

Time delay of evanescent electromagnetic waves and the analogy to particle tunneling

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The analogy between quantum tunneling of particles and evanescent electromagnetic waves can be used to study particle tunneling. Possibilities for the measurement of the time delay resulting from the transmission of waves through an evanescent region, say with lower dielectric constant, are discussed. In contrast to particle tunneling, an electromagnetic pulse can consist of many photons and can be probed in a noninvasive way. We emphasize the delay of the center of gravity of an electromagnetic pulse transmitted below cutoff through a portion of a waveguide with the same cross section as the adjacent propagating guides. The boundary conditions at the interface between the propagating and the evanescent region lead to the same transmission and reflection coefficients as for a square-barrier tunneling problem. For a pulse restricted to a narrow frequency range, the time delay depends only on the frequency derivative of the phase shift associated with transmission. The delay of a center of gravity is just that; there is no deeper physical sense which links the incoming center of gravity to the outgoing center of gravity. For sufficiently long evanescent regions, the delay is independent of thickness.

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I. INTRODUCTION

The analogy between electrons following the Schrödinger equation and electromagnetic (em) wave propagation has been a central ingredient in the development of quantum mechanics [1,2]. Furthermore, the analogy between the tunneling of electrons and the evanescent waves found in a low-dielectric-constant region separating two regions of higher dielectric constant, has been appreciated [3]. Such evanescent waves can be found in optics in frustrated internal reflection, or in microwaves in a guide with a frequency below cutoff. Control of the refractive index in the evanescent region can be used as a tool for modulating light [4].

The time elapsed while an electron tunnels through a potential barrier has been a subject of considerable theoretical attention. Starting with the very first investigation [5], many investigators followed the maximum of a wave packet as it tunnels through the barrier. Later on, the traversal time was analyzed by coupling a clock to the tunneling object, such as a spin precession in a magnetic field [6,7]. In the last decade, attention to the traversal-time problem has increased significantly. This increase stems in part from the emergence of new formulations for the problem, including the modulated barrier approach [8]. In this approach, the static potential barrier through which the particle tunnels is supplemented by a very small oscillatory term at frequency ω_m . At very low modulation frequencies, the particle does not have enough time to probe the oscillations of the barrier, which is then essentially a static barrier: it is the departure from adiabaticity as we go towards higher frequencies that reveals the traversal time. The traversal time determined by this approach is essentially the time spent by the particle *interacting* with the barrier, and this is not necessarily a delay suffered in going through a barrier. Indeed, as we shall try to make clear, the delay is delicately dependent on its exact definition, and not clearly

of physical significance. An alternative and also relatively recently developed approach for the traversal-time problem follows the *center of gravity* of a wave packet [9]. After the wave packet reaches the potential barrier, it leads to a transmitted and a reflected part, and the evolution of the center of gravity of the transmitted wave packet determines the traversal time, in a manner to be discussed later in this paper. There are a number of other approaches to the traversal time. Two of these are based, respectively, on the Feynman [10] and the Bohm [11] formulations of quantum mechanics and will not be discussed here. The reader is referred to recent reviews of the subject [12–15], but also warned that there is no clearly accepted consensus shared by these authors.

In contrast to the electronic case, where measuring the arrival time of a wave packet is a quantum-mechanically disturbing procedure, electromagnetic wave packets can be occupied by many identical photons. Capturing a few photons to make a measurement can leave most of the photons undisturbed to continue on their journey. Thus, measuring the separation in arrival time of a pulse's center of gravity, between two points, is a feasible measurement in the photon case. By contrast, wave packet delay measurements for an electron will not be easy. They are possible, perhaps, if we use a sequence of many identically launched electron wave packets and make measurements only on the side where the transmitted wave packets emerge. We stress, however, that *none* of the proposed and completed experiments, in the electron case, utilize time-delay measurements [16]. In this paper, where we analyze the electromagnetic case, we will emphasize pulse-delay measurement. Concern with the time required for electronic tunneling, as already indicated, immediately leads to equivalent questions for the evanescent em waves. Recent papers [17–19] explicitly emphasize this question. Yablonoitch has measured pulse-propagation delay through a periodic filter array, in a frequency range where propagation is blocked [20].

