The beat-frequency shift due to the gravity waves is then

$$\delta\omega_{12} = \omega_1 h_g(t) \; , \quad \kappa_1 = \kappa_2 \; , \tag{19}$$

where we have assumed $\omega_i \gg \Delta_i \; , \kappa_i$.

The appearance of a time dependence [Eq. (19)] in the steady-state solution may seem contradictory. However, if we allow time variation in the phase but not in the amplitude of A_i in Eqs. (3), one can easily show that a term $(-\dot{\phi}_i)$ should be added to Δ_i. Therefore, the steady-state solutions [Eqs. (5)–(10)] are valid if we identify Δ_i as the instantaneous frequency detuning of the mode i whose phase is slowly varying. Using Eq. (12) one can verify that the variation of the amplitudes r_i due to a minute frequency shift is insignificant.

Over a measurement time t_m that is small compared with the inverse of the tiny frequency shift $\delta\omega_{12}$, the signal-idler beat frequency accumulates a phase shift

$$\phi_g = \int_0^{t_m} \omega_1 h_g(t)\,dt \;=\; \frac{\omega_1}{\omega_g} h_0 \sin(\omega_g t_m) \; , \tag{20}$$

where the periodic nature of the gravity wave [Eq. (17)] is used.

IV. INTERNAL PHASE NOISE

In this section we calculate the phase noise of the signal-idler phase difference $\phi_{12} = \phi_1 - \phi_2$ inside the OPO cavity. As we shall see, the internal phase noise has two sources: the pump-laser phase noise and the phase diffusion noise. While the effect of the pump phase noise can be eliminated by matching the signal- and idler-cavity loss rates, the phase diffusion noise is unavoidable and it originates from the input vacuum modes at the signal and idler frequencies.

To obtain the phase noise, Eqs. (3) are modified to include the vacuum fluctuations and the field modes A_i become quantum annihilation operators [10, 17]:

$$\begin{aligned} \dot{A}_p &= -\kappa_p A_p - \chi A_1 A_2 + \sqrt{2\kappa_p}\,E_p + \sqrt{2\kappa_p}\,u_p \; , \\ \dot{A}_1 &= -(\kappa_1 - i\Delta_1)A_1 + \chi A_p A_2^\dagger + \sqrt{2\kappa_1}\,u_1 \; , \\ \dot{A}_2 &= -(\kappa_2 - i\Delta_2)A_2 + \chi A_p A_1^\dagger + \sqrt{2\kappa_2}\,u_2 \; , \end{aligned} \tag{21}$$

where u_i is the noise operator associated with the input vacuum fluctuation of mode i, and they have the usual nonzero correlations

$$\left\langle u_i(t) u_j^\dagger(t') \right\rangle = \delta_{ij}\,\delta(t-t') \; .$$

Writing $A_i(t) = r_i[1 + a_i(t)]\exp(i\phi_i)$, where a_i is the normalized small-signal annihilation operator, the quantum Langevin equations for an above-threshold OPO are linearized about the large-signal steady-state solutions, Eqs. (5)–(10), and solved for the time-dependent fluctuations a_i and $a_i^\dagger$. For a state with a high mean field $A_i = r_i(1+\mu_i)\exp[i(\phi_i+\psi_i)]$, the normalized amplitude fluctuation μ_i and the phase fluctuation ψ_i are related to a_i and $a_i^\dagger$ by

$$\mu_i = \tfrac{1}{2}\left(a_i + a_i^\dagger\right) \; , \quad \psi_i = \frac{1}{2i}\left(a_i - a_i^\dagger\right) \; . \tag{22}$$

For convenience we define for the real and imaginary parts of the vacuum input modes

$$\begin{aligned} \mu_i^u &= \frac{1}{2}\,(\tilde{u}_i + \text{H.c.}) \; , \quad \psi_i^u = \frac{1}{2i}\,(\tilde{u}_i - \text{H.c.}) \; , \\ \tilde{u}_i &= u_i e^{-i\phi_i} \; , \qquad i = 1,2 \; , \end{aligned} \tag{23}$$

where H.c. means Hermitian conjugate.

