PHYSICAL REVIEW A VOLUME 45, NUMBER 3 1 FEBRUARY 1992

Higher-order squeezing and photon statistics for squeezed thermal states

Paulina Marian
Laboratory of Physics, Department of Chemistry, University of Bucharest, Boulevard Carol I 13, Bucharest, R-70346, Romania
(Received 25 June 1991)

By use of the squeezed-number-state basis, we study several features of the squeezed thermal states. Higher-order squeezing properties are found to be entirely determined by the second-order ones. The photon-number distribution is expressed in closed form in terms of a Legendre polynomial. We point out that at a critical value of the squeeze parameter, pairwise oscillations of the photon-number distribution set in. Finally, a compact analytical expression of the exact lth-order correlation function is derived and examined.

PACS number(s): 42.50.Dv

I. INTRODUCTION

In recent years squeezed coherent states (SCS's) have been studied extensively [1]. The experimental success in generating SCS's at optical frequencies [2] spurs new attempts to obtain squeezed-state sources. Along this line, squeezing of thermal radiation has already been produced in a microwave Josephson-junction parametric amplifier [3]. However, there are few works investigating the properties of the squeezed thermal states (STS's) [4–9]. In their paper [4], Kim, de Oliveira, and Knight calculated the second-order correlation function, the characteristic and quasiprobability functions for both squeezed number states (SNS's) and STS's. They derived the photon-number distribution for a SNS and explained its large-scale macroscopic oscillations in terms of phase-space interference. Higher-order squeezing for SNS's and STS's was examined by Gong and Aravind [5].

In a Letter [6] the STS's were introduced within the formalism of thermofield dynamics to extend the concept of squeezed states to finite temperatures. Thermofield states have been applied to thermodynamic problems involving the amplification and attenuation of light [7].

In a very recent paper [10] studying higher-order squeezing properties and correlation functions for SNS's, we have derived analytical compact formulas for both moments and normally ordered moments of the quadrature operators. We have also shown that the photon-number distribution for a SNS is proportional to the square of a Gauss hypergeometric function.

The present work deals with the photon statistics and squeezing properties of STS's. Systematic use is made of the results for SNS's reported in our previous work [10], hereafter referred to as I. In Sec. II the expectation values of the field operators in STS's are found as well as the higher-order moments of the quadrature operators. The conditions for higher-order and intrinsic higher-order squeezing are derived and compared to the similar ones for SCS's [11] and SNS's [5,10]. The photon-number distribution is obtained in closed form in Sec. III. We find here that the squeeze parameter has a critical value for the onset of pairwise oscillations of the distribution. We also examine the behavior of the photon-number distribution for both weak and strong input chaotic fields. Section IV is devoted to the calculation and discussion of the lth-order correlation function. In Sec. V we summarize the results. Appendix A mentions two useful series of Gauss hypergeometric functions, while in Appendix B the sum of a nontrivial series involving Legendre polynomials is evaluated.

II. HIGHER-ORDER SQUEEZING

A STS is a mixed state whose density operator is a Bose-Einstein weighted sum of SNS projection operators

$$\rho_{\rm ST}=\sum_{m=0}^{\infty} W_m^T |m\rangle_{s\,s}\langle m| \ . \tag{1}$$

In Eq. (1), W_m^T is the photon-number distribution for the input chaotic field,

$$W_m^T=\frac{\bar{n}^{\,m}}{(\bar{n}+1)^{m+1}} \ , \tag{2}$$

with $\bar{n}$ the mean occupancy. The vector $|m\rangle_s$ describing a SNS arises from the action of the squeeze operator,

$$S(z)=\exp[\tfrac{1}{2}z(a^\dagger)^2-\tfrac{1}{2}z^*a^2] \tag{3}$$

on a Fock state $|m\rangle$

$$|m\rangle_s=S(z)|m\rangle \ . \tag{4}$$

In Eq. (3), $z=r\exp(i\theta)$, where $r=|z|$ is the squeeze parameter.

Note that the STS defined by Eq. (1) is the particular case $\gamma=0$ of the displaced squeezed thermal state (DSTS) described by the density operator

$$\rho_{\rm DST}=D(\gamma)\rho_{\rm ST}D^\dagger(\gamma) \ , \tag{5}$$

where

$$D(\gamma)=\exp(\gamma a^\dagger-\gamma^* a) \tag{6}$$

is a Weyl displacement operator. The normally ordered characteristic functions of the states (1) and (5) are derived in Refs. [4] and [9], respectively.

