Microwave scattering in an irregularly shaped cavity: Random-matrix analysis

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Microwave-scattering studies help to establish a link between classical chaos and the corresponding quantum dynamics. We present an analysis of recent experimental data using a theory based on random matrices that takes full account of quantal aspects and compare the results with the semiclassical approach. In our analysis purely wave-mechanical effects are seen, namely a non-Lorentzian correlation of the frequency spectrum, leading to a nonexponential decay in time.

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In a recent paper [1], Doron, Smilansky, and Frenkel studied experimentally, semiclassically, and through a detailed wave-mechanical calculation the scattering of microwaves in an irregularly shaped cavity, which was chosen so that a classical billiard with the same shape would show chaotic motion. In this way they presented the first experimental study of wave-mechanical manifestations of classical chaos in a scattering system. The relevant object to be understood, which contains the whole information on the scattering process, is the scattering matrix S as a function of the frequency ω. An important point to stress is that the Helmholtz and the Schrödinger equations are identical in their stationary forms, and that is the reason why such a setup is also suitable to provide a better understanding of "quantum-chaotic scattering."

The present Brief Report is a complement to the analysis of Ref. [1]. As to the model, we follow the random Hamiltonian approach to stochastic scattering [2], which takes full account of quantum aspects. The key features of this "quantum stochastic" theory, in comparison with the above-mentioned semiclassical one, were recently discussed in detail [3], and are, among others the following: The average correlation of S-matrix elements $C_{ij}(\delta\omega)=\langle S_{ij}(\omega)S^*_{ij}(\omega+\delta\omega)\rangle$, (where i,j label the channels and $\langle\ \rangle$ stands for the average over ω) is, in general, a *non-Lorentzian* function [4] parametrized by the average level spacing d and transmission coefficients T_i. Semiclassically, one has a *Lorentzian* $C_{ij}(\delta\omega)\propto\gamma/(\gamma-i\delta\omega)$ parametrized by the correlation length γ, which does not depend on the channel index. Note that the non-Lorentzian correlation in the frequency domain entails a nonexponential evolution in the time domain [4].

As to the physics, the main difference from Ref. [1] is the treatment of the absorption inside the cavity: In the experiment the cavity is coupled to a waveguide transmitting a single mode, or channel m. For this channel $S_{mm}(\omega)$ is measured. However, $|S_{mm}(\omega)|^2\neq 1$, which indicates absorption. Doron, Smilansky, and Frenkel [1] treated the S matrix as one dimensional, taking account of the absorption by shifting the frequency from the real axis. Then S becomes $S=S(\omega+i\alpha)$, and they found that $\alpha/\gamma=0.13$ fits the data well. This approach is in keeping with standard microwave theory, which allows the calculation of the damping parameter from the geometrical and material properties of the cavity.

Alternatively, we consider the absorption as due to a number N_p of "parasitic" channels, leading to an S matrix, which has a dimension greater than 1. The different parasitic channels may be seen as different excitations of the absorptive cavity wall. This approach is motivated by the case of neutron scattering through the compound nucleus, where absorption can also be modeled by inelastic channels [mainly of the (n,γ) type]. It should be stressed at this point that the experiment gives direct information only for the "main" channel coming in from the waveguide, for which $S_{mm}(\omega)$ is measured. For the parasitic channels only indirect information is provided.

Our results (expressed in the scaled frequency units of Ref. [1], i.e., $\omega=9.84\nu$, where ν is the measured frequency in GHz) are obtained through the following procedure. (i) We infer the necessary input parameters from the experiment. The average resonance spacing d is obtained by the Weyl formula with boundary corrections [5] calculated for the geometry of the experiment and is $d_W=0.27$. An experimental estimate of $d_{\rm expt}$ by counting the resonances in the "excitation function" S_{mm} gives $d_{\rm expt}=0.32$, which is rather an upper bound for d (because there may be resonances overlooked by the experiment, which leads to a larger d) and supports d_W as a good estimate for d. The transmission coefficient T_m of the main channel is calculated from the definition [2] $T_i=1-|\langle S_{ii}(\omega)\rangle|^2$ and its value is $T_m=0.97$, which is compatible with the value $T_m=1$ expected for billiards (no barriers). The transmission coefficients of the parasitic channels are obtained by fitting the parameters of the final formula of Ref. [2] to the experimental value $C_{mm}(0)=0.7$. Assuming the parasitic channels to have equal transmission coefficients T_p and choosing N_p, we get T_p, or rather $\sum_p^{N_p}T_p=N_pT_p$. By trying several cases, even with transmission coefficients that are different by as