The time-dependent solutions to Eqs. (21) have previously been obtained by Graham and Haken [10] and here we will reproduce only the relevant results concerning the phase diffusion of ϕ_{12}. The following assumptions are taken: $|\Delta_i|, |\kappa_1 - \kappa_2| \ll \kappa_1 + \kappa_2$ so that only the lowest order of Δ_i and $\kappa_1 - \kappa_2$ are retained.

There are two relevant phase parameters

$$\Phi = \psi_1 + \psi_2 \, , \quad \Psi = \frac{r_1}{r_2}\psi_1 - \frac{r_2}{r_1}\psi_2 \, . \tag{24}$$

Φ, the sum of the signal and idler phase fluctuations, follows the phase diffusion of the pump phase ψ_p adiabatically [10] such that

$$\left\langle [\Phi(t) - \Phi(0)]^2 \right\rangle = \left\langle [\psi_p(t) - \psi_p(0)]^2 \right\rangle = 2\gamma_p t \, , \tag{25}$$

where γ_p is the pump-laser linewidth. Ψ is given by [10]

$$\Psi(t) - \Psi(0) = \frac{\sqrt{2\kappa_1\kappa_2}}{C} \int_0^t d\tau \, (\psi_1^u - \psi_2^u)_\tau + \frac{2\sqrt{2\kappa_1\kappa_2}}{C} \int_0^t d\tau \int_0^\tau d\tau' \, e^{-(\kappa_1+\kappa_2)(\tau-\tau')} (\Delta_1\mu_1^u - \Delta_2\mu_2^u)_{\tau'} \, . \tag{26}$$

The effect of nonzero cavity detunings Δ_i is the coupling of the real part of the signal and idler vacuum modes μ_i^u into the phase Ψ. For times t longer than the photon cavity lifetimes $1/2\kappa_i$, the mean-square fluctuation of Ψ is given by

$$\left\langle [\Psi(t) - \Psi(0)]^2 \right\rangle = \frac{4\kappa_1\kappa_2}{P_s} \left(1 + 2 \frac{\Delta_1^2 + \Delta_2^2}{(\kappa_1 + \kappa_2)^2} \right) t \, , \tag{27}$$

where $P_s = P_1 + P_2 = 4C^2$ is the total average output power.

From Eqs. (24) we can express the beat-note phase fluctuation

$$\psi_{12} = \psi_1 - \psi_2 = \frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2} \Phi + \frac{2\sqrt{\kappa_1\kappa_2}}{\kappa_1 + \kappa_2} \Psi \, , \tag{28}$$

where relation (8) is used. The mean-square fluctuation of the internal phase is then obtained:

$$\begin{aligned} \Delta\psi_{12}^2(t) &= \langle [\psi_{12}(t) - \psi_{12}(0)]^2 \rangle \\ &= \left[\left(\frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2} \right)^2 2\gamma_p + \frac{16\kappa_1^2\kappa_2^2}{P_s(\kappa_1 + \kappa_2)^2} \left(1 + 2\frac{\Delta_1^2 + \Delta_2^2}{(\kappa_1 + \kappa_2)^2} \right) \right] t \, . \end{aligned} \tag{29}$$

The first term on the right-hand side of Eq. (29) is due to the phase diffusion of the pump laser, while the second term is the intrinsic OPO phase diffusion noise that is very much like the Schawlow-Townes laser phase-diffusion-noise term [10, 24]. For $\kappa_1 \approx \kappa_2$, the effect of the pump-laser phase diffusion is much reduced and one obtains

$$\Delta\psi_{12}^2(t) = \frac{4\kappa_1^2}{P_s} \left(1 + \frac{\Delta_1^2}{\kappa_1^2} \right) t, \quad \kappa_1 = \kappa_2 \, . \tag{30}$$