Since the squeeze operator is unitary, the set of vectors

$\{|m\rangle_s\}$ is orthonormal and complete just like the Fock states system $\{|m\rangle\}$. Consequently, the expectation value of an arbitrary operator A in a STS may be obtained evaluating the proper trace in the basis $\{|m\rangle_s\}$:

$$\langle A\rangle_{\rm ST}={\rm Tr}(\rho_{\rm ST}A)=\sum_{m=0}^{\infty}W_m^T\langle A\rangle_{\rm SN} . \quad (7)$$

Here, as well as in the following, $\langle A\rangle_{\rm SN}$ is the expectation value of the operator A in the SNS $|m\rangle_s$. It is suitable to choose Eq. (7) as the starting point of our calculations because it enables us to make use of the corresponding results for SNS's reported in I.

The lth-order moment of the creation operator $a^\dagger$ in a SNS is from I,

$$\langle (a^\dagger)^l\rangle_{\rm SN}=\begin{cases}(l-1)!!\exp\left[-i\dfrac{l\theta}{2}\right](\sinh r\cosh r)^{l/2}\,{}_2F_1\left[-\dfrac{l}{2},-m;1;2\right] & (l \text{ even}) ,\\ 0 \quad (l \text{ odd}) ,\end{cases} \quad (8)$$

where ${}_2F_1$ is a Gauss hypergeometric function. Inserting the expectation value (8) in Eq. (7) and taking into account Eq. (A1), we get

$$\langle (a^\dagger)^l\rangle_{\rm ST}=\begin{cases}(l-1)!!\exp\left[-i\dfrac{l\theta}{2}\right][(2\bar n+1)\sinh r\cosh r]^{l/2} & (l \text{ even}) ,\\ 0 \quad (l \text{ odd}) .\end{cases} \quad (9)$$

The particular case $l=2$ was obtained in Refs. [4] and [7]. Equation (9) has a very simple structure:

$$\langle (a^\dagger)^l\rangle_{\rm ST}=(l-1)!![\langle (a^\dagger)^2\rangle_{\rm ST}]^{l/2} . \quad (10)$$

In order to investigate higher-order squeezing we need to evaluate the Nth-order moment of the quadrature operator X_1 defined as

$$X_1=a+a^\dagger . \quad (11)$$

In Eq. (7) we introduce the following result of I:

$$\langle (\Delta X_1)^N\rangle_{\rm SN}=(N-1)!!|\alpha|^N\,{}_2F_1\left[-\frac{N}{2},-m;1;2\right] , \quad (12)$$

with

$$\alpha=\cosh r+\exp(i\theta)\sinh r . \quad (13)$$

By application of Eq. (A1) we find

$$\langle (\Delta X_1)^N\rangle_{\rm ST}=(N-1)!![(2\bar n+1)|\alpha|^2]^{N/2} , \quad (14)$$

and thus

$$\langle (\Delta X_1)^N\rangle_{\rm ST}=(N-1)!![\langle (\Delta X_1)^2\rangle_{ST}]^{N/2} . \quad (15)$$

The field is squeezed to any even order N in the quadrature X_1 if the Nth-order moment $\langle (\Delta X_1)^N\rangle$ is less than it is for a coherent state [11],

$$\langle (\Delta X_1)^N\rangle<(N-1)!! . \quad (16)$$

For the phase choice $\theta=\pi$ we find from the condition (16), when substituting Eq. (14), that squeezing to all orders sets in at the value

$$r_s=\tfrac{1}{2}\ln(2\bar n+1) \quad (17)$$

of the squeeze parameter. This value was first obtained by Fearn and Collett for second-order squeezing [7] and then by Gong and Aravind for Nth-order squeezing [5]. The value (17) does not depend on the order N of squeezing as a consequence of Eq. (15), which does not hold for a SNS.

It is worth mentioning that in their paper [4], Kim, de Oliveira, and Knight have obtained the P representation for a STS. They found a classical behavior of the STS for $r<r_s$, where a well-behaved P representation exists. The STS becomes nonclassical at $r=r_s$, where squeezing sets in.

To provide a fuller description of squeezing, we need to evaluate the normally ordered moments $\langle :(\Delta X_1)^N:\rangle$. As these quantities vanish for a coherent state, the condition for *intrinsic* higher-order squeezing is, according to Hong and Mandel [11],

$$\langle :(\Delta X_1)^N:\rangle<0 . \quad (18)$$

We recall from I the normally ordered moments for a SNS:

$$\langle :(\Delta X_1)^N:\rangle_{\rm SN}=(-1)^{N/2}(N-1)!!(1-|\alpha|^2)^{N/2}\times{}_2F_1\left[-\frac{N}{2},-m;1;2\right] . \quad (19)$$

Now, we insert Eq. (19) in Eq. (7) and use again the summation formula (A1) to obtain

$$\langle :(\Delta X_1)^N:\rangle_{\rm ST}=(-1)^{N/2}(N-1)!![1-(2\bar n+1)|\alpha|^2]^{N/2} , \quad (20)$$

and consequently

$$\langle :(\Delta X_1)^N:\rangle_{\rm ST}=(N-1)!!(\langle :(\Delta X_1)^2:\rangle)^{N/2} . \quad (21)$$

According to Eq. (20) the normally ordered moments in a STS are monotonic functions or r in contrast with the oscillatory character shown by the functions (19) for a SNS. Moreover, in the case $\theta=\pi$, when $\alpha=e^{-r}$, the squeezing is intrinsically of higher order for $r>r_s$ and odd values of $N/2$. Finally one should notice the identical structure of the higher-order moments (10), (15), and (21) as functions of the second-order ones.

