Nonlocal cancellation of dispersion

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Classically, two coincident light pulses propagating through two dispersive media will become broadened and their degree of coincidence will be reduced. When entangled photon pairs from parametric down-conversion are considered instead, it is found that the dispersion experienced by one photon can exactly cancel the dispersion experienced by the other in such a way that their coincidence is maintained. The dispersion cancellation is independent of the separation between the two photons and provides a further example of the nonlocal nature of the quantum theory.

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I. INTRODUCTION

A classical pulse of light propagating through a dispersive medium will experience a broadening that depends upon the local properties of the medium. If two such pulses are initially coincident but propagate in different directions through two separate media, then the dispersion of one pulse will be independent of the second. As a result, neither pulse represents a precisely defined time, and their degree of coincidence will be reduced. The independence of the two pulses follows directly from the local nature of Maxwell's equations and would hold equally well for any local theory.

In quantum optics, however, a coincident pair of photons produced by parametric down-conversion [1] corresponds to an entangled [2] state whose wave function cannot be factored into the product of two independent wave functions. This results in a variety of nonlocal effects, including violations [2,3] of Bell's inequality [4] for local hidden-variable theories and other inequalities satisfied by any semiclassical field theory [5,6]. Such effects have been observed in recent two-photon interferometer experiments [7–11] as well as earlier experiments [12,13] based upon the polarizations of the down-converted photons.

The propagation of a pair of entangled photons through two dispersive media will be considered here. It will be found that the dispersion experienced by one photon can be canceled out by the dispersion experienced by the other photon in such a way that the two photons remain coincident. This dispersion cancellation is independent of the separation between the two photons and provides a further example of the nonlocal nature of the quantum theory when dealing with entangled states.

II. SEMICLASSICAL DISPERSION

Although the classical theory of dispersion is well known [14], it may be useful to derive first the degree of coincidence of two classical light pulses in a form that is suitable for subsequent comparison with the quantum-theory predictions.

A light source will be assumed to emit two identical pulses that, for simplicity, will be taken to have negligible widths at the time $t=0$ of their emission. After passing through two identical narrow-band filters f_1 and f_2, the pulses propagate in different directions along paths 1 and 2 toward detectors D_1 and D_2.

The electric fields of the two pulses will be denoted by $E_1(x_1,t_1)$ and $E_2(x_2,t_2)$. A single linear polarization will be considered and it will be assumed that the two light beams have been sufficiently well collimated that they can be represented by plane waves, in which case the coordinates x_1 and x_2 can be taken to be one dimensional. It will also be assumed that the dispersive media along paths 1 and 2 have homogeneous indices of refraction that may not be the same in the two regions.

The fields at the location of the source can be written as

$$E_1(0,t_1)=E_0\delta(t_1)=\frac{E_0}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega_1 t_1}d\omega_1 , \tag{1}$$

where E_0 is a constant and a similar expression exists for $E_2(0,t_2)$. Both filters will be assumed to have a transmission coefficient $f(\omega)$ for incident fields given by

$$f(\omega)=e^{-(\omega-\omega_F)^2/2\sigma_F^2}, \tag{2}$$

where ω_F is the center frequency [15] of the filters and σ_F is their width (one standard deviation). After passing through the filters the fields then have the form

$$E_1(0,t_1)=\frac{E_0}{2\pi}\int_{-\infty}^{\infty}e^{-(\omega_1-\omega_F)^2/2\sigma_F^2}e^{-i\omega_1 t_1}d\omega_1 \tag{3}$$

with a similar expression for $E_2(0,t_2)$. The propagation delay between the source and filters has been assumed to be negligibly small in Eq. (3).

Since the filters have relatively small bandwidths, the wave number $k(\omega)$ as a function of the angular frequency ω can be expanded in a Taylor series about ω_F

$$k(\omega)=k_F+\alpha(\omega-\omega_F)+\beta(\omega-\omega_F)^2 . \tag{4}$$

Here k_F, α, and β are constants that may have different values in the two media. The second-order terms are sufficient to demonstrate the effects of interest, and higher-order terms, which are extremely small for a sufficiently narrow filter, have been neglected.