The phase diffusion can be further minimized by operating the OPO close to cavity resonance, in which case the phase noise ψ_{12} is mainly caused by the imaginary parts of the input signal and idler vacuum modes ψ_i^u [Eq. (26)]:

$$\Delta\psi_{12}^2(t) = \frac{4\kappa_1^2}{P_s} t, \quad \Delta_i \ll \kappa_i \, . \tag{31}$$

V. EXTERNAL PHASE NOISE

In calculating the external phase noise it is useful to specify the phase-measuring apparatus. Figure 2 shows a schematic of a synchronous detector that is suitable for a high-intensity field $E(t)$. Direct detection of the optical field is followed by a bandpass filter that centers at the beat frequency ω_{12}. The bandpass-filtered photocurrent $i(t)$, which contains the heterodyne beat of the signal and idler frequencies, is synchronously demodulated at the same beat frequency, and then lowpass filtered. The two quadrature signals $x(t)$ and $y(t)$ are then processed to yield the phase of the optical field $E(t)$.

As in the usual heterodyne detection [25], it is necessary to include the image-band vacuum modes u_3 and u_4 in order to obtain the correct shot-noise level. We assume in this case that the beat frequency is much larger than the cavity linewidths $\omega_{12} \gg \kappa_i$ so that the image-band vacuum modes are not affected by the parametric interaction. We further assume negligible internal losses so that κ_i is the output-coupling loss rate and P_s is the total detected power. The field external to the OPO cavity

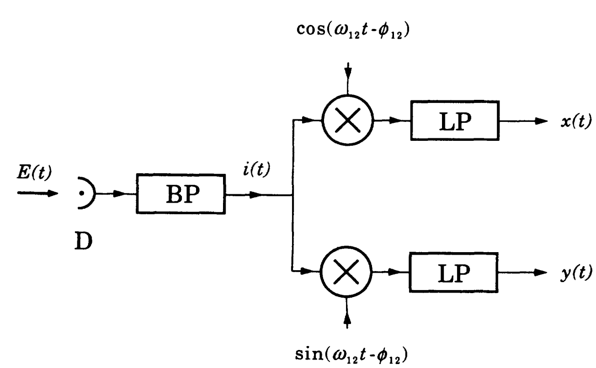

FIG. 2. Schematic of phase-measurement apparatus. D, detector; BP, bandpass filter; LP, lowpass filter. The bandpass-filtered photocurrent $i(t)$ is synchronously demodulated at $(\omega_{12}t - \phi_{12})$ to yield the amplitude $x(t)$ and phase $y(t)$ quadratures.

$$E(t)=\sqrt{2\kappa_1}\,r_1(1+a_1)e^{-i(\omega_1 t-\phi_1)}+\sqrt{2\kappa_2}\,r_2(1+a_2)e^{-i(\omega_2 t-\phi_2)}$$
$$-u_1e^{-i\omega_1 t}-u_2e^{-i\omega_2 t}-u_3e^{-i(2\omega_1-\omega_2)t}-u_4e^{-i(2\omega_2-\omega_1)t}\ . \tag{32}$$

The bandpass-filtered photocurrent is then given by

$$i(t)=2C^2\left[e^{-i(\omega_{12}t-\phi_{12})}\left(1+a_1+a_2^\dagger\right)+\text{H.c.}\right]-\sqrt{2}\,C\left[e^{-i(\omega_{12}t-\phi_{12})}(\tilde{u}_1+\tilde{u}_2^\dagger+\tilde{u}_3+\tilde{u}_4^\dagger)+\text{H.c.}\right]\ , \tag{33}$$

where

$$\tilde{u}_3=u_3e^{-i(2\phi_1-\phi_2)}\ ,\qquad \tilde{u}_4=u_4e^{-i(2\phi_2-\phi_1)}\ ,$$

and we assume unity detector quantum efficiency and set the electron charge to unity for convenience. The synchronously demodulated signals are defined by

$$x(t_m)=\frac{1}{t_m}\int_0^{t_m}dt\ i(t)\cos(\omega_{12}t-\phi_{12})\ , \tag{34}$$

$$y(t_m)=\frac{1}{t_m}\int_0^{t_m}dt\ i(t)\sin(\omega_{12}t-\phi_{12})\ . \tag{35}$$