III. PHOTON-NUMBER DISTRIBUTION

The photon-number distribution for the SNS $|m\rangle_s$ is written in I as

$$W_n^{\rm SN}(m)=\begin{cases}\dfrac{n!m!}{\left[\dfrac{n}{2}\right]!^2\left[\dfrac{m}{2}\right]!^2}\dfrac{1}{\cosh r}(\tfrac{1}{2}\tanh r)^{n+m}\left[{}_2F_1\left[-\dfrac{n}{2},-\dfrac{m}{2};\dfrac{1}{2};-(\sinh r)^{-2}\right]\right]^2 & (m,n \text{ even}),\\ \dfrac{n!m!}{\left[\dfrac{n-1}{2}\right]!^2\left[\dfrac{m-1}{2}\right]!^2}\dfrac{1}{(\cosh r)^3}(\tfrac{1}{2}\tanh r)^{n+m-2}\left[{}_2F_1\left[-\dfrac{n-1}{2},-\dfrac{m-1}{2};\dfrac{3}{2};-(\sinh r)^{-2}\right]\right]^2 & (m,n \text{ odd}),\\ 0 \text{ otherwise}. \end{cases} \tag{22}$$

The two-photon nature of the squeeze operator (3) accounts for the pairwise oscillations of the distribution (22). In Ref. [4] also discussed are the large-scale macroscopic oscillations of $W_n^{\rm SN}(m)$ analogous to those found for a SCS [12,13]. They are explained in the framework of the semiclassical theory by which the interfering probability amplitudes are interfering areas in phase space [4,12].

The photon-number distribution for a STS is obtained on account of Eq. (1) as

$$W_n^{\rm ST}=\langle n|\rho_{\rm ST}|n\rangle=\sum_{m=0}^{\infty}W_m^T W_n^{\rm SN}(m), \tag{23}$$

with W_m^T defined by Eq. (2). Now, by substituting Eq. (22) into Eq. (23) and using the series (A2), we get

$$W_n^{\rm ST}=\frac{(2n-1)!!}{n!}\left[1+\frac{2\bar{n}+1}{(\bar{n}+1)^2}(\sinh r)^2\right]^{-n-(1/2)}W_n^T \times {}_2F_1\left[-\frac{n}{2},-\frac{n-1}{2};-n+\frac{1}{2};v^2\right], \tag{24}$$

with

$$v\equiv\left[1-\left[\frac{\sinh(2r)}{\sinh(2r_s)}\right]^2\right]^{1/2} \quad [\mathrm{Im}(v)\ge 0], \tag{25}$$

where r_s is the critical value (17) of the squeeze parameter. The distribution (24) can be also expressed in terms of a Legendre polynomial, $P_n(1/v)$ [14],

$$W_n^{\rm ST}=\left[1+\frac{2\bar{n}+1}{(\bar{n}+1)^2}(\sinh r)^2\right]^{-n-(1/2)}W_n^T v^n P_n\left[\frac{1}{v}\right]. \tag{26}$$

Some particular cases are readily obtained from Eqs. (24) and (26):

$$W_n^{\rm ST}|_{r=0}=W_n^T, \tag{27}$$

$$W_n^{\rm ST}|_{r=r_s}=\frac{(2n-1)!!}{n!}\left[1+\frac{\bar{n}^2}{(\bar{n}+1)^2}\right]^{-n-(1/2)}W_n^T. \tag{28}$$

Note that the limit $\bar{n}=0$ of STS is the corresponding squeezed vacuum state (SVS). By specializing Eqs. (24)–(26) to $\bar{n}=0$ and using the particular value $P_n(0)$ of a Legendre polynomial [15], we recover the photon-number distribution for the SVS [16]:

$$W_n^{\rm SV}=\begin{cases}\dfrac{n!}{\left[\dfrac{n}{2}\right]!^2}\dfrac{1}{\cosh r}(\tfrac{1}{2}\tanh r)^n & (n \text{ even}),\\ 0 & (n \text{ odd}). \end{cases} \tag{29}$$

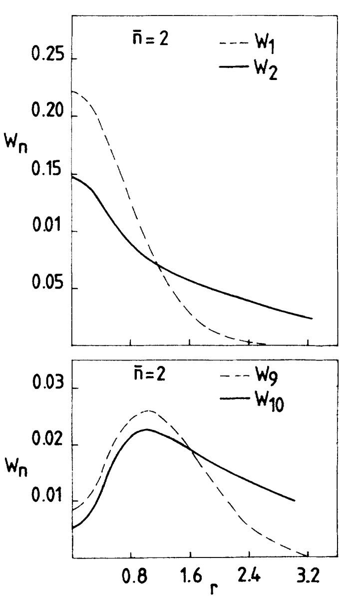

FIG.1. Plot of the functions W_1 (— — —), W_2 (——), and W_9 (— — —), W_{10} (——) vs the squeeze parameter for input mean photon number $\bar{n}=2$.