The comparison with the quantum-mechanical situation can be facilitated by introducing the small parameters ϵ_1 and ϵ_2 defined by

$$\omega_1=\omega_F+\epsilon_1 ,\qquad \omega_2=\omega_F-\epsilon_2 . \tag{5}$$

The reason for including the minus sign in the second equation will become apparent shortly. The dispersion relations can now be written as

$$k_1(\omega_1)=k_{F1}+\alpha_1\epsilon_1+\beta_1\epsilon_1^2 ,\qquad k_2(\omega_2)=k_{F2}-\alpha_2\epsilon_2+\beta_2\epsilon_2^2 . \tag{6}$$

It should be noted that the sign convention is such that $\alpha_1=\alpha_2$ and $\beta_1=\beta_2$ if the properties of the two media are identical.

After propagating through the medium in path 1, the electric field at detector D_1 becomes

$$E_1(x_1,t_1)=\frac{E_0}{2\pi}\int_{-\infty}^{\infty} e^{-\epsilon_1^2/2\sigma_F^2}e^{i(k_{F1}+\alpha_1\epsilon_1+\beta_1\epsilon_1^2)x_1} \times e^{-i(\omega_F+\epsilon_1)t_1}d\epsilon_1 , \tag{7}$$

where x_1 and x_2 now denote the distances between the detectors and the light source. Equation (7) can be integrated to give

$$E_1(x_1,t_1)=\frac{E_0}{2\pi^{1/2}a_1}\exp\left[-\frac{(\alpha_1x_1-t_1)^2(\sigma_0^2+i\beta_1x_1)}{4(\sigma_0^4+\beta_1^2x_1^2)}\right] , \tag{8}$$

where

$$a_1^2=\frac{1}{2\sigma_F^2}-i\beta_1x_1 , \tag{9}$$

$$\sigma_0^2=\frac{1}{2\sigma_F^2} \tag{10}$$

and an irrelevant phase factor has been dropped.

Multiplying Eq. (8) by its complex conjugate gives the intensity $I_1(x_1,t_1)$ of field 1

$$I_1(x_1,t_1)=\frac{E_0^2}{4\pi a_1^*a_1}e^{-(\alpha_1x_1-t_1)^2/2\sigma_1^2} \tag{11}$$

where the width σ_1 is given by

$$\sigma_1^2=\frac{(\sigma_0^4+\beta_1^2x_1^2)}{\sigma_0^2} . \tag{12}$$

The corresponding expressions for field 2 are

$$I_2(x_2,t_2)=\frac{E_0^2}{4\pi a_2^*a_2}e^{-(\alpha_2x_2-t_2)^2/2\sigma_2^2} , \tag{13}$$

$$a_2^2=\frac{1}{2\sigma_F^2}-i\beta_2x_2 , \tag{14}$$

$$\sigma_2^2=\frac{(\sigma_0^4+\beta_2^2x_2^2)}{\sigma_0^2} . \tag{15}$$

Detectors D_1 and D_2 will be assumed to be single-photon detectors, such as photomultiplier tubes, with detection efficiencies and pulse intensities sufficiently small that the probability of a detection event for any given light pulse is much less than one [16]. In a semiclassical field theory the probability of obtaining a count from either detector is then proportional to the local field intensity. The probability P of obtaining two such counts at times t_1 and $t_2=t_1+\tau$ is thus

$$P=\eta I_1(x_1,t_1)I_2(x_2,t_1+\tau) , \tag{16}$$

where η is a constant related to the detection efficiency. The overall probability $P(\tau)$ of detecting two photons at a time lag τ is then the integral of Eq. (16) over all time

$$P(\tau)=\frac{\eta E_0^4}{(4\pi)^2a_1^*a_1a_2^*a_2} \times\int_{-\infty}^{\infty} e^{-(\alpha_1x_1-t_1)^2/2\sigma_1^2}e^{-[\alpha_2x_2-(t_1+\tau)]^2/2\sigma_2^2}dt_1 . \tag{17}$$

