Suppression of fluorescence in a lossless cavity

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In this paper we theoretically investigate the behavior of a two-level atom in a lossless cavity driven by an external field. Using classical electrodynamics to describe the external field while quantizing the cavity field, we find that the cavity field is excited to a coherent state whose amplitude is equal to that of the external field, but shifted 180° in phase. This results in the disappearance of the atomic resonance fluorescence (i.e., the atom stops interacting with the fields). When we quantize the external field the effect persists. A fully quantized dressed-state approach provides some helpful insight and a nice analogy to another problem in which the resonance fluorescence vanishes.

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I. INTRODUCTION

The radiative properties of atoms inside a cavity have been the subject of intense investigation in recent years under the broad heading of cavity quantum electrodynamics (QED) [1]. Most well known are the effects of cavity enhanced and inhibited spontaneous emission in the perturbative regime [1–5], and the nonperturbative (strong-coupling) effect of "vacuum" Rabi splitting [6–9]. Due to experimental constraints, investigations in the nonperturbative regime have concentrated on many atoms in the cavity with an eye towards decreasing that number ultimately to one.

A major improvement in the capabilities of cavity QED experiments with single atoms is expected once the technologies of atomic cooling [10] and trapping [11] are incorporated into the experiments. Then some of the more exotic effects uncovered in theoretical work on cavity QED will be accessible in the laboratory. One model system that has received a lot of theoretical attention comprises a single atom coupled strongly to a single quantized cavity mode, driven by an external coherent field, and including cavity damping and spontaneous emission. This forms an archetypical model for a quantized dissipative dynamical system: a damped harmonic oscillator coupled to a damped two-level atom. By allowing for a flux of energy through the atom-cavity system, this system can evolve to a nonequilibrium steady state that exhibits many interesting and novel features. Such a system has been shown to produce nonclassical light; the cavity transmission shows photon antibunching, sub-Poissonian photon counting statistics, and squeezing [12]; also, the photon coincidence rate shows a novel nonclassical dependence on delay [13], and the incoherent portion of the optical spectra can exhibit squeezing-induced linewidth narrowing and squeezing-induced spectral holes [14]. The atom-cavity system also exhibits a single-atom absorptive optical bistability [15] and a bimodality in phase for strong driving fields [16].

In this paper we explore a modified version of the above model in an idealized limit. We drive the atom directly with an external coherent field and examine the situation of perfectly reflecting cavity mirrors (lossless cavity). A noteworthy feature arises in steady state. Even though a coherent field exists within the cavity in steady state, maintained by the perfectly reflecting mirrors, the atom decouples from this field, and the fluorescence turns off. For highly reflecting, but not perfect, mirrors the fluorescence is strongly suppressed. It is known that the fluorescence from atom inside a cavity is suppressed due to cavity enhancement of the spontaneous emission rate when the cavity linewidth is large compared with the atomic linewidth and the dipole coupling constant [5]. We treat the opposite limit in which the cavity linewidth is much less than the atomic linewidth and dipole coupling constant. The special significance of this case is that for perfect mirrors an analytical treatment can be given that includes quantum fluctuations—the cavity field is precisely a coherent state.

In Sec. II we analyze the lossless cavity using a quantized cavity field while treating the external driving field classically. In the following sections we quantize both fields and explore the problem from two related viewpoints. In Sec. III we generalize the results for a classical driving field to those for a quantized driving field and show how the two pictures are related. By examining the eigenstates of the two-mode problem we demonstrate the existence of trapping states—states that population can flow in to, but not out of. In a steady state, the population is distributed across these states coherently and the atom ceases to fluoresce. In Sec. IV we examine the problem from a dressed-atom point of view. In Sec. V we look at the same dressed-atom picture using a pair of composite field modes, one which interacts with the atom and one which decouples from it. In the final section we summarize the findings of the paper.

II. QUANTIZED CAVITY MODE WITH CLASSICAL DRIVING FIELD

We are interested in the interaction of a two-level atom with two radiation modes, one a mode of an idealized lossless cavity and the other a coherent external mode that drives the atom (see Fig. 1). The cavity mode and external field are tuned to resonance with the atomic transition frequency ω_0. The atom is damped at the rate γ by spontaneous emission to modes other than the privileged cavity mode. For an optical cavity in which the field mode subtends a small solid angle, γ is approximately equal to the Einstein A coefficient [9]. If the cavity mode subtends a large solid angle, γ is significantly smaller than the Einstein A coefficient [5]. In this section we quantize the cavity mode, with photon creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$, and we treat the coherent external field classically with amplitude $\mathcal{E}$.

The interaction between the atom and the cavity mode is described by the Jaynes-Cummings Hamiltonian (with $\hbar=1$) [17],

$$\hat{H}=\tfrac{1}{2}\omega_0\hat{\sigma}_z+\omega_0\hat{a}^\dagger\hat{a}+ig_a d(\hat{a}^\dagger\hat{\sigma}_--\hat{a}\hat{\sigma}_+)\ , \tag{1}$$

where $\hat{\sigma}_z$ is the atomic inversion operator, $g_a=(2\pi\omega_0/V_a)^{1/2}$ is a cavity volume factor with V_a the volume of the cavity, $d=\langle 2|e\mathbf{r}|1\rangle\cdot\boldsymbol{\varepsilon}$ is the projection of the transition dipole matrix element onto the polarization state of the cavity field, and $\hat{\sigma}_+$ ($\hat{\sigma}_-$) is the atomic raising (lowering) operator. The atomic operators obey the commutation relations $[\hat{\sigma}_+,\hat{\sigma}_-]=2\hat{\sigma}_z$ and $[\hat{\sigma}_z,\hat{\sigma}_\pm]=\pm\hat{\sigma}_\pm$. The Jaynes-Cummings model (JCM) with atomic dissipation and a coherent driving field is described by the master equation