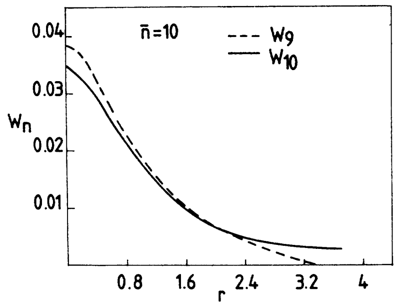

FIG. 2. Plot of the functions W_9 (— — —) and W_{10} (——) vs the squeeze parameter for $\bar{n}=10$.

This function displays typical pairwise oscillations for all values of the squeeze parameter r. This is not the case for a genuine STS ($\bar{n}>0$). A plot of the particular functions $W_1^{\rm ST}, W_2^{\rm ST}$ and $W_9^{\rm ST}, W_{10}^{\rm ST}$ versus the squeeze parameter r (Fig. 1, for $\bar{n}=2$) is relevant for the character of the photon-number distribution. As can be also seen from Fig. 2 ($W_9^{\rm ST}$ and $W_{10}^{\rm ST}$ versus r for $\bar{n}=10$), from a certain value of the squeeze parameter, the following inequality holds:

$$W_{2n}^{\rm ST}>W_{2n-1}^{\rm ST} \; . \tag{30}$$

The minimum value of the squeeze parameter, r_0, for which the photon-number distribution shows at least *one* oscillation is obtained by specializing the inequality (30) to $n=1$. Taking into account Eq. (24) we find

$$(\sinh r_0)^2=\frac{1}{2\bar{n}+1}\{\bar{n}^2-\tfrac{1}{2}+[(\bar{n}^2-\tfrac{1}{2})^2+2\bar{n}(\bar{n}+1)^2]^{1/2}\} \; . \tag{31}$$

From Eqs. (17) and (31) it is easy to show that, irrespective of $\bar{n}$,

$$r_0\geq r_s \; . \tag{32}$$

Pairwise oscillations do not occur in the absence of squeezing. It is also clear from Eqs. (17) and (31) that both r_0 and r_s increase when $\bar{n}$ increases. As expected, for the SVS we get $r_0=r_s=0$. In Table I we list the values of r_s, Eq. (17), and r_0, Eq. (31), for several input mean occupancies. Figures 3 and 4 present the photon-number distribution plotted for several values of the squeeze parameter. A closer inspection of the photon-number distribution is made in Fig. 5, where the values of the squeeze parameter are chosen slightly larger than r_0 in order to show the onset of the pairwise oscillations.

The contributions from SHS's with large m in Eq. (23)

TABLE I. Critical values of the squeeze parameter for higher-order squeezing (r_s) and pairwise oscillations (r_0) for several input mean occupancies $\bar{n}$.

$\bar{n}$	r_s	r_0
0.4	0.294	0.676
0.8	0.477	0.856
2	0.805	1.164
10	1.522	1.869
20	1.857	2.203

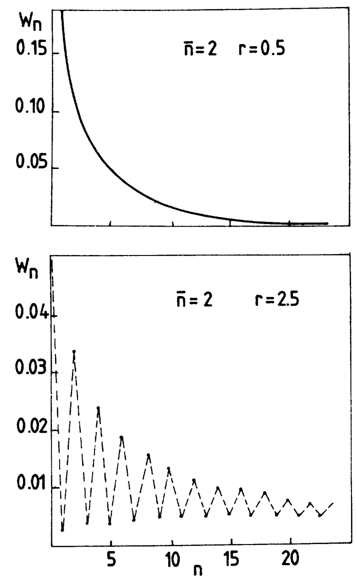

FIG. 3. Photon-number distribution at the squeeze parameters $r=0.5$ and 2.5 for $\bar{n}=2$.