By completing the square in the exponential, Eq. (17) can be put in the form

$$P(\tau)=\gamma\exp\left[-\frac{(\tau-\bar{\tau})^2}{2(\sigma_1^2+\sigma_2^2)}\right] , \tag{18}$$

where $\bar{\tau}=\alpha_2x_2-\alpha_1x_1$ and the constant γ has the value

$$\gamma=\frac{\eta E_0^4}{(4\pi)^2a_1^*a_1a_2^*a_2}\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2}\left[\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right]t^2\right]dt . \tag{19}$$

It can be seen from Eq. (18) that the classical coincidence distribution has a width σ_T given by

$$\sigma_T^2=\sigma_1^2+\sigma_2^2=\frac{2\sigma_0^4+(\beta_1^2+\beta_2^2)x^2}{\sigma_0^2} , \tag{20}$$

where it has been assumed for simplicity that $x_1=x_2=x$. In the limit of small dispersion or small distances this reduces to

$$\sigma_T^2=2\sigma_0^2=1/\sigma_F^2 \tag{21}$$

while for large distances and large dispersions

$$\sigma_T^2=\frac{(\beta_1^2+\beta_2^2)x^2}{\sigma_0^2}=2\sigma_F^2(\beta_1^2+\beta_2^2)x^2 . \tag{22}$$

No cancellation of the classical dispersion coefficients is possible since Eq. (22) corresponds to the sum of the squares of the individual coefficients. Although the special case of two Gaussian pulses was considered here, the Appendix shows that this is a general property of any classical or semiclassical theory. The lack of classical dispersion cancellation is a consequence of the independent propagation of the two pulses.

III. DISPERSION IN QUANTUM OPTICS

The propagation of light in two different media need not be independent if we consider a correlated or "entangled" quantum-mechanical state of two photons. The most general two-photon entangled state at the initial time $t=0$ has the form

$$|\psi(0)\rangle=\int d\omega_1\int d\omega_2 g(\omega_1,\omega_2)a^\dagger_{k_1}b^\dagger_{k_2}|0\rangle \ . \tag{23}$$

Here $g(\omega_1,\omega_2)$ is an arbitrary function, $|0\rangle$ is the vacuum state, and the operators $a^\dagger_{k_1}$ and $b^\dagger_{k_2}$ create photons in paths 1 and 2, respectively.

If the photon pair is created from an initial photon of energy $\hbar\omega_0$, as is the case in parametric down-conversion [1], then energy conservation limits the possible form of the state vector to

$$|\psi(0)\rangle=c_N\int_0^{\omega_0}d\omega_1 f^2(\omega_1)a^\dagger_{k_1}b^\dagger_{k_0-k_1}|0\rangle \ . \tag{24}$$

Here c_N is a suitable normalization constant, and we have used

$$\omega_2=\omega_0-\omega_1 \ . \tag{25}$$

The passage of both photons through the two filters has been reflected by the inclusion of the factor of $f^2(\omega_1)$. The center frequency ω_F was chosen to be $\omega_0/2$, in which case $f(\omega_2)=f(\omega_1)$ from Eqs. (2) and (25). This choice simplifies the calculations somewhat, but the same general results can be obtained without it.

Equation (25) can be rewritten by introducing a small parameter ϵ defined by

$$\omega_1=\omega_0/2+\epsilon \ , \qquad \omega_2=\omega_0/2-\epsilon \ . \tag{26}$$

Equation (26) is analogous to the semiclassical equation (5) except that now energy conservation requires $\epsilon_2=\epsilon_1=\epsilon$. The dispersion relations of Eq. (6) can now be written as

$$k_1(\omega_1)=k_{F1}+\alpha_1\epsilon+\beta_1\epsilon^2 \ , \qquad k_2(\omega_2)=k_{F2}-\alpha_2\epsilon+\beta_2\epsilon^2 \ . \tag{27}$$

It will be simplest to work in the Heisenberg picture, in which case the positive-frequency component [17] of the electric field operator has the form

$$E^+(x_1,t_1)=i\sum_{\omega_1}\left[\frac{2\pi\hbar\omega_1}{V}\right]^{1/2}e^{i(k_1x_1-\omega_1t_1)} \ . \tag{28}$$

The probability P of detecting two photons at times t_1 and $t_2=t_1+\tau$ is proportional to

$$P=\eta'\langle\psi(0)|E_1^-(x_1,t_1)E_2^-(x_2,t_1+\tau)\times E_2^+(x_2,t_1+\tau)E_1^+(x_1,t_1)|\psi(0)\rangle \ , \tag{29}$$

where the constant η' is related to the detection efficiency.

