Preparation of a pure atomic state

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We study the resonant interaction of a two-level atom with one mode of the electromagnetic field. We calculate the entropy associated with the atomic state evolving from arbitrary initial conditions. We show that pure atomic states can be generated from an arbitrary initial mixed atomic state. Also, we observe and discuss a significant difference in the long-time behavior of the entropy for an initial number state and a coherent state.

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In a recent Letter [1], the possibility of generating atomic pure states from an atom interacting with one mode of the field in a lossless cavity was reported. An initial pure atomic state, after the "collapse" time, evolved into another pure state, independent of the initial one, with the field being initially in a coherent state.

The "purity" of the state was determined by observing the time evolution of $\mathrm{tr}(\rho_{\mathrm{atom}}^2)$. Another way of looking at this problem [2] is via the entropy $-\mathrm{tr}(\rho_{\mathrm{atom}}\ln\rho_{\mathrm{atom}})$. In the present Brief Report, we ask ourselves two questions.

(a) Is it possible to generate pure atomic states, starting from a mixed atom state?

Before going into our second question, we must point out that we assume in the present Brief Report (as well as in Ref. [1]) that the field is initially in a pure state. At the present time we know that pure states of the electromagnetic field can be generated in lossless cavities, in theory. As examples, we can mention number states and cotangent states, known to be stationary states of a lossless one-photon micromaser, with the atoms initially in the upper state [3] or in a coherent superposition [4] of the two atomic levels, respectively. Also one can get the even states as stationary states of a two-photon lossless micromaser [5]. Now we may ask the next question.

(b) Is the long-time behavior of the entropy very different if our initial state of the field is a coherent or a number state?

In order to answer these questions, we start with the Hamiltonian of a two-level atom interacting with a one field mode, that is,

$$H=\hbar\omega a^{\dagger}a+\sum_{i=a,b}\hbar\omega_i|i\rangle\langle i|+\hbar g(a|a\rangle\langle b|+|b\rangle\langle a|a^{\dagger}) , \tag{1}$$

where a and $a^{\dagger}$ are the usual field operators and $|i\rangle\langle i|$ and $|i\rangle\langle j|$ are the populations and polarization atomic operators. Let us assume, to keep our model simple, that the cavity is tuned to the atomic transition, that is, $\omega=\omega_a-\omega_b$, where a and b are the upper and lower levels, respectively. The total density matrix for the coupled system at time t is defined as

$$\rho_{\mathrm{atom}\otimes\mathrm{field}}(t)=U(t)\rho_{\mathrm{atom}\otimes\mathrm{field}}(0)U^{\dagger}(t) . \tag{2}$$

The operator $U(t)$ is the well-known temporal evolution operator for the Jaynes-Cummings model, given by

$$U(t)=\begin{bmatrix}\mathcal{C} & -i\mathcal{S}\\ -i\mathcal{S}^{\dagger} & \overline{\mathcal{C}}\end{bmatrix} , \tag{3}$$

with

$$\mathcal{C}=\cos(gt\sqrt{aa^{\dagger}}),\quad \overline{\mathcal{C}}=\cos(gt\sqrt{a^{\dagger}a}) ,$$
$$\mathcal{S}=\frac{\sin(gt\sqrt{aa^{\dagger}})}{\sqrt{aa^{\dagger}}}a . \tag{4}$$

If initially the atom and field are decoupled, we can write $\rho_{\mathrm{atom}\otimes\mathrm{field}}(0)=\rho_{\mathrm{atom}}(0)\otimes\rho_{\mathrm{field}}(0)$. The initial atomic state is represented by the initial atomic density matrix:

$$\rho_{\mathrm{atom}}(0)=\begin{bmatrix}\rho_{aa} & \rho_{ab}\\ \rho_{ba} & \rho_{bb}\end{bmatrix} . \tag{5}$$

If we trace Eq. (2) with respect to the field variables, we get

$$\rho_{\mathrm{atom}}(t)=\mathrm{tr}_{\mathrm{field}}[U(t)\rho_{\mathrm{atom}}(0)\otimes_{\mathrm{field}}(0)U^{\dagger}(t)] , \tag{6}$$

which we can write as

$$\rho_{\mathrm{atom}}(t)=\begin{bmatrix}\alpha(t) & \gamma(t)\\ \gamma^{*}(t) & \beta(t)\end{bmatrix} , \tag{7}$$