By retaining only the dc terms, the demodulated, lowpass-filtered quadrature signals are obtained:

$$x(t_m)=\frac{1}{t_m}\int_0^{t_m}dt\ \left[2C^2\left(1+\mu_1+\mu_2\right)-\sqrt{2}\,C\left(\mu_1^u+\mu_2^u+\mu_3^u+\mu_4^u\right)\right]\ , \tag{36}$$

$$y(t_m)=\frac{1}{t_m}\int_0^{t_m}dt\ \left[2C^2\psi_{12}-\sqrt{2}\,C(\psi_{12}^u+\psi_{34}^u)\right]\ , \tag{37}$$

where $\psi_{ij}^u=\psi_i^u-\psi_j^u$ and ψ_i^u is defined by Eqs. (23). The mean quadrature signals are easily obtained:

$$\langle x(t_m)\rangle=2C^2\ ,\qquad \langle y(t_m)\rangle=0\ . \tag{38}$$

Figure 2 indicates that the local oscillator for demodulation includes ϕ_{12}, as in the usual phase-locked loop in which the phase of interest is tracked. This choice permits the out-of-phase quadrature to have a zero mean. For a large in-phase signal $\langle x\rangle^2 \gg \langle\Delta x^2\rangle,\langle\Delta y^2\rangle$ the mean-square fluctuation of the measured phase is given by

$$\Delta\phi_{12}^2(t_m)=\langle\Delta y^2(t_m)\rangle\,/\,\langle x(t_m)\rangle^2\ . \tag{39}$$

Assuming that the OPO is operated at cavity resonance and with equal cavity-loss rates, i.e., $\Delta_i=0$ and $\kappa_1=\kappa_2$, the minimum internal phase noise [Eq. (31)] is obtained. We can then write, using Eqs. (26) and (28),

$$y(t_m)=\frac{\sqrt{2}\,C}{t_m}\int_0^{t_m}dt\ \Bigg(\int_0^t d\tau\ 2\kappa_1\psi_{12}^u(\tau)-\psi_{12}^u(t)-\psi_{34}^u(t)\Bigg)\ , \tag{40}$$

which yields

$$\langle\Delta y^2(t_m)\rangle=\langle y^2(t_m)\rangle=\frac{2C^2}{t_m}(1-\kappa_1t_m+\tfrac{2}{3}\kappa_1^2t_m^2)\ . \tag{41}$$

The phase noise is then given by

$$\Delta\phi_{12}^2(t_m)=\frac{2}{P_st_m}(1-\kappa_1t_m+\tfrac{2}{3}\kappa_1^2t_m^2)\ . \tag{42}$$

From Eqs. (26) and (28) one observes that the internal phase fluctuation ψ_{12} depends on the time history of ψ_{12}^u. Therefore one cannot simply add the phase diffusion noise and the photon shot noise, which is caused by ψ_i^u, as if they were independent noise sources.

It is useful to evaluate the above phase-noise result for a limiting case: a signal which consists of two uncorrelated coherent states of equal amplitudes at the frequencies ω_1 and ω_2, plus the two image-band vacuum modes u_3 and u_4. This can be represented in our analysis by setting $\kappa_i=0$, i.e., the cavity is totally reflecting and the internal field is completely isolated from the external field. Equation (42) then yields the simple result

$$\Delta\phi_{12}^2(t_m)=2/P_st_m\ ,\qquad \kappa_i=0\ . \tag{43}$$

The mean-square photon-number fluctuation in this case is well known and is given by $\Delta N^2(t_m)=N(t_m)=P_st_m$, which yields a number-phase uncertainty product

$$\Delta N^2(t_m)\Delta\phi_{12}^2(t_m)=2\ ,\quad \kappa_i=0\ . \tag{44}$$

The above uncertainty product is four times larger than the usual single-mode homodyne case [26] because there are four modes in this heterodyne measurement: two signal and two image-band vacuum modes.