Combining Eqs. (24)–(28) gives the approximate expression

$$E_2^+(x_2,t_1+\tau)E_1^+(x_1,t_1)|\psi(0)\rangle=-c_N\left[\frac{2\pi\hbar\omega_F}{V}\right]\int_{-\infty}^{\infty}d\epsilon\, e^{-\epsilon^2/\sigma_F^2}e^{i[(k_{F1}+\alpha_1\epsilon+\beta_1\epsilon^2)x_1-(\omega_F+\epsilon)t_1]}\times e^{i[(k_{F2}-\alpha_2\epsilon+\beta_2\epsilon^2)x_2-(\omega_F-\epsilon)(t_1+\tau)]}|0\rangle \ . \tag{30}$$

The narrow bandwidth of the filters allowed the slowly varying factors of ω_1 and ω_2 in the square roots to be approximated by ω_F. The range of the integral was also extended to $\pm\infty$.

The integral in Eq. (30) can be evaluated to give

$$E_2^+(x_2,t_1+\tau)E_1^+(x_1,t_1)|\psi(0)\rangle=\frac{c_N\pi^{1/2}}{a'}\left[\frac{2\pi\hbar\omega_F}{V}\right]\exp\left[-\frac{(\tau-\bar{\tau})^2(1/\sigma_F^2+i(\beta_1+\beta_2)x)}{4(1/\sigma_F^4+(\beta_1+\beta_2)^2x^2)}\right]|0\rangle \ . \tag{31}$$

An irrelevant phase factor has been dropped, and it has been assumed once again that $x_1=x_2=x$. The constants a' and $\bar{\tau}$ are given by

$$a'^2=1/\sigma_F^2-i(\beta_1+\beta_2)x \ , \tag{32}$$

$$\bar{\tau}=(\alpha_2-\alpha_1)x \ . \tag{33}$$

Multiplying Eq. (31) by its Hermitian conjugate gives

$$P'(\tau)=\frac{\eta' c_N^2}{(a'^*a')}\left[\frac{2\pi\hbar\omega_F}{V}\right]^2\pi e^{-(\tau-\bar{\tau})^2/2\sigma_T'^2} \ , \tag{34}$$

where

$$\sigma_T'^2=\frac{1/\sigma_F^4+(\beta_1+\beta_2)^2x^2}{1/\sigma_F^2} \ . \tag{35}$$

The quantum-mechanical coincidence distribution thus has a width that approaches

$$\sigma_T'^2=1/\sigma_F^2=2\sigma_0^2 \tag{36}$$

when the dispersion or distances are negligibly small. In the opposite limit of large dispersion the quantum-mechanical width becomes

$$\sigma_T'^2 = \frac{(\beta_1+\beta_2)^2 x^2}{2\sigma_0^2} . \tag{37}$$

Unlike the classical result, the quantum-mechanical width depends upon the square of the sum of the individual dispersion coefficients, which gives rise to an interference term, $2\beta_1\beta_2$.

A comparison of Eqs. (20) and (21) with Eqs. (36) and (37) shows that the quantum-mechanical results are the same as the classical results in the limit of negligible dispersion or whenever $\beta_1=\beta_2$.

The interference term is due to the fact that the two-photon probability amplitudes are coherently summed before being squared and is somewhat analogous to the earlier two-photon interferometer experiments [3,7,8,10,11]. The summation of probability amplitudes is, of course, a fundamental property of quantum mechanics and gives rise to nonlocal effects in this case. In particular, a medium may have a negative dispersion coefficient if the frequency of the light is near that of a resonant atomic transition [14,17]. If the properties of the two media are such that $\beta_1=-\beta_2$, then the dispersion encountered by one of the photons will cancel that encountered by the other photon. The two photons can remain totally coincident after traversing two dispersive media, aside from the intrinsic spread σ_0, which goes to zero in the limit of large bandwidth and is negligibly small in most experiments.