We will discuss some experimental possibilities for measuring a time related to evanescent em wave propagation. Electromagnetic wave measurements present several advantages over measurements on tunneling particles. Even with the current availability of femtosecond measurements, the time associated with electron tunneling through solid state barriers is believed to be rather short, and difficult to investigate. Electromagnetic waves provide us with much longer times, more accessible to experiment. In addition, electron tunneling can be complicated by electron-electron correlations effects, such as in the experiment of Gueret, Marclay, and Meier [21] on GaAs/Al_xGa_{1-x}As heterostructures. Furthermore, the electrodynamic “environment” coupled to the tunneling capacitor may affect the tunneling process [22]. Most theoretical approaches to the traversal-time problem attempt to determine the time taken for a particle to cross a static potential barrier, where these complications are neglected. It is therefore highly desirable to study an experimental system which can be described as an analog to tunneling, but where the listed effects can be disregarded. Below, we distinguish three experimental possibilities, and then focus on the time delay of an em pulse propagating through a wave guide with an evanescent region.

II. EXPERIMENTAL POSSIBILITIES

A. Amplitude-modulated incident wave

In the electronic case we can have two waves at different incident energies superimposed to give a modulation in the incident amplitude [12, 23]. This was precisely the object of a recent microwave experiment by Ranfagni *et al.*, whose wave-guide configuration is discussed briefly in Sec. III A. If the incident amplitude varies slowly compared to the interaction time of transmitted waves with the barrier, we can expect a transmitted wave which consists simply of the incident amplitude, currently applicable, multiplied by the complex transmission coefficient $|t(\omega)|e^{i\alpha(\omega)}$. Due to the frequency dependence of this latter quantity, the transmitted waves at ω and $\omega + \Delta\omega$ have slightly different amplitudes, and are shifted in their relative phase by an amount $\Delta\alpha$ compared to the incident components. Since the timing of the outgoing peaks is independent of the relative magnitude of the two components, the envelope of the transmitted signal will be shifted in time by an amount

$$\tau_\alpha = \frac{\Delta\alpha}{\Delta\omega}, \quad (1)$$

which can be identified as the time delay for amplitude modulated waves. In the limit of small $\Delta\omega$, this becomes the derivative, $d\alpha/d\omega$.

Within the evanescent region we will have a superposition of two exponentially decaying waves. They will be comparable in magnitude at the far end, where the combination of the two exponentials is needed to match to the outgoing wave. Now let us specialize to the case of an evanescent section long enough to attenuate the wave ap-

preciably (*opaque*, hereafter). Then at the incident end, one of the two exponentials will be dominant, and can be matched to the combination of incident and reflected wave on the incident side. Thus, we have a contribution to the phase change $\Delta\alpha$ from the matching at the incident interface. Within the evanescent region the dominant decaying mode will have no phase variation. There is then, an additional contribution to the phase change $\Delta\alpha$ from the matching at the outgoing end. Therefore the phase change, $\Delta\alpha$, will depend only on the matching at the interfaces to the cutoff section, and will be independent of the length of that section. Thus, the effective velocity, derived from the delay in Eq. (1), can be made as large as desired. That result, while perhaps puzzling, is not in any clear sense a paradox. Our result represents exactly that which was calculated: A delay in the appearance of a peak. As stressed in Ref. [23], a peak in the outgoing wave is not, in any direct and simple physical sense identifiable with a peak in the incoming wave. Furthermore a wave, as discussed here, with two components, has very little information, and we cannot identify each peak as an independent information carrying feature. Our effective peak crossing velocity is not, in any simple sense, a signal velocity. To give a simple but perhaps not irrelevant analogy: consider a lighthouse, sending out a rotating beam of light. The velocity of the spot projected on a ship can exceed c ; one spot is not the source of a later adjacent spot. Nevertheless, the ease with which large velocities can be found in this field, e.g., Refs. [24] and [25], is one of its poorly resolved mysteries.