$$\frac{d\hat{\rho}}{dt}=-i[\hat{H},\hat{\rho}]+\mathcal{E}d(\hat{\sigma}_-e^{i\omega_0 t}-\hat{\sigma}_+e^{-i\omega_0 t},\hat{\rho})+\frac{\gamma}{2}(2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+-\hat{\sigma}_+\hat{\sigma}_-\hat{\rho}-\hat{\rho}\hat{\sigma}_+\hat{\sigma}_-)\ , \tag{2}$$

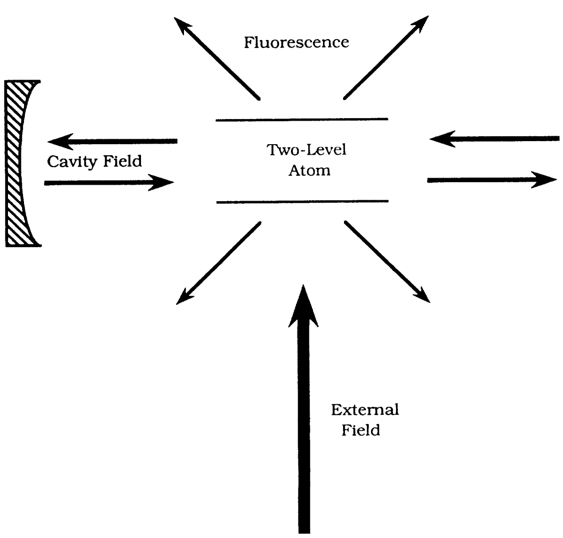

FIG. 1. The externally driven two-level atom-in-a-cavity system under consideration in this paper.

where $\hat{\rho}$ is the reduced density operator for the atom-cavity system and γ is the spontaneous emission rate. Arbitrary phases for the driving field and the dipole coupling constant may be absorbed into the definition of the operators. Thus, there is no loss of generality in using real quantities $\mathcal{E}$ and d.

We can remove the explicit time dependences in Eq. (2) by transforming to the interaction picture. If we let

$$\tilde{\rho}=\hat{\mathcal{U}}^\dagger\hat{\rho}\hat{\mathcal{U}}\ , \tag{3}$$

where $\hat{\mathcal{U}}=e^{-i\hat{H}_0 t}$ and $\hat{H}_0=\omega_0(\frac{1}{2}\hat{\sigma}_z+\hat{a}^\dagger\hat{a})$, then we find

$$\frac{d\tilde{\rho}}{dt}=g_a d[\hat{a}^\dagger\hat{\sigma}_--\hat{a}\hat{\sigma}_+,\tilde{\rho}]+\mathcal{E}d(\hat{\sigma}_--\hat{\sigma}_+,\tilde{\rho})+\frac{\gamma}{2}(2\hat{\sigma}_-\tilde{\rho}\hat{\sigma}_+-\hat{\sigma}_+\hat{\sigma}_-\tilde{\rho}-\tilde{\rho}\hat{\sigma}_+\hat{\sigma}_-)\ . \tag{4}$$

Equation (4) describes an idealized lossless optical cavity. To include the effects of a leaky output mirror, a cavity-damping term $\kappa(2\hat{a}^\dagger\tilde{\rho}\hat{a}-\hat{a}^\dagger\hat{a}\tilde{\rho}-\tilde{\rho}\hat{a}^\dagger\hat{a})$ must be added to Eq. (4). Here, 2κ is the cavity decay rate. Before exploring the solution to Eq. (4), it is worthwhile to briefly investigate the origin of the suppression of fluorescence in a damped cavity in the weak-field limit. In this way we may see what the restrictions are for finite κ and see the connection to the enhanced spontaneous emission work of Heinzen *et al.* [5]. We proceed by writing down the approximate Maxwell-Bloch equations derived from Eq. (4) with the addition of the cavity-damping term, for weak driving fields. These semiclassical equations are

$$\dot{\alpha}=-\kappa\alpha+gv\ , \tag{5a}$$

$$\dot{v}=-\frac{\gamma}{2}v-(g\alpha+\mathcal{E})\ , \tag{5b}$$

where $\alpha=\langle\hat{a}\rangle$ is the mean intracavity field and $v=\langle\hat{\sigma}_-\rangle$ is the atomic polarization. For weak fields, we assume the atomic population remains mostly in the ground state so that the inversion is $\langle\hat{\sigma}_z\rangle\approx-1$. Note that the total field seen by the atom, $g\alpha+\mathcal{E}$ in Eq. (5b), is the sum of the external driving field plus the intracavity field.