Returning to the main result of the external phase noise, Eq. (42) can be written as

$$\Delta\phi_{12}^2(t_m)=\frac{2}{P_st_m}[\tfrac{5}{8}+\tfrac{2}{3}(\kappa_1t_m-\tfrac{3}{4})^2]. \tag{45}$$

At $\kappa_1t_m=\frac{3}{4}$ the minimum phase noise is obtained with a mean-square phase fluctuation

$$\Delta\phi_{12}^2(\text{min})=5/4P_st_m\ ,\qquad \kappa_1t_m=\tfrac{3}{4}\ . \tag{46}$$

This minimum phase noise is smaller than the shot noise of two uncorrelated coherent states, Eq. (43), indicating a time-dependent squeezing of the external phase noise of the OPO's signal-idler beat frequency. For times longer than the cavity decay times $\kappa_1 t_m \gg 1$ the phase noise is primarily due to the internal signal-idler phase diffusion and is equal to $(4\kappa_1^2/3P_s)t_m$, which is smaller than the internal phase noise [Eq. (31)] by a factor of 3. For times shorter than the cavity decay times $\kappa_1 t_m \ll 1$ the internal phase diffusion is frozen and the external phase noise is simply the usual photon shot noise $2/P_s t_m$. The $1/t_m$ dependence for short times and the t_m dependence for long times suggest that the minimum phase noise is obtained in the intermediate region of $\kappa_1 t_m \sim 1$. Indeed, the shot noise and the phase diffusion noise interfere to yield the minimum phase noise $\Delta\phi_{12}(\text{min})$ at $\kappa_1 t_m = \frac{3}{4}$.

Equation (20) shows that the optimum integration time $t_m \sim \pi/2\omega_g$ yields a signal $\phi_g = (\omega_1/\omega_g)h_0$. This optimum measurement time is chosen to be about a quarter of the expected period of the gravity waves. For shorter times the induced phase shift has not been accumulated. For longer times the phase shift has already reached its maximum value and in order to maintain the minimum phase noise (at $\kappa_1 t_m = \frac{3}{4}$) the cavity finesse has to be increased (smaller κ_1) without additional benefits. For a unity signal-to-noise ratio, the minimum detectable gravity-wave strain is given by

$$h_{\text{min}} = \frac{\omega_g}{\omega_1}\left(\frac{2}{P_s t_m}(1 - \kappa_1 t_m + \tfrac{2}{3}\kappa_1^2 t_m^2)\right)^{1/2} ,$$

$$\omega_g t_m \approx \pi/2 \,. \quad (47)$$

By choosing a cavity-loss rate given by $\kappa_1 t_m = \frac{3}{4}$, the phase noise is minimized to yield

$$h_{\text{min}} \approx \frac{2\kappa_1}{\omega_1}\sqrt{1/P_s t_m} \,, \quad \omega_g t_m \approx \pi/2 \,, \quad \kappa_1 t_m = \tfrac{3}{4} \,. \quad (48)$$

Noting that $2\kappa_1$ is the cavity power-decay rate, Eq. (48) shows that the OPO cavity interferometer has the same sensitivity as either a passive Michelson interferometer [4, 5] or an active two-laser interferometer [3] for a given size and finesse of the apparatus and detected optical power. For example, if we take $\omega_g/2\pi = 100$ Hz, $\omega_1/2\pi = 3\times10^{14}$ Hz, $\hbar\omega_1 P_s = 1$ W, output coupling of 1%, and an arm length of 2.5 km, then we have $2\kappa_1 = 600$ s^{-1}, and $h_{\text{min}} \approx 3\times10^{-21}$.