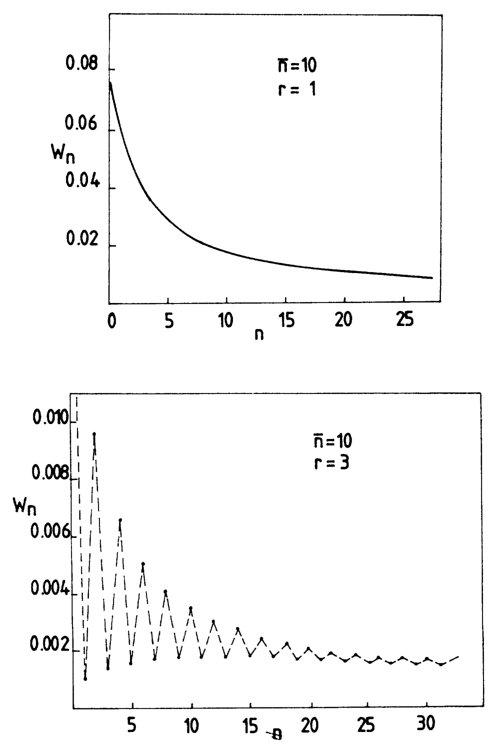

FIG. 4. Photon-number distribution at the squeeze parameters $r=1$ and 3 for $\bar{n}=10$.

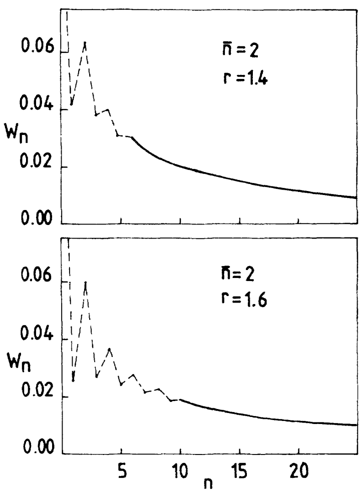

FIG. 5. Photon-number distribution for $\bar{n}=2$ at the squeeze parameters $r=1.4$ and 1.6.

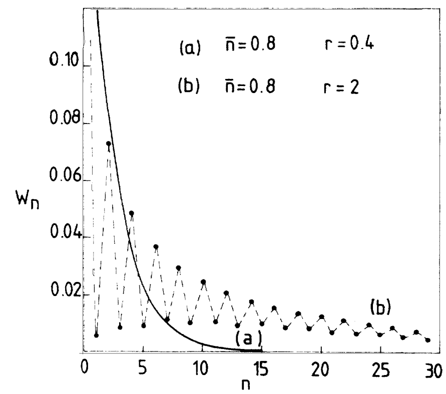

FIG. 6. Photon-number distribution for $\bar{n}=0.8$ at the squeeze parameters $r=0.4$ (——) and $r=2$ (– – –).

are important and cannot be disregarded for both large (Fig. 4) and small values of $\bar{n}$, as shown in Fig. 6, where the distribution is plotted for $\bar{n}=0.8$. For every $\bar{n}$, oscillations occur only for $r>r_0$ [17]. A remnant of the typical behavior of the chaotic photon-number distribution W_n^T is seen to survive for $r<r_0$.

IV. HIGHER-ORDER CORRELATION FUNCTIONS

The correlation functions are essential for the definition of optical coherence [18]. For a one-mode field the lth-order correlation function $\langle (a^\dagger)^l a^l \rangle$ can be calculated using the photon-number distribution W_n,

$$\langle (a^\dagger)^l a^l \rangle = l! \sum_{n=l}^{\infty} \begin{bmatrix} n \\ l \end{bmatrix} W_n \; . \qquad (33)$$

For a chaotic state Eq. (33) gives

$$\langle (a^\dagger)^l a^l \rangle_T = l! (\langle a^\dagger a \rangle_T)^l \; . \qquad (34)$$

Accordingly, the chaotic field has only first-order coherence.

In the case of a STS, Eq. (33) becomes after inserting the expression (26),

$$\langle (a^\dagger)^l a^l \rangle_{\rm ST} = l! \sum_{n=l}^{\infty} \left\{ \begin{bmatrix} n \\ l \end{bmatrix} \left[1 + \frac{2\bar{n}+1}{(\bar{n}+1)^2} (\sinh r)^2 \right]^{-n-(1/2)} \times W_n^T v^n P_n \left[\frac{1}{v} \right] \right\} \; . \qquad (35)$$

The series on the right-hand side of Eq. (35) is of the type (B1). Its sum is evaluated in Appendix B and, according to Eq. (B9), we get

$$\langle (a^\dagger)^l a^l \rangle_{\rm ST} = l! (\langle n \rangle_{\rm ST})^l y^l P_l \left[\frac{1}{y} \right] , \qquad (36)$$

where $\langle n \rangle_{\rm ST}$ is the average photon number in a STS,

$$\langle n \rangle_{\rm ST} = \bar{n} + (2\bar{n}+1)(\sinh r)^2 \qquad (37)$$

and

$$y \equiv \frac{1}{\langle n \rangle_{\rm ST}} (\langle n_s \rangle_{\rm ST} - \langle n \rangle_{\rm ST})^{1/2} \quad [\mathrm{Im}(y) \geq 0] \; . \qquad (38)$$