B. Modulated-dielectric-constant measurements

We can follow the procedure discussed in Ref. [8] and modulate the dielectric constant in the evanescent region and search for the frequency, as we increase frequency, at which the adiabatic transmission calculation fails. This requires a physical structure that will allow application of a modulation field, say to a section of wave guide. For an effective experiment, at least one of the two frequencies (signal, modulation) should be variable. If the modulation field propagates beyond the region in which we intend to modulate the dielectric constant ϵ , it should not matter. Presumably we have taken care that only the evanescent region is filled with a field-dependent dielectric. The wave-guide side walls should not short out the modulation field. We can eliminate side walls by using outward radial propagation between two parallel slabs. Launch the wave at the inner radius into a high ϵ region, with some angular variation, to permit cutoff effects beyond a subsequent interface, encountered in the outward propagation. For example, near the inner radius of the parallel conducting planes we can launch the outgoing wave from two antennas, diametrically opposed in position, and driven 180° out of phase. Or n antennas, spaced periodically around the inner radius, and with suitable chosen relative phasing. There is an alternative way [26] to avoid shorting by the wave-guide side walls, shown in Fig. 1. Here only a portion of the dielectric in the guide is modulated. This approach does present some complications. First of all the slab has an effect on the precise

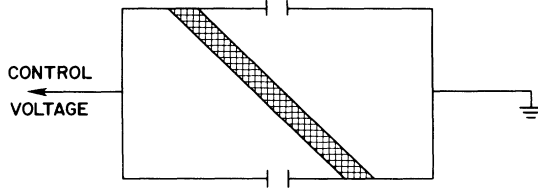


FIG. 1. Rectangular wave guide with material (cross-hatched) whose dielectric constant can be controlled through an applied voltage. The wave-guide walls have slits, located at a position where the mode involved has no current flow across the slit position. The slits are there to prevent short-circuiting of the control voltage by the guide walls.

location where the slits shown in Fig. 1 can be placed; i.e., the slab perturbs the mode. The slab also complicates the interface reflections generated at the ends of the evanescent region.

Alternatively, as suggested by Platzman [27], we can have a modulation field supplied through the same wave guide as the signal. For easy interpretation the modulation signal should be relatively uniform within the evanescent region. This can be achieved by using a fixed modulation frequency just above cutoff. That, however, will cause most of the incident modulation signal to be reflected. Attempts to introduce “matching” at the modulation signal frequency must be done carefully, lest they interfere with the behavior at the signal frequency.

C. Pulse-delay measurements

When transmitting an electromagnetic pulse through an evanescent region, the higher frequency components of the pulse will be attenuated less than the lower ones. Therefore, the emerging pulse will have a higher average frequency. This, in turn, implies a higher average group velocity in the direction perpendicular to the low-index slab. Thus in the photon case, as in the electron case, the evanescent region acts as an effective accelerator. If a time delay is measured between points sufficiently far from the evanescent region the insertion of the low dielectric slab has actually speeded up the motion of the pulse. This is the effect characterized for electron by Eqs. (3), in Ref. [28]. As in the electronic case discussed in Ref. [28], we can expect that the em-wave delay between two points, separated by an evanescent region, depends on the group velocities in the two semi-infinite regions, supplemented by a term which (in the opaque or highly attenuating case) depends only on the phase changes at the interfaces. Providing a detailed discussion of that is the primary purpose of our next section.

III. TIME DELAY OF AN em PULSE IN A WAVE GUIDE

A. Matching conditions

In this section, we will show that the matching conditions imposed on the electric and magnetic field for transverse electric (TE) modes, and for transverse magnetic

(TM) modes, at an interface between a propagating and an evanescent region are equivalent to those of the electron tunneling problem. The experiments invoked by the authors of Ref. [18] utilized an evanescent region obtained not via a change in dielectric constant, but through a section of narrower wave guide. That, of course, generates additional complicating reflections arising from the geometrical discontinuity, and does not seem to be the easiest way to supply interpretable results. While discussing Ref. [18], we also note that this work presented a comparison of pulse delay to the results for the particle tunneling case obtained from the modulated barrier analysis. It is not clear why the modulated barrier analysis should give results relatable to delay measurements.