VI. DISCUSSION

An above-threshold OPO has the following quantum-noise characteristics for the output intensities [12–14]. The signal and idler intensities, when measured by separate detectors, are strongly correlated and yield a sub-shot-noise difference intensity spectrum for frequencies within the cavity linewidth. Within the same frequency bandwidth there is excess noise in the individual and summed intensity spectra. For frequencies large compared with the cavity linewidth, however, both the sum and difference intensities are shot-noise limited. For the proposed OPO gravity-wave detector with its long arm lengths, the cavity linewidth is assumed to be much smaller than the signal-idler beat frequency. Therefore it cannot take advantage of the sub-shot-noise intensity correlation between the signal and idler beams nor is it affected by the excess noise in the sum intensity at low frequencies.

Implicit in our OPO analysis is that the internal losses of the OPO cavity are much smaller than the output-coupling losses. In the case that the internal losses are not negligible, the internal phase diffusion is caused by the vacuum modes that enter through the output-coupling mirror and those that enter through the internal losses. The output power that is detected through the coupling mirror is reduced by the ratio of the internal losses to the total losses. Consequently, the phase noise is increased and the detection sensitivity is reduced. This is similar to squeezing experiments in which internal losses reduce the correlation that is detected through the output-coupling mirror [12–15, 17, 27].

In obtaining our result, Eq. (42), the near-resonance condition $\Delta_i \approx 0$ is assumed. Equation (26) shows that a nonzero Δ_i couples the real parts of the vacuum modes μ_1^u and μ_2^u into the internal phase noise. This leads to an increase in both the internal and external phase noise. Therefore, the near-resonance condition is required for obtaining the lowest phase noise of the OPO interferometer.

For the passive interferometer, it is known that, in principle, the detection sensitivity can be enhanced by injecting squeezed vacuum into the unused input port [4, 5]. The spontaneous emission noise in the active two-laser interferometer can be suppressed if a correlated-emission laser is used, which reduces the phase diffusion between the two emitted radiations [3]. For the proposed OPO gravity-wave interferometer, it is also possible to improve the phase-measurement sensitivity by using squeezed light.

Under the ideal conditions of negligible internal losses and near-resonance operation, the external phase noise is entirely due to the input vacuum phase noises ψ_{12}^u and ψ_{34}^u [Eq. (40)]. By injecting quadrature-squeezed vacuum at the four frequencies into the OPO, such that $\psi_1^u \approx \psi_2^u$ and $\psi_3^u \approx \psi_4^u$, the external phase fluctuation can be much reduced. For a 10-dB quadrature-squeezed vacuum input, this yields about a tenfold improvement in the detection sensitivity. The two pairs of vacuum modes 1-2 and 3-4 are symmetrically displaced in frequency about the degenerate frequency $(\omega_1 + \omega_2)/2$. Therefore, the quadrature-squeezed vacuum can be generated with a similar but much smaller OPA [19] that is pumped by the same laser with frequency ω_p. The squeezed spectrum of a small OPA can extend to several tens of MHz and hence overlap the required beat-frequency range of the proposed OPO gravity-wave detector. In this way a single OPA can be used to generate all four quadrature-squeezed vacuum modes.

In practice, however, the use of quadrature-squeezed vacuum inputs presents some severe technical challenges. As the input vacuum phase noise is squeezed, the corresponding input amplitude noise increases [19]. Therefore it is necessary to maintain a tight cavity resonance $\Delta_i = 0$; otherwise, a significant amount of the input amplitude noise would be coupled into the phase fluctuation of the beat frequency [see Eq. (26)]. In addition, the internal losses of the OPO interferometer have to be much smaller than the output coupling, as is required in most squeezing experiments [12–15, 27].