In Eq. (38), $\langle n_s \rangle_{\rm ST}$ is the mean photon number for the special value (17) of the squeeze parameter,

$$\langle n_s \rangle_{\rm ST} \equiv \langle n \rangle_{\rm ST} |_{r=r_s} = \bar{n}(\bar{n}+1) \; . \qquad (39)$$

The lth-order degree of coherence for a STS,

$$g_{\rm ST}^{(l)}(0) \equiv \frac{\langle (a^\dagger)^l a^l \rangle_{\rm ST}}{(\langle a^\dagger a \rangle_{\rm ST})^l} = l! y^l P_l \left[\frac{1}{y} \right] , \qquad (40)$$

can be also expressed in terms of a Gauss hypergeometric function [14]:

$$g_{\rm ST}^{(l)}(0) = (2l-1)!! \, {}_2F_1 \left[-\frac{l}{2}, -\frac{l-1}{2}; -l+\frac{1}{2}; y^2 \right] . \qquad (41)$$

In the limit $\bar{n}=0$, Eqs. (37), (38), and (41) yield the lth-order degree of coherence for a SVS,

$$g_{\rm SV}^{(l)}(0) = (2l-1)!! \, {}_2F_1 \left[-\frac{l}{2}, -\frac{l-1}{2}; -l+\frac{1}{2}; -(\sinh r)^{-2} \right] . \qquad (42)$$

This formula is equivalent to Eq. (41) from I.

It is interesting to study the dependence of the function (40) on the squeeze parameter r, for a fixed input mean occupancy $\bar{n}$. As expected, for $r=0$ we recover the result (34) for a chaotic state,

$$g_T^{(l)}(0) = l! \; . \qquad (43)$$

For strong squeezing ($r \to \infty$ and, accordingly, $y \to 0$) Eq.

(41) gives a value independent of the initial field intensity,

$$\lim_{r\to\infty} g_{\rm ST}^{(l)}(0)=(2l-1)!! \; . \tag{44}$$

We notice that for $r=r_s$, when $y=0$, one obtains

$$g_{\rm ST}^{(l)}(0)|_{r=r_s}=(2l-1)!! \tag{45}$$

and

$$\frac{d}{dr}[g_{\rm ST}^{(l)}(0)]\bigg|_{r=r_s}=(2l-3)!!\frac{l(l-1)}{\bar{n}(\bar{n}+1)} \quad (l\geq 2) \; . \tag{46}$$

For $r<r_s$ the variable (38) is real and less than unity, while for $r>r_s$ it becomes imaginary just like the variable (25). Due to the fact that all the zeros of a Legendre polynomial lie on the real axis between -1 and $+1$, there is no zero of the polynomial (40) in the range of values of r specified above. Consequently, there is no extremum point in this range, excepting $y=0$, because

$$\frac{d}{dy}[g_{\rm ST}^{l}(0)]\neq 0 \quad \text{for all } y, \text{ excepting } y=0 \; . \tag{47}$$

However, owing to Eq. (46), $g_{\rm ST}^{(l)}(0)$ as a function of r has no extremum at the point $r=r_s$, when $y=0$. Therefore the only extremum of this function arises from the condition

$$\frac{dy}{dr}=0 \; , \tag{48}$$

and it is a maximum achieved at

$$(\langle n\rangle_{\rm ST})_{\rm max}=2\langle n_s\rangle_{\rm ST}=2\bar{n}(\bar{n}+1) \; , \tag{49}$$

that is,

$$(y)_{\rm max}=i(\langle n_s\rangle_{\rm ST})^{-1/2}, \quad (r)_{\rm max}=\sinh^{-1}(\bar{n}^{1/2}) \; . \tag{50}$$

This maximum is significant especially for $\bar{n}<1$ as shown in Fig. 7 for the input mean photon number $\bar{n}=0.4$. It becomes less marked for strong initial chaotic field intensity, as plotted in Fig. 8 for $\bar{n}=10$.

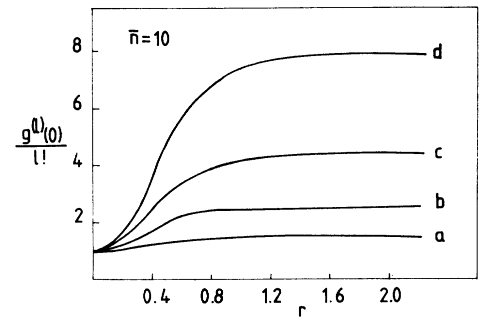

FIG. 8. As in Fig. 7, for $\bar{n}=10$.