Figure 2 describes a wave guide of uniform cross section. Between positions $z = z_1$ and $z = z_2$, hereafter labeled region II, the wave guide is filled with a material characterized by ϵ_{II} and μ_{II} , such that waves are evanescent for the frequency range of the pulse. The dielectric constant ϵ_I and the permeability constant μ_I on both sides of region II (regions I and III) are chosen so that the pulse propagates there.

In the propagating region, we look for wavelike solutions [$\exp(ikz - i\omega t)$] of Maxwell's equations for the electric field $\mathbf{E} \equiv (\mathbf{E}_t, E_z)$ and the magnetic field $\mathbf{B} \equiv (\mathbf{B}_t, B_z)$ [29]:

$$ik\mathbf{E}_t + i\frac{\omega}{c}\mathbf{e}_z \times \mathbf{B}_t = \nabla_t E_z, \quad \mathbf{e}_z \cdot (\nabla_t \times \mathbf{E}_t) = i\frac{\omega}{c}B_z, \quad (2a)$$

$$ik\mathbf{B}_t - i\mu\epsilon\frac{\omega}{c}\mathbf{e}_z \times \mathbf{E}_t = \nabla_t B_z, \quad \mathbf{e}_z \cdot (\nabla_t \times \mathbf{B}_t) = -i\mu\epsilon\frac{\omega}{c}E_z, \quad (2b)$$

$$\nabla_t \cdot \mathbf{E}_t + ikE_z = 0, \quad \nabla_t \cdot \mathbf{B}_t + ikB_z = 0. \quad (2c)$$

In the evanescent region (II), the em fields have the same form, except that k is replaced by $i\kappa$. For a wave guide with perfectly conducting walls, the boundary conditions are

$$E_z|_s = 0, \quad \frac{\partial B}{\partial n}|_s = 0, \quad (3)$$

on the surface of the wave guide. We consider the matching problem for the TE ($E_z = 0$) and the TM ($B_z = 0$) waves at the two boundaries between the propagating and the evanescent region. The displacement current D_z

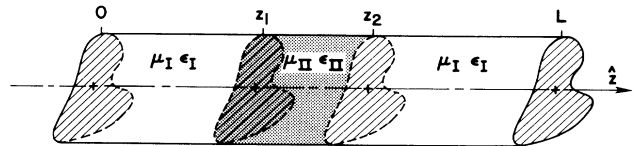


FIG. 2. Wave guide of uniform section with an evanescent region: the two propagating regions, characterized by ϵ_I, μ_I are separated by an evanescent region (ϵ_{II}, μ_{II}), located between $z = z_1$ and $z = z_2$. The position of the center of gravity of the pulse is measured at positions $z = 0$ and $z = L$.

and B_z , as well as the transverse components \mathbf{E}_t and \mathbf{H}_t , are continuous at $z = z_1$ and $z = z_2$. Equations (2a) and (2b) can be rewritten in term of these variables, and utilizing $\gamma^2 = \mu\epsilon\omega^2/c^2 - k^2$, in the form:

$$\mathbf{H}_t = \left\{ \begin{array}{l} ck/\mu\omega \\ \epsilon\omega/c k \end{array} \right\} \mathbf{e}_z \times \mathbf{E}_t \quad \left\{ \begin{array}{l} \text{TE} \\ \text{TM} \end{array} \right. \quad (4a)$$

$$\left. \begin{array}{l} \mathbf{H}_t \\ \mathbf{E}_t \end{array} \right\} = i \frac{k}{\gamma^2} \left\{ \begin{array}{l} \mu^{-1} \\ \epsilon^{-1} \end{array} \right\} \nabla_t \left\{ \begin{array}{l} B_z \\ D_z \end{array} \right\} \quad \left\{ \begin{array}{l} \text{TE} \\ \text{TM} \end{array} \right. , \quad (4b)$$

$$(\nabla_t^2 + \gamma^2) \left\{ \begin{array}{l} B_z \\ D_z \end{array} \right\} = 0 \quad \left\{ \begin{array}{l} \text{TE} \\ \text{TM} \end{array} \right. . \quad (4c)$$

With the boundary conditions of Eq. (3), Eq. (4c) describes an eigenvalue problem. Note that, in general, the eigenvalues γ_λ ($\lambda = 1, 2, \dots$) for TE and TM modes are not equal, due to the difference in boundary conditions on B_z and D_z [see Eq. (3)]. With a given solution of the boundary value problem in Eq. (4c), we can obtain \mathbf{H}_t and \mathbf{E}_t from Eqs. (4a) and (4b).