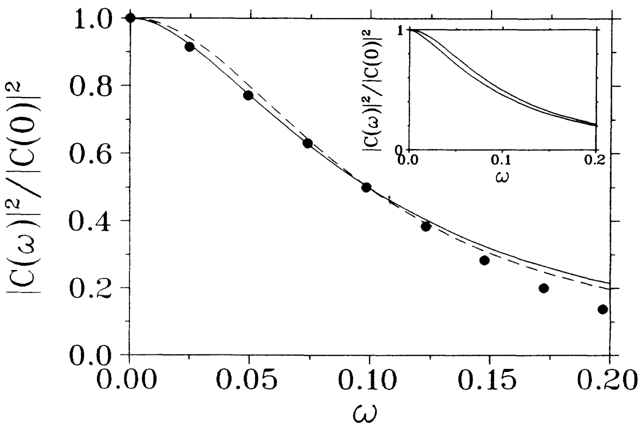

FIG. 1. Normalized S-matrix correlation function as a function of scaled frequency. Closed circles, experiment; solid line, prediction from random matrix theory for $N_p = 20$; dashed line, Lorentzian with $\gamma = 0.1$ as predicted by semiclassical theory. Inset: same as above, a comparison of random matrix predictions for $N_p = 1$ (lower curve) and $N_p = 20$ (upper curve). For details, see text.

much as an order of magnitude, we checked that both the assumption of equal transmission coefficients and the choice of N_p have a weak influence on our final results, provided that $C_{mm}(0)$ is fitted as explained above. For a representative example, this will be shown below explicitly. (ii) We use these input data to calculate within the quantal stochastic model the following observables: the squared correlation function $|C_{mm}(\delta\omega)|^2$ (see Fig. 1) and a quantity closely related to the Wigner-Smith time delay [6] in the main channel $\Delta t_{mm} = -iS^*_{mm}(\partial/\partial\omega)S_{mm}$. We restrict ourselves to the cases $N_p = 1$ and $N_p \to \infty$ (for the quantities of interest here, this is practically realized already for $N_p = 20$, because, upon increasing N_p further, there is no significant change in their numerical values). For $N_p = 1$ we found $\overline{\Delta t_{mm}} d_W = 2.4$ (with $\sum_p T_p = 0.7$, and the overbar denoting ensemble average [3]), and for $N_p \to \infty$ we found $\overline{\Delta t_{mm}} d_W = 2.0$ (with $\sum_p T_p = 0.6$). Experimentally, one has $\langle \Delta t_{mm} \rangle d_W = 2.8$. For the quantity $d\phi_{mm}/d\omega$, with $S_{mm}(\omega) = a_{mm}(\omega)\exp[i\phi_{mm}(\omega)]$, studied in [1], there is no prediction presently available from stochastic theory.

The following points are noteworthy. (i) Once $\sum_p T_p$ is fitted to (and d, T_m extracted from) the data, other experimental data such as $\langle \Delta t_{mm} \rangle$ can be satisfactorily reproduced. (ii) The fit of $\sum_p T_p$ involves $C_{mm}(0)$, i.e., the autocorrelation function at a *single* point, from which $C_{mm}(\delta\omega)$ is uniquely determined at *all* points, again according to Ref. [2]; the resulting functional shape of $|C_{mm}(\delta\omega)|^2$, thus being a specific prediction of this formalism, fits the data very well. This can be seen from Fig. 1. (iii) The dependence on N_p, qualitatively in the shape of $|C_{mm}(\delta\omega)|^2$ and quantitatively in the value of Δt_{mm} (a 25% difference between $N_p = 1$ and $N_p \to \infty$), is weak, as claimed above. (iv) In the formalism of Ref. [2], there is a natural way of describing absorptive (or inelastic) processes with the aid of parasitic channels. This may be a good approach also in the context of other random-scattering or transport systems.

With the present experimental data it is not possible to decide for the dashed or the solid curves in Fig. 1. The statistical analysis is not trivial, since for the autocorrelation function not all measured points are independent [7]. The statistical errors depend strongly on how many independent points one has in the sample, and this number is model dependent [8]. For small ω, although the stochastic theory fits the data perfectly, the Lorentzian cannot be totally discarded. For large values of ω, the statistical errors increase, and both curves are compatible with the experimental points.

By way of conclusion, we argue that the model put forward in Ref. [2] is, for the time being, among the main candidates for describing chaotic scattering of waves. More precise empirical data, both in energy and time domain, are necessary to establish a better connection between the alternative models, and microwave experiments seem to be a good tool for obtaining them.

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