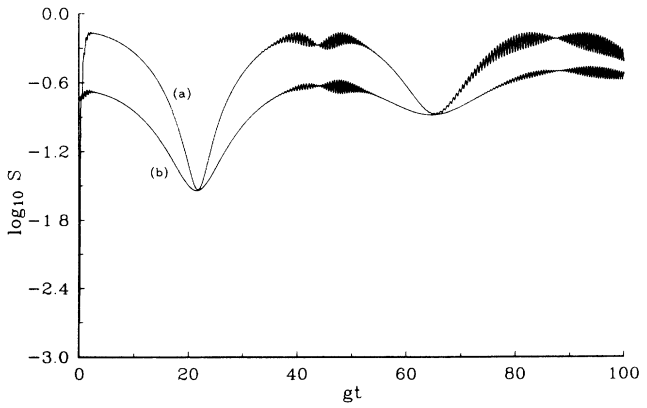

FIG. 1. Entropy vs gt for the atom initially in (a) a pure state $\rho_{a,a}=\rho_{a,b}=0$ and (b) a mixed state $\rho_{a,a}=0.6$ and $\rho_{a,b}=0.2$ and the field initially in a coherent state with $|z|^2=49$.

where the following conditions are satisfied:

$$\alpha(t)+\beta(t)=1, \quad |\gamma(t)|^2 \leq \alpha(t)\beta(t) . \tag{8}$$

The entropy of a quantum state is defined as

$$S=-\mathrm{tr}(\rho \ln \rho) . \tag{9}$$

Since the trace is invariant under a similarity transformation, we can go to a basis in which the atomic density matrix is diagonal, and write Eq. (9) as

$$S=-\sum_{j=+,-} \lambda_j \ln \lambda_j . \tag{10}$$

In the case of $\rho_{\mathrm{atom}}(t)$ defined in (7), the λ's are given by

$$\lambda_{\pm}=\tfrac{1}{2}\{1 \pm \sqrt{1-4[\alpha(t)\beta(t)-|\gamma(t)|^2]}\} . \tag{11}$$

We can write, on the other hand, the expressions for the field density matrix. A straightforward calculation shows that

$$\begin{aligned}\rho_{\mathrm{field}}(t)=&\rho_{aa}[\mathcal{C}\rho(0)\mathcal{C}+\mathcal{S}^{\dagger}\rho(0)\mathcal{S}] \\ &+\rho_{bb}[\overline{\mathcal{C}}\rho(0)\overline{\mathcal{C}}+\mathcal{S}\rho(0)\mathcal{S}^{\dagger}] \\ &+i\rho_{ab}[\mathcal{C}\rho(0)\mathcal{S}^{\dagger}-\mathcal{S}^{\dagger}\rho(0)\overline{\mathcal{C}}] \\ &+i\rho_{ba}[\overline{\mathcal{C}}\rho(0)\mathcal{S}-\mathcal{S}\rho(0)\mathcal{C}] .\end{aligned} \tag{12}$$

If the initial field condition is characterized by

$$\rho(0)=\sum_{n,m} \rho_{nm}(0)|n\rangle\langle m| , \tag{13}$$

then

$$\begin{aligned}\rho_{nm}(t)=&\rho_{aa}[a_{n+1m+1}\rho_{nm}(0)+b_{nm}\rho_{n-1m-1}(0)]+\rho_{bb}[a_{nm}\rho_{nm}(0)+b_{n+1m+1}\rho_{n+1m+1}(0)] \\ &+i\rho_{ab}[c_{n+1m+1}\rho_{nm+1}(0)-c_{mn}\rho_{n-1m}(0)]+i\rho_{ba}[c_{nm}\rho_{nm-1}(0)-c_{m+1n+1}\rho_{n+1m}(0)] ,\end{aligned} \tag{14}$$

where

$$\begin{aligned}a_{nm}&=\cos(\phi\sqrt{n})\cos(\phi\sqrt{m}) , \\ b_{nm}&=\sin(\phi\sqrt{n})\sin(\phi\sqrt{m}) , \\ c_{nm}&=\cos(\phi\sqrt{n})\sin(\phi\sqrt{m}) ,\end{aligned} \tag{15}$$

with $\phi=gt$.

In a similar way, we can evaluate the general expressions for the coefficients of the atomic density matrix:

$$\begin{aligned}\alpha(t)=&\sum_n [\rho_{aa}\cos^2(\phi\sqrt{n+1})+\rho_{bb}\sin^2(\phi\sqrt{n})]\rho_{nn}(0) \\ &+\left[i\rho_{ab}\sum_n \cos(\phi\sqrt{n+1})\sin(\phi\sqrt{n+1})\rho_{nn+1}(0)+\text{c.c.}\right] ,\end{aligned} \tag{16}$$

$$\begin{aligned}\gamma(t)=&\sum_n [i\rho_{aa}\cos(\phi\sqrt{n+1})\sin(\phi\sqrt{n})\rho_{nn-1}(0)-i\rho_{bb}\sin(\phi\sqrt{n+1})\cos(\phi\sqrt{n})]\rho_{n+1n}(0) \\ &+\rho_{ab}\sum_n \cos(\phi\sqrt{n+1})\cos(\phi\sqrt{n})\rho_{nn}(0)+\rho_{ba}\sum_n \sin(\phi\sqrt{n+1})\sin(\phi\sqrt{n})\rho_{n+1n-1}(0) .\end{aligned} \tag{17}$$

With the above results, we can now proceed to calculate the behavior of the entropy for various initial pure states of the field.