Consider the propagation of a single mode λ corresponding to the eigenvalue γ_λ (for a rectangular wave guide with side wall dimensions a and b , $\gamma_{mn} = \pi\sqrt{m^2/a^2 + n^2/b^2}$). Given a wave which is incident on the left-hand side of the barrier, the fields in region I (II) consist of a superposition of $\pm k \equiv \sqrt{\mu_I \epsilon_I \omega^2 / c^2 - \gamma_\lambda^2}$ ($\mp \kappa \equiv \sqrt{\gamma_\lambda^2 - \mu_{II} \epsilon_{II} \omega^2 / c^2}$) contributions, with respective coefficients A_I and B_I (A_{II} and B_{II}). On the right-hand side of the evanescent portion (region III), the wave has only a $+k$ component with coefficient A_{III} . The conti-

nunity of B_z (D_z) for the TE (TM) mode, at $z = z_1$ and $z = z_2$ yields the conditions:

$$A_I e^{ikz_1} + B_I e^{-ikz_1} = A_{II} e^{-\kappa z_1} + B_{II} e^{\kappa z_1} , \quad (5a)$$

$$A_{II} e^{-\kappa z_2} + B_{II} e^{\kappa z_2} = A_{III} e^{ikz_2} , \quad (5b)$$

which is equivalent to the continuity condition of the wave function for the electron tunneling problem. From Eq. (4b), the continuity of \mathbf{H}_t (\mathbf{E}_t) for the TE (TM) mode becomes

$$ik \left\{ \begin{array}{l} \mu_I^{-1} \\ \epsilon_I^{-1} \end{array} \right\} [A_I e^{ikz_1} - B_I e^{-ikz_1}] \\ = -\kappa \left\{ \begin{array}{l} \mu_{II}^{-1} \\ \epsilon_{II}^{-1} \end{array} \right\} \times [A_{II} e^{-\kappa z_1} - B_{II} e^{\kappa z_1}] \quad \left\{ \begin{array}{l} \text{TE} \\ \text{TM} \end{array} \right. \quad (6a)$$

$$-\kappa \left\{ \begin{array}{l} \mu_{II}^{-1} \\ \epsilon_{II}^{-1} \end{array} \right\} [A_{II} e^{-\kappa z_2} - B_{II} e^{\kappa z_2}] \\ = ik \left\{ \begin{array}{l} \mu_I^{-1} \\ \epsilon_I^{-1} \end{array} \right\} \times A_{III} e^{ikz_2} \quad \left\{ \begin{array}{l} \text{TE} \\ \text{TM} \end{array} \right. . \quad (6b)$$

Note that Eqs. (6a) and (6b) are equivalent to the continuity of the derivative of the wave function for an electron crossing a rectangular barrier if $\mu_I = \mu_{II}$ (TE) and $\epsilon_I = \epsilon_{II}$ (TM). The remaining boundary conditions on \mathbf{E}_t (\mathbf{H}_t) for TE (TM) waves turn out to be equivalent to Eqs. (5a) and (5b). From these conditions, the transmission and reflection coefficients associated with the evanescent region have the form:

$$t(\omega) \equiv |t(\omega)| e^{i\alpha(\omega)} = \frac{e^{-ik(z_2-z_1)}}{\cosh[\kappa(z_2-z_1)] i(\kappa'/k' - k'/\kappa') \sinh[\kappa(z_2-z_1)]/2} , \quad (7a)$$

$$r(\omega) \equiv |r(\omega)| e^{i\beta(\omega)} = \frac{-i(\kappa'/k' + k'/\kappa') \exp(2ikz_2) \sinh[\kappa(z_2-z_1)]}{2 \cosh[\kappa(z_2-z_1)] + i(\kappa'/k' - k'/\kappa') \sinh[\kappa(z_2-z_1)]} , \quad (7b)$$

where $k'/\kappa' = \mu_{II} k / \mu_I \kappa$, ($= \epsilon_{II} k / \epsilon_I \kappa$), for the TE (TM) mode.