In free space there is no intracavity field and Eq. (5b) yields in steady state

$$v_f=-\frac{\mathcal{E}}{\gamma/2}\ . \tag{6}$$

The intensity of the fluorescence is proportional to $|v_f|^2=(2\mathcal{E}/\gamma)^2$. In the cavity, however, the steady-state solutions are

$$\alpha_c=\frac{g}{\kappa}v_c\ , \tag{7a}$$

$$v_c=-\frac{\mathcal{E}}{\gamma/2}\Bigg/\left[1+\frac{2g^2}{\kappa\gamma}\right]\ . \tag{7b}$$

The intensity of the fluorescence from the atom is modified by a factor of $(1+2g^2/\kappa\gamma)^{-2}$ with respect to the free-space result. The condition for a significant reduction in the fluorescent intensity in a cavity compared to the free-space case is

$$\frac{\kappa\gamma}{2g^2} \ll 1 \ . \tag{8}$$

Note that this can be achieved by having either κ very small or γ very small.

Let us explore the two limiting cases, $\kappa \to 0$ and $\gamma \to 0$. For $\kappa \to 0$, the master equation is given by Eq. (4). The semiclassical equations yield $v_c = 0$ and $\alpha_c = -\mathcal{E}/g$. We show below that for $\kappa \to 0$, $(|1\rangle\langle 1|)(|-\mathcal{E}/g\rangle\langle -\mathcal{E}/g|)$ is the steady-state solution valid for all $\mathcal{E}$, not just for weak driving fields ($|1\rangle$ is the unexcited state of the atom, from which there is no fluorescence; thus, for $\kappa \to 0$ the intensity of the fluorescence is zero for all strengths of the driving field). If κ is not exactly zero, but is much smaller than all the other rates, the condition for observing strong suppression of fluorescence is given by Eq. (8). In this case, the fluorescence will not be suppressed for arbitrarily strong driving fields. Eventually the atom will saturate ($|v_c| \to 0$) and the fluorescent intensity will be the same as in free space (dominated by incoherent scattering). But for the atom in a cavity, much stronger driving fields will be required to reach saturation. Note that the suppression factor $\kappa\gamma/2g^2$ is also the suppression factor of the intracavity field on the lower branch of absorptive optical bistability. This reduction of the intracavity field is due to reradiation from the atom (absorption) and suppresses the fluorescence.

For $\gamma \to 0$, Eqs. (5) yield $v_c = -\kappa\mathcal{E}/g^2 \neq 0$ and the field approaches $\alpha_c = -\mathcal{E}/g$. The polarization does not go to zero as it does for $\kappa \to 0$, but it can still be arbitrarily small compared to the free-space case; $|v_c|/|v_f| = \kappa\gamma/2g^2$ goes to zero as γ goes to zero. When $\kappa \gg g \gg \gamma$ with $\kappa\gamma/2g^2 \ll 1$, we have the suppression of fluorescence noted by Heinzen *et al.* [5] in the cavity-enhanced spontaneous emission regime. In this bad-cavity limit, the master equation does not yield a simple steady-state solution valid for all driving fields. In contrast, in the $\kappa \to 0$ regime we now show that a simple exact analytical solution is obtainable and valid for all values of the driving field.

We are interested in the steady-state solution to Eq. (4). An easy way to obtain this is to first eliminate the classical driving field by transforming $\tilde{\rho}$ as follows (see also Ref. [18]):

$$\tilde{P} = \hat{\mathcal{D}}^\dagger(-\mathcal{E}/g_a)\tilde{\rho}\hat{\mathcal{D}}(-\mathcal{E}/g_a) \ , \tag{9}$$

where $\hat{\mathcal{D}}(\alpha) = e^{(\alpha\hat{a}^\dagger - \alpha^*\hat{a})}$ is the displacement operator $[\hat{\mathcal{D}}(\alpha)|0\rangle = |\alpha\rangle$, where $|0\rangle$ is the vacuum state and $|\alpha\rangle$ is a coherent state]. Using

$$\hat{\mathcal{D}}^\dagger(-\mathcal{E}/g_a)\hat{a}\hat{\mathcal{D}}(-\mathcal{E}/g_a) = \hat{a} - \mathcal{E}/g_a \ , \tag{10}$$

Eq. (4) becomes

$$\frac{d\tilde{P}}{dt} = g_a d[\hat{a}^\dagger\hat{\sigma}_- - \hat{a}\hat{\sigma}_+, \tilde{P}] + \frac{\gamma}{2}(2\hat{\sigma}_-\tilde{P}\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\tilde{P} - \tilde{P}\hat{\sigma}_+\hat{\sigma}_-) \ , \tag{11}$$

which is the master equation for the JCM including spontaneous emission but in the absence of an external driving field.

The Jaynes-Cummings interaction Hamiltonian

$$\hat{H}_{\rm JC} = i g_a d(\hat{a}^\dagger\hat{\sigma}_- - \hat{a}\hat{\sigma}_+) \tag{12}$$

has the atom-cavity dressed eigenstates for $N \geq 1$ quanta[17]

$$|N(\pm)\rangle = \frac{1}{\sqrt{2}}(|N,1\rangle \mp i|N-1,2\rangle) \ , \tag{13a}$$

with the ground state

$$|g\rangle = |0,1\rangle \ , \tag{13b}$$

where $|N,i\rangle$ $(i=1,2)$ denotes the free-field–free-atom product state $|N\rangle \otimes |i\rangle$. The total Jaynes-Cummings Hamiltonian [Eq. (1)] has eigenvalues

$$E_{N(\pm)} = N\omega_0 \pm g_a d\sqrt{N} \ , \tag{14}$$

which give rise to the familiar dressed-state ladder of coupled atom-cavity states (see Fig. 2). Equation (11) describes spontaneous transitions between these states. Once population is distributed among these states, the spontaneous transitions proceed downward until all the population eventually ends up in the ground state $|g\rangle$. Thus,

$$\tilde{P}_{\rm SS} = |g\rangle\langle g| = |0,1\rangle\langle 0,1| \ . \tag{15}$$