If the two detectors are not equidistant from the source, Eq. (37) can be generalized to

$$\sigma_T'^2 = \frac{(\beta_1 x_1+\beta_2 x_2)^2}{2\sigma_0^2} . \tag{38}$$

It can be seen that the cancellation will be complete whenever $\beta_1 x_1=-\beta_2 x_2$ and that the properties of the two media can be matched by varying the ratio of x_1 to x_2. It should be kept in mind, however, that x_1 and x_2 are the distances from the source and cannot have negative values. Equations (22) and (37) are not equivalent in general even when β_1 and β_2 are both positive.

The cancellation of dispersion is clearly due to the anticorrelation of the frequency components of the two photons, which originate from a monochromatic pump photon. One might ask whether or not such frequency correlations may have the same effect in the classical case. It is shown in the Appendix that no classical theory can reproduce these effects due to the incoherent addition of classical probabilities.

It should be noted that the initial state of Eq. (24) corresponds to a situation in which the emission time of either photon has a very large uncertainty in the quantum-mechanical sense [3]. As a result, it is somewhat meaningless to consider the effects of dispersion on a single photon of a pair. In any event, the cancellation of dispersion coefficients in Eqs. (37) and (38) applies only to the degree of coincidence between the two photons. Nonlocal effects in general are limited to joint-measurement probabilities.

IV. SUMMARY

It has been shown that the dispersion experienced by one photon of an entangled pair can be canceled by the dispersion experienced by the other in such a way that the two photons remain coincident. This is in contrast to the classical situation where the dispersion experienced by a light pulse is dependent only upon the local properties of the medium through which it is propagating.

These results are due to the coherent superposition of quantum-mechanical probability amplitudes, which have no classical analogy. For an entangled pair of photons this produces nonlocal phenomena that can only be understood by viewing the effects of dispersion on the two-photon system rather than on each photon individually.

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APPENDIX

The text considered the propagation of classical Gaussian pulses and showed that their dispersion does not cancel. The purpose of this Appendix is to show that this is a general property of any classical or semiclassical field and that the dispersion cancellation is thus an inherently quantum effect. The degree of coincidence of the counts from two photodetectors will be investigated by calculating the average squared difference σ_J^2 in detection times. A classical inequality for σ_J will be derived, and it will be shown that no classical or semiclassical theory can produce total coincidence between two beams of light propagating through two dispersive media.

In any classical or semiclassical theory the electric field at a detector located a distance x_1 from the source in medium 1 will have some specific value as a function of time, which will be denoted by $E_1(x_1,t_1)$. This is the case even if the field is unknown and stochastic in nature, in which case it may be described by a classical probability distribution. The same is true for the field $E_2(x_2,t_2)$ at a detector in medium 2, and for stochastic fields the two fields may be correlated. It will be convenient to consider the case in which both fields are nonzero only for $0<t<T$, where T is some constant. This condition can be established for any type of light source by the appropriate use of a shutter.

The probability that the two detectors will both produce an output at times t_1 and t_2 is given by the joint probability

$$P_J(t_1,t_2)=\eta' E_1^*(x_1,t_1)E_1(x_1,t_1)E_2^*(x_2,t_2)E_2(x_2,t_2) , \tag{A1}$$

where η' is a constant related to the detection probability. It can be noted immediately that P_J factors into the form

$$P_J(t_1,t_2)=P_1(t_1)P_2(t_2) , \tag{A2}$$

where P_1 and P_2 are proportional to the individual intensities. This factorization of the joint probability for any given set of fields is a general consequence of locality and corresponds to the fact that the two detectors operate independently. When averaged over a correlated probability distribution for E_1 and E_2, Eqs. (A1) and (A2) can still give correlated results of a purely causal nature, however, as is the case for local hidden-variable theories in general [18].

It will be convenient to normalize the constant η' in such a way that

$$\int_0^T P_1(t_1)dt_1 = 1 \tag{A3}$$

with a similar expression for P_2. The same results can be obtained without such a normalization if the equations that follow are divided by the integral in Eq. (A3).