B. Time delay

We are now in a position to calculate the time delay for an em wave packet. To specify the position of the packet along the z -direction at a given time, we follow the method of Hauge *et al.* [9, 28], relying on the fact that (i) initially the incident wave packet is located far from the barrier region; (ii) after dwelling in the evanescent region, the transmitted and reflected components of the wave packet do not overlap with this region. The location of the wave packet is taken to be the average value of z after weighting by the square of a field component. For TE or TM waves we choose to weight with $|B_z|^2(\mathbf{r}, t)$ or $|D_z|^2(\mathbf{r}, t)$, respectively, so that we can treat both modes on the same footing (for a wave packet with a narrow

frequency range, the choice of em field components, used as a weighting factor, does not matter).

For the waves to be propagating (evanescent) in regions I and III (region II) it is necessary that the central frequency ω_0 of the pulse and its spectral width satisfy:

$$\omega_0 \equiv \frac{\int_{\omega_{cI}}^{\infty} d\omega \omega |A(\omega)|^2}{\int_{\omega_{cI}}^{\infty} d\omega |A(\omega)|^2} \quad \text{such that} \quad \omega_{cI} < \omega_0 < \omega_{cII} , \quad (8a)$$

$$\Delta\omega^2 \equiv \frac{\int_{\omega_{cI}}^{\infty} d\omega (\omega - \omega_0)^2 |A(\omega)|^2}{\int_{\omega_{cI}}^{\infty} d\omega |A(\omega)|^2} \quad \text{such that} \quad \Delta\omega \ll \omega_{cII} - \omega_{cI} , \quad (8b)$$

where $A(\omega)$ is the pulse amplitude, and the cutoff fre-

quency is defined as $\omega_{cI} = c\gamma_\lambda/\sqrt{\mu_I\epsilon_I}$ (similarly for ω_{cII}). A word of caution is in order here. Unless the pulse has a sharp boundary in the frequency domain, $A(\omega)$ will have a small tail at frequencies $\omega > \omega_{cII}$. As a result, in the opaque limit [$\kappa(z_2 - z_1) \gg 1$], the transmitted wave packet will have exponentially attenuated components at frequencies below cutoff, as well as a contribution from the high-frequency tail of *propagating* waves. It is therefore important for a time-delay experiment that this latter contribution is negligible.

Choosing a pulse centered at the position $z = 0$ at $t = 0$, and requiring that the pulse is located far from the evanescent region at time $\tau = 0$ imposes the additional

constraints, expressed here only for the TE case:

$$\langle z \rangle_0 = \frac{\int_{-\infty}^{+\infty} d\mathbf{r}^3 z |B_z|^2}{\int_{-\infty}^{+\infty} d\mathbf{r}^3 |B_z|^2} = 0, \quad (9a)$$

$$\Delta z = \left(\frac{\int_{-\infty}^{+\infty} d\mathbf{r}^3 (z - \langle z \rangle)^2 |B_z|^2}{\int_{-\infty}^{+\infty} d\mathbf{r}^3 |B_z|^2} \right)^{1/2} \ll z_1. \quad (9b)$$

With a Fourier analysis similar to that of Refs. [9, 28], making the substitution $\int_{\omega_{cI}}^{\infty} d\omega A(\omega) \dots \rightarrow \int_0^{\infty} dk (d\omega/dk) A(\omega(k)) \dots$, we rewrite this as

$$\langle z \rangle_0 = \int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk)^2 |A|^2 (d\xi/d\omega) / \int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |A|^2 = 0, \quad (10a)$$

$$\Delta z = \int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |d((d\omega/dk)A)/d\omega|^2 / \int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |A|^2 \ll z_1, \quad (10b)$$

where $\xi(\omega)$ is the phase of the complex amplitude $A(\omega)$.