This is easily verified by setting $d\tilde{P}/dt = 0$ in Eq. (11) and substituting Eq. (15) into the right-hand side to show that it is explicitly zero. If we invert the transformation (9), we obtain

$$\tilde{\rho}_{\rm SS} = \hat{\mathcal{D}}(-\mathcal{E}/g_a)|0,1\rangle\langle 0,1|\hat{\mathcal{D}}^\dagger(-\mathcal{E}/g_a) = |-\mathcal{E}/g_a, 1\rangle\langle -\mathcal{E}/g_a, 1| \ . \tag{16}$$

Thus, we find that in the presence of an external coherent field $\mathcal{E}$ driving the atom, the steady-state configuration of the atom in a lossless cavity is a coherent state for the intracavity field of amplitude $-\mathcal{E}/g_a$, with the atom in its lower state $|1\rangle$. The consequence of this configuration is that the atom will no longer fluoresce.

To summarize, from an initial vacuum state for the cavity mode and the driven atom in the lower state $|1\rangle$, a

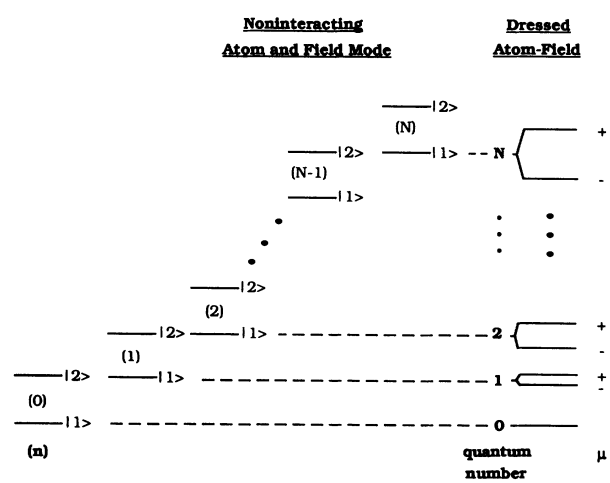

FIG. 2. Energy-level pictures for a two-level atom interacting with a single quantized field.

coherent field of amplitude $-\mathcal{E}/g_a$ builds up in the cavity. When this amplitude is reached, the field configuration is maintained by the perfectly reflecting cavity mirrors ($\kappa=0$). In steady state, the atom, which radiated the cavity field, returns to its lower state $|1\rangle$ and remains there even in the presence of the nonzero cavity field and external driving field (which now cancel at the site of the atom). Alternatively, one could say that in the presence of the external driving field $\mathcal{E}$, the field mode that couples to the atom is $\hat{a}'=\hat{\mathcal{D}}(-\mathcal{E}/g_a)\hat{a}\hat{\mathcal{D}}^\dagger(-\mathcal{E}/g_a)=\hat{a}+\mathcal{E}/g_a$; this can be seen directly by combining the first two terms in Eq. (4). The bare ground state $|g\rangle_a=|0_a,1\rangle$ (where the subscript labels the mode a) is replaced by the ground state $|g\rangle_{a'}=|0_{a'},1\rangle=\hat{\mathcal{D}}(-\mathcal{E}/g_a)|0_a,1\rangle$. Preparing the system initially in the bare ground state $|g\rangle_a$ distributes population coherently over the new Jaynes-Cummings dressed states $|N(\pm)\rangle_{a'}=\hat{\mathcal{D}}(-\mathcal{E}/g_a)|N(\pm)\rangle_a$. This population then makes its way to the ground state $|g\rangle_{a'}$ via fluorescence. Once this state is reached, the system remains there; the atom does not fluoresce and an intracavity coherent state of amplitude $-\mathcal{E}/g_a$ is present. Note that if the cavity itself is driven by an external field or if energy is allowed to leak out of the cavity, this result does not hold (the intracavity field is not then in a coherent state). Significant suppression of the fluorescence occurs under a variety of other conditions, however what distinguishes the present situation is that the suppression is completely to zero and the field driving the atom is exactly a coherent state.

III. QUANTIZED CAVITY MODE AND A QUANTIZED DRIVING FIELD

To simplify the analysis in Sec. II, we did not quantize the driving field. In this section we will quantize both fields to show that the results of Sec. II still hold even in the fully quantized model. In order to compare with the treatment in Sec. II, we quantize the driving field by placing a cavity of infinite length in the direction of the driving field around the atom. We characterize one mode of this cavity with the photon creation and annihilation operators $\hat{b}^\dagger$ and $\hat{b}$, with $[\hat{b}^\dagger,\hat{b}]=1$. The cavity volume factor for this mode is $g_b=(2\pi\omega_0/V_b)^{1/2}$. Thus, as the cavity becomes infinite in extent, $g_b\to 0$. Now, if the state of the b mode is a coherent state $|\beta\rangle_b$ with $|\beta|^2=\bar{n}$, then $g_b\beta$ will remain constant in the simultaneous limit $V_b\to\infty$ and $\bar{n}\to\infty$. Therefore, in the limit of an infinite cavity, we need the classical limit $\bar{n}\to\infty$ in order for the field to interact with the atom. The Jaynes-Cummings interaction Hamiltonian $\hat{H}_{\rm JC}=ig_bd_b(\hat{b}^\dagger\hat{\sigma}_--\hat{b}\hat{\sigma}_+)$ is then replaced by the semiclassical interaction Hamiltonian $\hat{H}_C=i\mathcal{E}d_{\mathcal{E}}(\hat{\sigma}_--\hat{\sigma}_+)$, where we take $g_b\beta\to\mathcal{E}$ in the limit $V_b\to\infty$ and $\bar{n}\to\infty$.