Since the fields are nonzero only in the interval $0<t<T$, we can consider a large number of systems prepared in accordance with the same probability distribution and measure the ensemble average of various quantities. Alternatively, one can suppose that a real experimental system contains a shutter controlled in such a way as to produce a sequence of time intervals over which averages can be performed. The latter situation will be assumed to be the case here, since it is equivalent to what is generally measured experimentally. The averages so obtained will then correspond to the actual experimental measurements without any assumption of ergodicity or stationarity [6].

For stochastic fields there are two sources of randomness or variation in the measurements. For a given value of the fields $E_1(x_1,t_1)$ and $E_2(x_2,t_2)$ there will be some randomness associated with the detection probability of Eq. (A1). Additional randomness may result from any variations of the fields from one time interval to the next. The variation due to the detection process itself will be considered first, after which the additional randomness due to any variations in the fields will be taken into account.

For any specific value of the fields, the variation in the detection time in path 1 is given by

$$\sigma_1^2 = \int_0^T dt_1 \int_0^T dt_2 P_J(t_1,t_2)(t_1-\bar{t}_1)^2 = \int_0^T P_1(t_1)(t_1-\bar{t}_1)^2 dt_1 , \tag{A4}$$

where the average value of t_1 over the interval is given by

$$\bar{t}_1 = \int_0^T P_1(t_1) t_1 dt_1 \tag{A5}$$

with a similar expression for field 2. The average squared difference in detection times is given by

$$\sigma_J^2 = \int_0^T dt_1 \int_0^T dt_2 P_J(t_1,t_2)(t_1-t_2)^2 . \tag{A6}$$

Equation (A6) can be rewritten as

$$\sigma_J^2 = \int_0^T dt_1 \int_0^T dt_2 P_1(t_1)P_2(t_2) \times[(t_1-\bar{t}_1)-(t_2-\bar{t}_2)+(\bar{t}_1-\bar{t}_2)]^2 \tag{A7}$$

or

$$\sigma_J^2 = \int_0^T dt_1 \int_0^T dt_2 P_1(t_1)P_2(t_2) \times[\Delta t_1^2+\Delta t_2^2-2\Delta t_1\Delta t_2+2\Delta t_1(\bar{t}_1-\bar{t}_2) -2\Delta t_2(\bar{t}_1-\bar{t}_2)+(\bar{t}_1-\bar{t}_2)^2] , \tag{A8}$$

where the notation $\Delta t_1 = t_1-\bar{t}_1$ has been used. The first term gives

$$\int_0^T P_1(t_1)(t_1-\bar{t}_1)^2 dt_1 \int_0^T P_2(t_2)dt_2 = \sigma_1^2 \tag{A9}$$

with a similar expression for the second term. The third term reduces to

$$-2\int_0^T P_1(t_1)(t_1-\bar{t}_1)dt_1 \int_0^T P_2(t_2)(t_2-\bar{t}_2)dt_2 = -2(\bar{t}_1-\bar{t}_1)(\bar{t}_2-\bar{t}_2)=0 . \tag{A10}$$

The fourth and fifth terms are similarly zero, while the last term gives just $(\bar{t}_1-\bar{t}_2)^2$.

Combining these results gives

$$\sigma_J^2 = \sigma_1^2+\sigma_2^2+(\bar{t}_1-\bar{t}_2)^2 . \tag{A11}$$

The minimum value occurs when $\bar{t}_1=\bar{t}_2$ and in general

$$\sigma_J^2 \geq \sigma_1^2+\sigma_2^2 . \tag{A12}$$

The results obtained above correspond to a fixed set of fields. Any variation in the electric fields from one time interval to the next can be taken into account by averaging over the correlated probability distribution for the two fields, which will be denoted by angular brackets. In that case Eq. (A6) becomes

$$\langle \sigma_J^2 \rangle = \left\langle \int_0^T dt_1 \int_0^T dt_2 P_J(t_1,t_2)(t_1-t_2)^2 \right\rangle , \tag{A13}$$

where P_J is now a function of the fields. The results again reduce to

$$\langle \sigma_J^2 \rangle \geq \langle \sigma_1^2 \rangle + \langle \sigma_2^2 \rangle \geq \sigma_{\min,1}^2+\sigma_{\min,2}^2 , \tag{A14}$$

where $\sigma_{\min,1}$ and $\sigma_{\min,2}$ are the minimum widths that can be achieved for any given set of fields. It should be emphasized that σ_1 and σ_2, as defined by Eqs. (A1) and (A4), include only the uncertainty associated with the detection process and do not include the additional uncertainty associated with any variations in the fields.