For short times, the position of the wave packet is given by

$$\langle z \rangle_{\tau \rightarrow 0} \simeq \tau \frac{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk)^2 |A|^2}{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |A|^2}, \quad (11)$$

which means that the wave packet propagates with a velocity $d\omega/dk$. For long times, the position of the transmitted and reflected wave packets yields:

$$\langle z \rangle_{\tau \rightarrow \infty}^T \simeq \frac{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk)^2 |tA|^2 (\tau - d\xi/d\omega - d\alpha/d\omega)}{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |tA|^2}, \quad (12a)$$

$$\langle z \rangle_{\tau \rightarrow \infty}^R \simeq \frac{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk)^2 |rA|^2 (d\xi/d\omega + d\beta/d\omega - \tau)}{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |rA|^2}, \quad (12b)$$

where α and β are the phases introduced in Eqs. (7a) and (7b).

To determine the time delay, we propagate the incident wave packet forward in time, using Eq. (11), until it reaches the beginning of the evanescent region ($z = z_1$). Similarly, we propagate the transmitted wave packet backwards in time with Eq. (12a) until it reaches the end of the evanescent region ($z = z_2$). The time delay for tunneling is then the difference between the respective times associated with these two events. The delay due to reflection is obtained straightforwardly by propagating the reflected wave packet back to the position $z = z_1$. With this procedure, we get:

$$\tau_T = \frac{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |tA|^2 (d\alpha/dk + d\xi/dk + z_2)}{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk)^2 |tA|^2} - z_1 \frac{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |A|^2}{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk)^2 |A|^2}, \quad (13a)$$

$$\tau_R = \frac{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |rA|^2 (d\beta/dk + d\xi/dk - z_1)}{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk)^2 |rA|^2} - z_1 \frac{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk) |A|^2}{\int_{\omega_{cI}}^{\infty} d\omega (d\omega/dk)^2 |A|^2}, \quad (13b)$$

which is equivalent to the result of Refs. [9, 28], except that the particle velocity $\hbar k/m$ is replaced by the group velocity. In a proposed experiment, the time taken for the pulse to cross from $z = 0$ to $z = L$ is measured, and the time of travel through the two propagating regions, $[0, z_1]$ and $[z_2, L]$ (the velocity in I and III may be different) is subtracted from the measurement to yield the delay associated with transmission, to be compared with Eqs. (13a). Similarly, the time to travel through the region $[0, z_1]$ as an incident wave packet, and through the region $[z_1, 0]$ as a reflected wave packet is subtracted.

To conclude, we consider the simple situation considered in Ref. [28], where the frequency width of the wave packet is small enough that the integrals are dominated by the contribution around ω_0 . A narrow frequency band implies that the resulting wave packet must have a large spatial extent, forcing the detection apparatus to be placed far from the evanescent region to ensure that the center of gravity of the complete wave packet is measured. As stated in Sec. IIC, the high-frequency components will have better transmission than lower frequencies, leading to an effective acceleration of the packet. Thus, if a delay is defined in terms of the *change* of the time required for propagation from $z = 0$ to $z = L$, due to the insertion of an evanescent region, the acceleration effects can yield a negative delay. If we invoke Eq. (13a) to calculate τ_T , we must make allowances for the exact form of $A(\omega)$. Otherwise, we will calculate the propagation time, from $z = z_2$ to $z = L$ incorrectly. Note that if we reduce $\Delta\omega$, we reduce the spread of velocities which give rise to the acceleration effect. At the same

time, however, we lengthen the wave packet, and therefore the minimal allowed distance between $z = z_2$ and $z = L$. This, in turn, emphasizes the effect of the velocity variation, within $\Delta\omega$, and we have not gained in simplicity by reducing $\Delta\omega$. Nevertheless, if we follow the literature, and make the unwarranted assumption that for small enough $\Delta\omega$ the integrands in Eq. (13a) can be taken to be independent of ω , we do get a simpler answer. We also make the additional assumption that $\epsilon_I = \epsilon_{II}$ ($\mu_I = \mu_{II}$) for TE (TM) waves. From Eqs. (7a) and (7b), we can extract the phase of the transmission and reflection coefficients:

$$\alpha = -k(z_2 - z_1) - \tan \left[\left(\frac{\kappa}{2k} - \frac{k}{2\kappa} \right) \tanh[\kappa(z_2 - z_1)] \right], \quad (14a)$$

$$\beta = 2kz_2 - \tan \left[\left(\frac{\kappa}{2k} - \frac{k}{2\kappa} \right) \tanh[\kappa(z_2 - z_1)] \right] - \frac{\pi}{2}. \quad (14b)$$