We will now surround the atom with two cavities of different, finite lengths in order to define two quantized modes, an a mode and a b mode. Again we want to treat the idealized case of no cavity damping for either mode. Strictly speaking, this poses a problem as to just how the b mode (the driving mode) is excited in order to start the interaction, but we will ignore this technicality. The goal of this model is to shed a different light onto our previous model involving a *classical* driving field.

In the interaction picture the master equation for the two-mode, on-resonance, Jaynes-Cummings model with atomic dissipation is an obvious extension of Eq. (4):

$$\frac{\partial\tilde{\rho}}{\partial t}=g_ad[\hat{a}^\dagger\hat{\sigma}_--\hat{a}\hat{\sigma}_+,\tilde{\rho}]+g_bd[\hat{b}^\dagger\hat{\sigma}_--\hat{b}\hat{\sigma}_+,\tilde{\rho}]+\frac{\gamma}{2}(2\hat{\sigma}_-\tilde{\rho}\hat{\sigma}_+-\hat{\sigma}_+\hat{\sigma}_-\tilde{\rho}-\tilde{\rho}\hat{\sigma}_+\hat{\sigma}_-)\ . \tag{17}$$

Instead of a displacement, as in Eq. (9), we combine the modes a and b via a rotation

$$\hat{a}=\hat{A}\cos\theta-\hat{B}\sin\theta\ , \tag{18a}$$

$$\hat{b}=\hat{A}\sin\theta+\hat{B}\cos\theta\ , \tag{18b}$$

with

$$\tan\theta=\frac{g_b}{g_a}\ . \tag{18c}$$

The new composite modes A and B are independent, $[\hat{A},\hat{B}^\dagger]=0=[\hat{A},\hat{B}]$, and satisfy the commutation relations $[\hat{A},\hat{A}^\dagger]=1=[\hat{B},\hat{B}^\dagger]$. The unitary transformation that realizes the rotation (18) is given by

$$\hat{\mathcal{U}}(\theta)=e^{\theta(\hat{A}^\dagger\hat{B}-\hat{B}^\dagger\hat{A})}\ ; \tag{19}$$

this can be shown by using [19]

$$e^{\theta\hat{X}}\hat{Y}e^{-\theta\hat{X}}=\hat{Y}+\theta[\hat{X},\hat{Y}]+\frac{\theta^2}{2!}[\hat{X},[\hat{X},\hat{Y}]]+\cdots \tag{20a}$$

and noting that

$$[\hat{A}^\dagger\hat{B}-\hat{B}^\dagger\hat{A},\hat{A}]=-\hat{B} \tag{20b}$$

and

$$[\hat{A}^\dagger\hat{B}-\hat{B}^\dagger\hat{A},\hat{B}]=\hat{A}\ . \tag{20c}$$

Thus, we have

$$\hat{a}=\hat{\mathcal{U}}\hat{A}\hat{\mathcal{U}}^\dagger \tag{21a}$$

and

$$\hat{b}=\hat{\mathcal{U}}\hat{B}\hat{\mathcal{U}}^\dagger\ . \tag{21b}$$

Under this unitary transformation, Eq. (17) becomes

$$\frac{d\tilde{\rho}}{dt}=gd[\hat{A}^\dagger\hat{\sigma}_--\hat{A}\hat{\sigma}_+,\tilde{\rho}]+\frac{\gamma}{2}(2\hat{\sigma}_-\tilde{\rho}\hat{\sigma}_+-\hat{\sigma}_+\hat{\sigma}_-\tilde{\rho}-\tilde{\rho}\hat{\sigma}_+\hat{\sigma}_-)\ , \tag{22}$$

where we have defined $g=(g_a^2+g_b^2)^{1/2}$. This equation is exactly analogous to Eq. (11) except that now it describes a two-quantized-mode problem. We see that the composite B mode does not interact with the atom; only the composite A mode does.

It is interesting to note that the transformation Eq. (19) is commonly used to combine two modes on a lossless beam splitter [20]. Thus the atom, in a sense, acts like a beam splitter, combining the two modes a and b into the composite modes A and B. This "beam splitter" interacts with the A mode, via the usual Jaynes-Cummings interaction, and ignores the B mode.

By Eqs. (20), the two-mode Fock state $|n_a,m_b\rangle \equiv |n\rangle_a \otimes |m\rangle_b$ in the (a,b)-mode representation is related to the two-mode Fock state $|n_A,m_B\rangle \equiv |n\rangle_A \otimes |m\rangle_B$ in the (A,B)-mode representation via the unitary transformation $\hat{\mathcal{U}}$ as

$$|n_a,m_b\rangle = \hat{\mathcal{U}}|n_A,n_B\rangle \ . \tag{23}$$

(Note: The subscripts a,b and A,B do not label the photon numbers n and m, but rather they label the kets and identify the mode they represent.) Because of Eq. (18), we find that, expressed in terms of the a,b modes, Eq. (19) also takes the form

$$\hat{\mathcal{U}}(\theta) = e^{\theta(\hat{a}^\dagger\hat{b} - \hat{b}^\dagger\hat{a})} \ . \tag{24}$$

The choice of representation used is determined by whether $\hat{\mathcal{U}}(\theta)$ acts on an (a,b)- or (A,B)-mode state.