In the following we focus our attention on the second-order degree of coherence, a parameter which is relevant to the photon statistics. For $l=2$, Eq. (40) reduces to

$$g_{\rm ST}^{(2)}(0)=3+\frac{\langle n\rangle_{\rm ST}-\langle n_s\rangle_{\rm ST}}{(\langle n\rangle_{\rm ST})^2} \; , \tag{51}$$

in agreement with the previous result of Kim, de Oliveira, and Knight [19]. The maximum value of $g_{\rm ST}^{(2)}(0)$ is given by Eqs. (51) and (49) as

$$[g_{\rm ST}^{(2)}(0)]_{\rm max}=3+\frac{1}{4\bar{n}(\bar{n}+1)} \; . \tag{52}$$

For large input intensities $g_{\rm ST}^{(2)}(0)\simeq 3$; but at weak initial fields $\bar{n}<1$, this maximum is well pronounced, as shown in Fig. 9, which includes also the case $\bar{n}=0$. From Eqs. (51), (37), and (39) it follows that for arbitrary r,

$$g_{\rm ST}^{(2)}(0)\geq 2 \; . \tag{53}$$

Therefore the photon statistics in a STS is superchaotic for every r. Squeezing enhances the large fluctuations in the intensity of the chaotic field.

V. SUMMARY

We have analyzed several general properties of the STS's. Some significant expectation values in STS's have

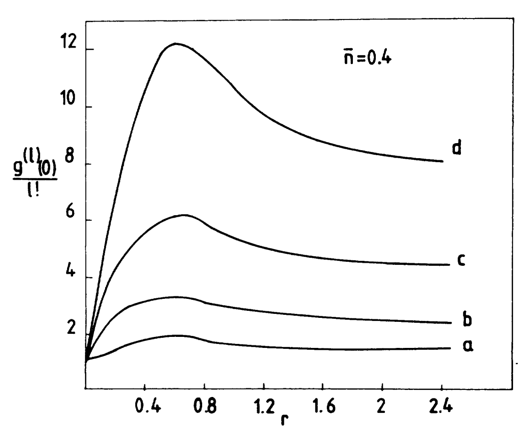

FIG. 7. Plot of the functions $g^{(l)}(0)/l!$ vs the squeeze parameter when $\bar{n}=0.4$ and $l=2$ (curve a), $l=3$ (curve b), $l=4$ (curve c), and $l=5$ (curve d).

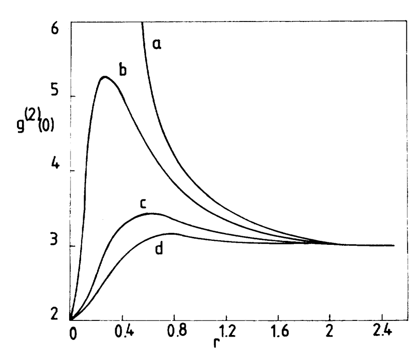

FIG. 9. Second-order degree of coherence vs the squeeze parameter for $\bar{n}=0$ (curve a), $\bar{n}=0.1$ (curve b), $\bar{n}=0.4$ (curve c), and $\bar{n}=0.8$ (curve d).

been obtained by performing analytically Bose-Einstein-weighted sums of the corresponding ones in SNS's. We have derived in this way higher-order squeezing properties and found them to be entirely determined by the second-order ones. The photon-number distribution was expressed in terms of a Legendre polynomial and studied as a function of the squeeze parameter. The onset squeeze parameter for pairwise oscillations was derived by simple algebraic means. We have succeeded in writing a compact formula for the lth-order degree of coherence $g^{(l)}(0)$. As a function of the squeeze parameter, $g^{(l)}(0)$ has a unique maximum point independent of l.

We finally remark that for strong squeezing both squeezing properties and normalized correlation functions to all orders do not depend on the initial chaotic field intensity, being those found for SVS's [10,11].

APPENDIX A: USEFUL SERIES OF GAUSS HYPERGEOMETRIC FUNCTIONS

We give the sums of two series involving Gauss hypergeometric functions needed in Secs. II and III. The first one is [20]

$$\sum_{n=0}^{\infty} \frac{s^n (c)_n}{n!}\, {}_2F_1(-n,a;c;u) = (1-s)^{a-c}(1-s+su)^{-a} , \tag{A1}$$

where

$$|s|<1, \quad |s(1-u)|<1 ,$$

and $(c)_n \equiv \Gamma(c+n)/\Gamma(c)$ is the Pochhammer symbol. The second one is a series of products of two Gauss functions [21]

$$\sum_{n=0}^{\infty} C_\lambda^n s^n\, {}_2F_1(-n,b;-\lambda;u_1)\, {}_2F_1(-n,\beta;-\lambda;u_2) = (1+s)^{\lambda+b+\beta}(1+s-su_1)^{-b}(1+s-su_2)^{-\beta}\, {}_2F_1\left[b,\beta;-\lambda;-\frac{u_1 u_2 s}{(1+s-su_1)(1+s-su_2)}\right] , \tag{A2}$$

with

$$C_\lambda^n \equiv \frac{(-1)^n \Gamma(n-\lambda)}{\Gamma(-\lambda)} . \tag{A3}$$