As a result, in the opaque limit [$\kappa(z_2 - z_1) \gg 1$], the time delay for transmission and reflection depend only on the derivative of the corresponding phase shifts:

$$\begin{aligned} \tau_T &= \frac{d\alpha}{d\omega} + \frac{z_2 - z_1}{d\omega/dk} \\ &= \frac{1}{2} \left[1 + \tan^2 \left(\frac{\kappa}{2k} - \frac{k}{2\kappa} \right) \right] \\ &\quad \times \left[\left(\frac{\kappa}{k^2} + \frac{1}{\kappa} \right) \frac{d\kappa}{d\omega} - \left(\frac{1}{k} + \frac{k}{\kappa^2} \right) \frac{dk}{d\omega} \right], \end{aligned} \quad (15a)$$

$$\tau_R = \frac{d\beta}{d\omega} - \frac{2z_1}{d\omega/dk} = \tau_T, \quad (15b)$$

where all quantities are evaluated at $\omega = \omega_0$. In this particular case, the time delays reduce to the “energy derivative” of the phase shifts associated with the transmission and reflection process. In this case, which corresponds to quantum tunneling through a square barrier, τ_T and τ_R are identical. Note that τ_T is independent of the width of the evanescent region. Translated to an effective velocity for crossing the evanescent region, this time can correspond to arbitrary large velocities in the opaque limit. Similar problems were discussed in Sec. II A. In the present case, in contrast to that in Sec. II A, we are not really measuring an actual delay between z_1 and z_2 , but one between 0 and L , extrapolated back to z_1 and z_2 . It is therefore, perhaps, a weaker surprise.

IV. SUMMARY AND CONCLUSION

Several possibilities for the characterization of times for em waves passing through evanescent regions have been discussed. These include the delay of peaks of amplitude modulated waves, the dependence of the propagation on the frequency of an imposed modulation, and pulse-delay measurements. For this last category, we have demonstrated that one-dimensional particle tunneling is in direct analogy with em waves in a wave guide of arbitrary, but uniform, cross section, including a section of guide below the cutoff frequency. The proposed experiment requires the preparation of an em pulse with a tunable frequency, to be “fitted” between the two characteristic cutoff frequencies associated with the two materials filling the wave guide. The time delay essentially depends on the frequency derivative of the phase shift associated with the transmission process, weighted by $A(\omega)$. We find that an evanescent region can also be used to accentuate the high-frequency component of the pulse. In this sense, the evanescent region acts like an effective accelerator.

We emphasize that the delay of the center of gravity is exactly that, and no more. There is no deeper physical sense in which the incoming center of gravity is mapped into the outgoing center of gravity. To stress this we provide the following example. Consider a wave packet which has travelled an appreciable distance in region I of Fig. 2. The higher-frequency components move faster, and will therefore dominate in the front end of the wave packet. It is also these higher-frequency components which will be transmitted more effectively through the evanescent region (II). Thus the transmitted packet will come preferentially from the front end of the incident packet. The center of gravity of this transmitted packet, therefore, may have little or no causative relation to the center of gravity of the incident packet. The transmitted peak can, in fact, emerge before the transmitted peak has arrived. The analysis of experiments which follow pulses [18] must allow for this sort of effect, e.g., by invoking the complex value of $A(\omega)$.

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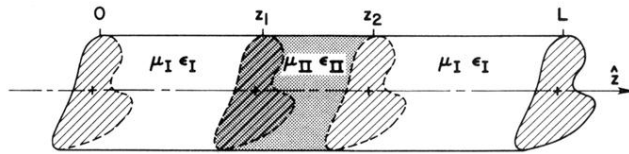


FIG. 2. Wave guide of uniform section with an evanescent region: the two propagating regions, characterized by ϵ_I , μ_I are separated by an evanescent region (ϵ_{II} , μ_{II}), located between $z = z_1$ and $z = z_2$. The position of the center of gravity of the pulse is measured at positions $z = 0$ and $z = L$.