Formally we can write Eq. (22) as

$$\frac{d\tilde{\rho}}{dt} \equiv \hat{\mathcal{L}}_{A\sigma}\tilde{\rho} \ , \tag{25}$$

where in general, the Louivillian super operator $\hat{\mathcal{L}}$ might contain operators of both modes A and B, as well as atomic σ operators. Here the subscripts $A\sigma$ denote the fact that the B mode does not interact with the atom. Formally, the solution of Eq. (25) is

$$\tilde{\rho}(t) = e^{\hat{\mathcal{L}}_{A\sigma}t}\tilde{\rho}(0) \ , \tag{26}$$

which simply states that only the A mode and the atom cause $\tilde{\rho}(0)$ to evolve, while the B-mode parts maintain their initial values. If $\tilde{\rho}(0)$ was a product of density matrices, i.e., $\tilde{\rho}(0) = \tilde{\rho}_{A\sigma}(0) \otimes \tilde{\rho}_B(0)$, where $\tilde{\rho}_B(0) = \mathrm{Tr}_{A\text{-mode}}\mathrm{Tr}_{\text{atom}}[\tilde{\rho}(0)]$ and $\tilde{\rho}_{A\sigma}(0) = \mathrm{Tr}_{B\text{-mode}}[\tilde{\rho}(0)]$, then $\tilde{\rho}(t)$ would remain a product of density matrices, $\tilde{\rho}(t) = \tilde{\rho}_{A\sigma}(t) \otimes \tilde{\rho}_B(0)$ for all times. Here $\tilde{\rho}_{A\sigma}(t) = e^{\hat{\mathcal{L}}_{A\sigma}t}\tilde{\rho}_{A\sigma}(0)$ is the solution to Eq. (25) with $\tilde{\rho}(t)$ replaced by $\tilde{\rho}_{A\sigma}(t)$. Expressed with respect to the original modes, $\tilde{\rho}(t)$ will not, in general, factorize as such when $\tilde{\rho}(0)$ is given as a product of (a,b)-mode and atomic density matrices $[\tilde{\rho}(0) = \tilde{\rho}_a(0) \otimes \tilde{\rho}_b(0) \otimes \tilde{\rho}_\sigma(0)]$. This is because the Louivillian operator $\hat{\mathcal{L}}_{ab\sigma}$ expressed in terms of the (a,b) modes will not preserve the factorization due to the mixing of the modes by the rotation operator $\hat{\mathcal{U}}$. We do, however, know something about the steady state when expressed in terms of the new composite modes. Since Eq. (22) describes a Jaynes-Cummings interaction between the A mode and the atom, all atom-A-mode terms in the density matrix must decay to the dressed ground state $|g_A\rangle = |0_A,1\rangle$. Since the B mode is noninteracting, we have $\tilde{\rho}_B(t) = \mathrm{Tr}_{A\text{-mode}}\mathrm{Tr}_{\text{atom}}[\tilde{\rho}(t)] = \tilde{\rho}_B(0)$. Thus the steady state will be, with respect to the (A,B) modes.

$$\begin{aligned}\tilde{\rho}_{\text{SS}} &= |g_A\rangle\langle g_A| \otimes \tilde{\rho}_B(0) \\ &= |0_A,1\rangle\langle 0_A,1| \otimes \tilde{\rho}_B(0) \ .\end{aligned} \tag{27}$$

Let us choose a particular example. To mimic the single quantized mode description of Sec. II, we choose as our initial state

$$\tilde{\rho}(0) = |0_a,\beta_b,1\rangle\langle 0_a,\beta_b,1| \ , \tag{28}$$

i.e., a vacuum for the a mode, the b mode in a coherent state

$$|\beta\rangle_b = e^{-|\beta|^2/2} \sum_{m=0}^{\infty} \frac{\beta^m}{\sqrt{m!}} |m\rangle_b \ ,$$

and the atom initially in its lower state $|1\rangle$. With respect to the (A,B) modes, we have

$$\tilde{\rho}(0) = \hat{\mathcal{U}}|0_A,\beta_B,1\rangle\langle 0_A,\beta_B,1|\hat{\mathcal{U}}^\dagger \ . \tag{29}$$

For this initial state we can actually calculate $\tilde{\rho}(0)$ quite easily using the displacement operator. Writing $|\beta\rangle_B = \hat{\mathcal{D}}_B(\beta)|0\rangle_B$ and $\hat{\mathcal{U}}\hat{\mathcal{D}}_B(\beta) = \hat{\mathcal{U}}\hat{\mathcal{D}}_B(\beta)\hat{\mathcal{U}}^\dagger\hat{\mathcal{U}}$ and noting that $\hat{\mathcal{U}}|0_A,0_B,1\rangle = |0_A,0_B,1\rangle$ (since the vacuum is rotationally invariant) and

$$\hat{\mathcal{U}}(\theta)\hat{\mathcal{D}}_B(\beta)\hat{\mathcal{U}}^\dagger(\theta) = e^{\hat{\mathcal{U}}(\theta)(\beta\hat{B}^\dagger - \beta^*\hat{B})\hat{\mathcal{U}}^\dagger(\theta)} \ , \tag{30a}$$

we find [using Eqs. (18b) and (20b)]

$$\hat{\mathcal{U}}(\theta)\hat{\mathcal{D}}_B(\beta)\hat{\mathcal{U}}^\dagger(\theta) = \hat{\mathcal{D}}_A(\beta\sin\theta)\hat{\mathcal{D}}_B(\beta\cos\theta) \ . \tag{30b}$$