Another source of phase noise is the fluctuations of radiation pressure on the mirrors (radiation-pressure error) [4, 5]. In the two-arm OPO this radiation-pressure error is proportional to the fluctuation in the internal signal-idler intensity difference. (Strictly speaking, the proportionality is only approximate for equal end-mirror masses because the signal and idler frequencies are slightly different. However, if the end mirrors have slightly different masses in the same ratio as that of the signal and idler frequencies, the proportionality is exact.) It is well known that the signal and idler intensities are perfectly correlated outside a lossless cavity, but the maximum amount of difference intensity squeezing is only 50% inside the cavity [28]. Therefore the radiation-pressure error in the two-arm OPO is unavoidable and, unlike the two-frequency interferometer analyzed by Bondurant and Shapiro [5], the two-arm OPO cannot surpass the standard quantum limit in position sensing [4, 5]. Under normal operating conditions in which the available optical powers are far below the optimal amounts [4], the sensitivity of the OPO interferometer is primarily limited by the photon shot noise and the signal-idler phase diffusion and is worse than the standard quantum limit. The sensitivity can be improved with the use of quadrature-squeezed vacuum inputs as discussed above, but at the expense of increased amplitude noise which leads to an increase in the radiation-pressure error. The tradeoff is optimized when the two noise sources become equal. This is basically the same situation as in the passive interferometer [4], and hence one expects that in principle the standard quantum limit can be reached by both types of interferometer at about the same average photon number.

As in all interferometric gravity-wave detectors, the proposed OPO interferometer requires that the arms' lengths be highly stabilized. While its detection sensitivity is the same (assuming no squeezed light or correlated-emission laser is used) as that of the passive Michelson interferometer and the active two-laser interferometer, the OPO gravity-wave detector has some practical advantages. The use of beat-frequency detection reduces the effects of certain technical noise sources that have a high noise content at low frequencies. The phase noise is immune to the pump-laser frequency jitter and phase fluctuation if equal signal- and idler-loss rates are maintained. In a practical system, a pump linewidth of less than 1 Hz should be adequate. The nonresonant parametric interaction in a highly transparent nonlinear crystal means that the OPO can have very low internal losses and a high finesse of the OPO cavity can be obtained. Remaining losses are primarily due to scattering and absorption losses at the crystal and mirror surfaces. Current optical-coating capability shows $< 10^{-5}$ fractional scattering and absorption loss per surface for mirrors. Suitable nonlinear crystals such as potassium titanyl phosphate (KTP), which can be type-II phase-matched, has a high nonlinearity, a high damage threshold, and very low losses at the 1-μm-wavelength region. In addition, it has been demonstrated that a cw doubly resonant KTP OPO can be continuously tuned [23], even through the frequency-degenerate point, suggesting that the disadvantage of added complexity in this OPO arrangement can be overcome.

In summary, we have shown that an OPO with orthogonal signal and idler paths can be utilized as an active gravity-wave detector. The gravity-wave induced phase shift of the beat frequency is monitored by synchronous detection and a comparison with a stable rf source. The external phase noise is calculated using a linearized quantum analysis and is shown to exhibit time-dependent squeezing. The phase fluctuation is found to be comparable to the usual passive Michelson interferometer and the active two-laser interferometer, and the detection sensitivity can be improved with the injection of quadrature-squeezed vacuum.

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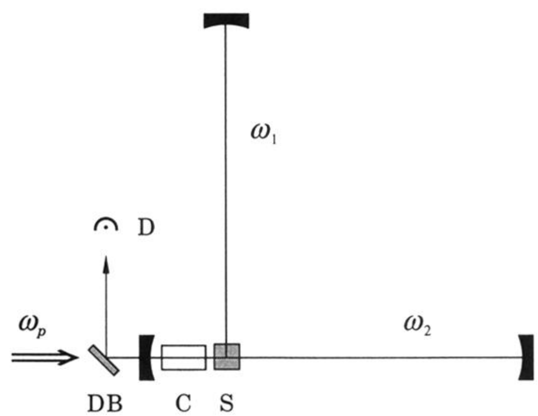

FIG. 1. Schematic of an OPO gravity-wave detector: signal (ω_1) and idler (ω_2) waves are separated (S) to propagate along orthogonal paths. ω_p, pump frequency; C, crystal; DB, dichroic beamsplitter; D, detector.