It is apparent from Eqs. (A12) and (A14) that the effects of dispersion on σ_J cannot cancel between two classical fields. This is a consequence of the fact that there is some randomness associated with the detection process itself for which the joint probability of Eq. (A1) factors into the product of two independent probabilities. Correlations between the two fields for the stochastic case can at best ensure that $\bar{t}_1=\bar{t}_2$ for each time interval.

The results obtained above correspond to single-photon detectors with low detection efficiencies for which Eq. (A1) applies. Alternatively, one can consider the limit of high field intensities and continuous classical measurements, in which case all of the above equations still hold if they are reinterpreted as statistical moments of the joint intensity distribution.

There is one special case that needs to be considered and that is the possibility that some specific set of fields may cause σ_1 and σ_2 to be zero individually, in which case the classical results of Eqs. (A12) and (A14) would agree with the quantum prediction of zero σ_J for a trivial reason. This possibility can be avoided by considering the field at two different locations x_1 and $x'_1 \neq x_1$. It is straightforward to show that no classical field theory can cause the widths σ_1 and σ_2 to be zero at both locations simultaneously [19]. We can then consider the sum of the widths measured at these two locations and show that it obeys an equation analogous to Eq. (A14), where the corresponding (summed) values of σ_1 and σ_2 are nonzero. It follows from this that no classical or semiclassical theory can maintain total coincidence between two light beams traversing two dispersive media.

The difference between the classical and quantum results can be illustrated by considering a classical situation in which the frequency of one field is totally correlated (or anticorrelated) with the frequency of the other field. For stochastic fields there would then be various probabilities for these correlated pairs of frequencies to occur. In the quantum-optics case, the corresponding probability amplitudes are coherently added, which gives rise to the dispersion cancellation discussed in the text. But in the classical case the results must be incoherently averaged over each such pair of correlated frequencies, which is equivalent to a superposition of probability amplitudes with random phases. It is the incoherent addition of classical probabilities that prevents classically correlated frequencies from giving any dispersion cancellation.

It should be noted that σ_J, the root mean square of the difference in detection times, can be very different from the coherence time [20]. For example, in a stationary thermal field the detector counts are not coincident at all (σ_J is infinite), but the correlation time may be finite. As a result, Eqs. (A12) and (A14) are not relevant to the effects of dispersion on coherence times. Wang, Magill, and Mandel [21] have shown that all the statistical properties of a stationary thermal field are unaffected by propagation through a dispersive medium. As an example of this, if a thermal beam of light is split in two and sent through two identical dispersive media, the times at which the fluctuation maxima occur may be altered by the dispersion, but the fluctuations will still be the same in the two beams, leaving the factor-of-two peak in the coincidence curve (Hanburry Brown and Twiss effect [22]) unaltered. In the author's opinion, classical effects of that kind should not be viewed as a cancellation of dispersion, since the dispersion is clearly taking place, even though the correlation time is unaffected.

This Appendix began with the observation that classical fields have some definite value at each moment in time even though those values may be unknown and described by a classical probability distribution. That assumption (objective realism) cannot be made in the case of quantum fields, for which reason the results of this Appendix do not apply to the predictions of quantum optics. A similar situation was recently discussed with regard to two-photon interferometry [6].

To summarize the results of this Appendix, it has been shown that the degree of coincidence as measured by σ_J must satisfy the classical inequalities of Eqs. (A12) and (A14) and that no classical theory can give total coincidence between two light beams propagating through two dispersive media. The quantum-mechanical results of Eq. (38) need not satisfy this inequality and can give total coincidence in the limit of large bandwidth.

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[15] Taking ω_F to be positive in the classical calculations gives complex fields but simplifies the analysis and makes it more analogous to the quantum-mechanical case. It is straightforward to show that the same results are obtained using real classical fields.

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zero. The propagation of Gaussian packets is calculated in the text and gives a large width at $x_1' \neq x_1$, from which it follows that the width σ_1 cannot be zero simultaneously at two different locations in a dispersive medium.
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