Thus,

$$\tilde{\rho}(0) = |\beta_{sA},\beta_{cB},1\rangle\langle\beta_{sA},\beta_{cB},1| \ , \tag{31a}$$

where

$$\beta_s = \beta\sin\theta \tag{31b}$$

and

$$\beta_c = \beta\cos\theta \ . \tag{31c}$$

From the analogy with lossless beam splitters, this is the well-known result that mixing a vacuum mode and a coherent state on a beam splitter yields a product of rotated coherent states (in the new modes) upon output [21]. In this example, $\tilde{\rho}(0) = \tilde{\rho}_A(0) \otimes \tilde{\rho}_B(0) \otimes \tilde{\rho}_\sigma(0)$, where $\tilde{\rho}_A(0) = |\beta_s\rangle_{AA}\langle\beta_s|$, $\tilde{\rho}_B(0) = |\beta_c\rangle_{BB}\langle\beta_c|$, and $\tilde{\rho}_\sigma(0) = |1\rangle\langle 1|$. As noted above Eq. (27), the density matrix $\tilde{\rho}(t)$ will evolve as $\tilde{\rho}(t) = \tilde{\rho}_{A\sigma}(t) \otimes \tilde{\rho}_B(0)$, where $\tilde{\rho}_{A\sigma}(0) = \tilde{\rho}_A(0) \otimes \tilde{\rho}_\sigma(0)$, and at steady state will settle into

$$\tilde{\rho}_{\text{SS}} = |0_A,\beta_{cB},1\rangle\langle 0_A,\beta_{cB},1| \ . \tag{32}$$

We can express this in terms of the (a,b) modes by using the inverse of Eq. (23) with $\hat{\mathcal{U}}$ now written in its (a,b) form. We then have [analogous to Eq. (29)]

$$\tilde{\rho}_{\text{SS}} = \hat{\mathcal{U}}^\dagger(\theta)|0_a,\beta_{cb},1\rangle\langle 0_a,\beta_{cb},1|\hat{\mathcal{U}}(\theta) \ . \tag{33}$$

Again, we write $|0_a,\beta_{cb},1\rangle = \hat{\mathcal{D}}_b(\beta_c)|0_a,0_b,1\rangle$ and [noting that $\hat{\mathcal{U}}^\dagger(\theta) = \hat{\mathcal{U}}(-\theta)$] we proceed exactly as we did following Eq. (29), but now with $(A,B) \to (a,b)$, $\theta \to -\theta$, and $\beta \to \beta_c$ to yield

$$\tilde{\rho}_{\text{SS}} = |\tilde{\alpha}_a,\tilde{\beta}_b,1\rangle\langle\tilde{\alpha}_a,\tilde{\beta}_b,1| \ , \tag{34a}$$

with

$$\tilde{\alpha} = -\beta_c\sin\theta = -\beta\cos\theta\sin\theta \tag{34b}$$

and

$$\tilde{\beta}=\beta_c\cos\theta=\beta\cos^2\theta\ , \tag{34c}$$

analogous to Eqs. (31). Thus, beginning with the initial state of the system as $|\psi(0)\rangle=|0_a,\beta_b,1\rangle$, the system settles down in steady state to $|\psi_{SS}\rangle=|\tilde{\alpha}_a,\tilde{\beta}_b,1\rangle$, with a coherent state of magnitude $\tilde{\alpha}$ in the a mode, a coherent state of magnitude $\tilde{\beta}$ in the b mode, and with the atom in its lower state $|1\rangle$.

That this is the generalization of Eq. (16) can be seen as follows. As discussed earlier, as $g_b\to 0$ and $\beta\to\infty$, we require $g_b\beta\to\mathcal{E}$ in order that a field in an infinite cavity interacts with the atom. Formally, we realize this "thermodynamic" limit by replacing $g_b\hat{b}$ in $\hat{H}_b^{\text{quantum}}=ig_bd(\hat{b}^\dagger\hat{\sigma}_- - \hat{b}\hat{\sigma}_+)$ by $\mathcal{E}$, to yield $\hat{H}_b^{\text{quantum}}\to\hat{H}_b^{\text{semiclass}}=i\mathcal{E}d(\hat{\sigma}_- - \hat{\sigma}_+)$. Using $\tilde{\rho}_{SS}=|\psi_{SS}\rangle\langle\psi_{SS}|$, Eq. (33) allows us to write $|\psi_{SS}\rangle$ as

$$|\psi_{SS}\rangle=\hat{\mathcal{U}}^\dagger(\theta)|0_a,\beta\cos\theta,1\rangle=\hat{\mathcal{U}}(-\theta)|\psi'_{SS}\rangle\ . \tag{35}$$