APPENDIX B: SUMMATION OF A SERIES OF LEGENDRE POLYNOMIALS

We evaluate the sum of the series

$$G_l(t,u) = \sum_{n=l}^{\infty} \binom{n}{l} t^n P_n(u) \quad (l=0,1,2,\ldots) . \tag{B1}$$

In the particular case $l=0$, for

$$|t| < \min|u \pm \sqrt{u^2-1}| , \tag{B2}$$

the sum of the series (B1) is the generating function of the Legendre polynomials [22]

$$G_0(t,u) = (1-2tu+t^2)^{-1/2} . \tag{B3}$$

Accordingly, Cauchy's integral theorem gives the well-known integral representation of a Legendre polynomial,

$$P_n(u) = \frac{1}{2\pi i} \int_{(0+)} \frac{G_0(\xi,u)}{\xi^{n+1}} d\xi . \tag{B4}$$

The integration here is to be taken along a path observing the condition (B2) and circling the origin in the counterclockwise sense. On the other hand, from Eq. (B1) we can write

$$G_l(t,u) = \frac{1}{l!} t^l \frac{\partial^l G_0}{\partial t^l} . \tag{B5}$$

Applying again Cauchy's integral theorem, we get

$$G_l(t,u) = \frac{1}{2\pi i} t^l \int_{(t+)} \frac{G_0(z,u)}{(z-t)^{l+1}} dz . \tag{B6}$$

The integration contour here encloses the point t in the counterclockwise sense. It is straightforward to derive the following factorization of the generating function (B3):

$$G_0(t+\tau,u) = G_0(t,u) G_0(\xi,\eta) \tag{B7}$$

where

$$\xi = \tau G_0(t,u) , \quad \eta = (u-t) G_0(t,u) . \tag{B8}$$

An obvious change of variable in the integral (B6) followed by the factorization (B7) leads, via Eq. (B4), to the remarkable formula

$$G_l(t,u) = G_0(t,u)[t G_0(t,u)]^l P_l((u-t)G_0(t,u)) . \tag{B9}$$

[1] For a review on squeezed coherent states, see, for example, D. F. Walls, Nature **306**, 141 (1983); R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987).

[2] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985); R. M. Shelby, M. D. Levenson, S. H. Perlmutter, R. G. De Voe, and D. F. Walls, *ibid.* **57**, 691 (1986).

[3] B. Yurke *et al.*, Phys. Rev. Lett. **60**, 764 (1988).

[4] M. S. Kim, F. A. M. de Oliveira, and P. L. Knight, Phys. Rev. A **40**, 2494 (1989).

[5] J. J. Gong and P. K. Aravind, in *Coherence and Quantum Optics VI,* edited by J. H. Eberly, L. Mandel, and E. Wolf (Plenum, New York, 1989), p. 425.
[6] A. Kireev, A. Mann, M. Revzen, and H. Umezawa, Phys. Lett. A **142**, 215 (1989).
[7] H. Fearn and M. Y. Collett, J. Mod. Opt. **35**, 553 (1988).
[8] A. Vourdas, Phys. Rev. A **39**, 206 (1989).
[9] H. Ezawa, A. Mann, K. Nakamura, and M. Revzen, Ann. Phys. (N.Y.) **209**, 216 (1991).
[10] Paulina Marian, Phys. Rev. A **44**, 3325 (1991).
[11] C. K. Hong and L. Mandel, Phys. Rev. A **32**, 974 (1985).
[12] W. Schleich, D. F. Walls, and J. A. Wheeler, Phys. Rev. A **38**, 1177 (1988).
[13] A. Vourdas and R. M. Weiner, Phys. Rev. A **36**, 5866 (1987).
[14] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (McGraw-Hill, New York, 1953), Vol. 1, Sec. 3.6, Eqs. (15) and (16).
[15] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (Ref. [14]), Sec. 3.6, Eq. (15) for $z=0$.
[16] J. Hollenhorst, Phys. Rev. D **19**, 1669 (1979), the last equation in Appendix.
[17] In Ref. [4] a plot of the photon-number distribution is given [Fig. 4(b)] for $\bar{n}=2$ and $r=3$, i.e., for $r > r_0$. Strong pairwise oscillations occur at this value of r.
[18] R. J. Glauber, Phys. Rev. **130**, 2529 (1963).
[19] M. S. Kim, F. A. M. de Oliveira, and P. L. Knight, Phys. Rev. A **48**, 2494 (1989), Eq. (3.10); see also Fig. 1 where $g^{(2)}(0)$ is plotted for $\bar{n}=25$. The specified maximum is not significant, as expected from our Eq. (52).
[20] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (Ref. [14]), last equation from Sec. 2.5.1.
[21] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (Ref. [14]), Sec. 2.5.2, Eq. (12).
[22] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (Ref. [14]), Sec. 3.6, Eq. (33).