In Fig. 1, we have plotted the entropy versus gt for an initial pure atomic state (a) $\rho_{a,a}=\rho_{a,b}=0$ and a mixed state (b) $\rho_{a,a}=0.6, \rho_{a,b}=0.2$ when the field is initially in a coherent state with $|z|^2=49$. By comparing the minima and maxima of these curves, we see that their behavior is the same, independent of the initial preparation of the atom. Also, the first minimum is the deeper one, thus generating an approximately pure atomic state, which agrees with Ref. [1] [curve 1(a)], but also shows that one can generate an approximately pure atomic state, starting from a mixed atomic state [curve 1(b)], thus answering the first question mentioned previously.

However, we can generate "better" pure atomic states (with lower entropy) by using pure $|N\rangle$ or Fock states. This is shown in Fig. 2, where we have plotted the en-

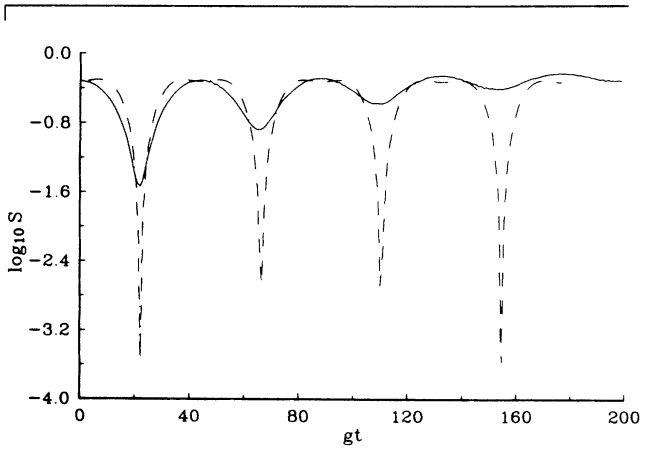

FIG. 2. Entropy vs gt for initial mixed atomic state $\rho_{a,a}=0.5$ and $\rho_{a,b}=0.1$ when the field is initially in a number state $|N=49\rangle$ (dashed line) and a coherent state $|z|^2=49$ (solid line).

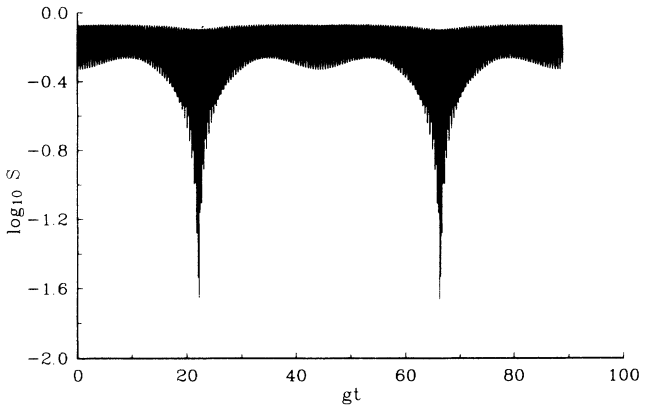

FIG. 3. Entropy vs gt for initial mixed atomic state $\rho_{a,a}=0.6$ and $\rho_{a,b}=0.2$ when the field is initially in a number state $|N=49\rangle$.

velope curve for the entropy versus gt, for the case of an initial coherent state (a) and an initial $|N\rangle$ state (b), both with $\langle N\rangle=49$. We immediately notice that we can generate atomic states with the entropy much closer to zero when we start from an $|N\rangle$ state for the field. Also, there is a fundamental difference in the long-time behavior. Curve (a) goes to $\log_{10}2$, which corresponds to the situation of the maximum entropy $\rho_{a,a}=\rho_{b,b}=\frac{1}{2}$. On the other hand, curve (b) (initial $|N\rangle$ state) is periodic in time, the period being proportional to $\sqrt{N+1}$. Since, at the beginning, we made the rotating-wave approximation in our model, we can understand this dramatic difference in behavior if we think that for an initial $|N\rangle$ state, for later times, the only allowed states are $|N+1\rangle$ and $|N-1\rangle$ depending on the initial preparation of the atom. On the other hand, when the field is initially in a coherent state, during its time evolution, the system is allowed to go through any $|N\rangle$ state, thus the number of available states in this last case is much larger, and the system is more "thermodynamic," tending to maximize its entropy. Finally, in Fig. 3 we show the same as in Fig. 2, but for a different atomic mixture, showing that the initial atomic preparation has no effect on the time dependence of the entropy.

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