Here, $|\psi'_{SS}\rangle=|0_a,1\rangle\otimes|\beta\cos\theta\rangle_b$ is the analog of $|g\rangle$ in Eq. (13c), the ground state of the single quantized mode (the a-mode) problem. But now $|\beta\cos\theta\rangle_b$ represents a slightly depleted pump, i.e., a coherent state of amplitude $\beta\cos\theta$, reduced from its initial value of β. If we note that as $g_b\to 0$, $\theta\approx\tan\theta=g_b/g_a$, and if we perform the thermodynamic limit by formally replacing $g_b\hat{b}$ with $\mathcal{E}$, we may write

$$\hat{\mathcal{U}}^\dagger(\theta)=\hat{\mathcal{U}}(-\theta)=e^{-\theta(\hat{a}^\dagger\hat{b}-\hat{b}^\dagger\hat{a})}$$
$$\to\exp\left[-\frac{\mathcal{E}}{g_a}(\hat{a}^\dagger-\hat{a})\right]=\hat{\mathcal{D}}(-\mathcal{E}/g_a)\ . \tag{36}$$

Further, $|\beta\cos\theta\rangle_b\to|\beta\rangle_b$ and does not change throughout the calculation. It represents the undepleted pump mode which we now can treat as classical and drop from $|\psi'_{SS}\rangle$. Thus

$$|\psi_{SS}\rangle\to\hat{\mathcal{D}}(-\mathcal{E}/g_a)|0_a,1\rangle=|(-\mathcal{E}/g_a)_a,1\rangle \tag{37a}$$

and

$$|\tilde{\rho}_{SS}\rangle\to|(-\mathcal{E}/g_a)_a,1\rangle\langle(-\mathcal{E}/g_a)_a,1|\ , \tag{37b}$$

which is Eq. (16).

IV. DRESSED-ATOM APPROACH IN THE (a,b)-MODE REPRESENTATION

In this section we will use a dressed-atom formalism [22] with both field modes quantized to develop some insight into the results of the preceding section. The dressed states of the atom-field system represented in Fig. 1 are the eigenstates of the Hamiltonian (with $\hbar=1$)

$$\hat{H}=\tfrac{1}{2}\omega_0\hat{\sigma}_z+\omega_a\hat{a}^\dagger\hat{a}+\omega_b\hat{b}^\dagger\hat{b}$$
$$+ig_ad(\hat{a}^\dagger\hat{\sigma}_- - \hat{a}\hat{\sigma}_+)+ig_bd(\hat{b}^\dagger\hat{\sigma}_- - \hat{b}\hat{\sigma}_+) \tag{38}$$

[the Hamiltonian used in Eq. (17)].

If we label the bare-atom eigenstates as $|i\rangle$ $(i=1,2)$ and the bare-field states as $|n\rangle_a$ and $|m\rangle_b$ (Fock states with n,m equal to the number of photons in mode a,b), then we may take as our basis states the free-atom–free-field product states $|n\rangle_a\otimes|m\rangle_b\otimes|i\rangle\equiv|n_a,m_b,i\rangle$. If the two field modes are nearly resonant with the $|1\rangle$ to $|2\rangle$ atomic transition, then the states $|n,N-n,1\rangle$, $n=0,1,2,\ldots,N$ and $|n,N-n-1,2\rangle$, $n=0,1,2,\ldots,N-1$, where $N=n+m$, are all nearly degenerate and are strongly mixed by the atom-field interaction. The rotating-wave approximation (RWA) ignores any mixing of states that differ in the total photon number $n+m$. This is equivalent to ignoring non-energy-conserving transitions such as absorbing an a or b photon when making a transition from $|2\rangle$ to $|1\rangle$, or emitting an a or b photon when making a transition from $|1\rangle$ to $|2\rangle$. Since the dressing of the atom by the two radiation modes mixes the nearly degenerate states, the dressed states of the atom-field system are linear combinations of the free-atom–free-field product states, in the form

$$|N(\mu)\rangle=\sum_{n=0}^{N}C_\mu(n,N-n,1)|n,N-n,1\rangle$$
$$+\sum_{n=0}^{N-1}C_\mu(n,N-n-1,2)|n,N-n-1,2\rangle\ , \tag{39}$$

with $\mu=0,\pm1,\pm2,\ldots,\pm N$.

In Fig. 2 we represent the transition from the bare-atom picture to the dressed-atom picture via energy-level diagrams for a two-level atom interacting with a *single* radiation mode. In Fig. 3 we show the equivalent picture representing a two-level atom interacting with *two* radiation modes, as described in this paper. For the bare-atom picture, the atomic states appear for every combination of photon numbers n and m that add up to the total quantum number $0,1,2,\ldots,N$. There are 0 quanta of energy when the atom is in the ground state with no photons in either field. There is 1 quantum of energy when the atom is in the excited state with no photons present, or in the ground state with 1 photon in either field, and so forth up the quantum ladder to N quanta of energy in the system. The figure shows schematically the $2N+1$ energy levels that will be mixed together by the interaction. In the dressed-atom picture we see the splitting of the degeneracies into $2N+1$ dressed levels.

From the action of the various operators on the product states,

$$\hat{\alpha}^\dagger|k,i\rangle_\alpha=\sqrt{k+1}|k+1,i\rangle_\alpha,\quad \alpha=a,b\ , \tag{40a}$$

$$\hat{\alpha}|k,i\rangle_\alpha=\sqrt{k}\,|k-1,i\rangle_\alpha\ , \tag{40b}$$

$$\hat{\sigma}_\pm|1\rangle_\alpha=\begin{cases}|2\rangle_\alpha\\ 0\ ,\end{cases} \tag{40c}$$

$$\hat{\sigma}_\pm|2\rangle_\alpha\begin{cases}0\\ |1\rangle_\alpha\ ,\end{cases} \tag{40d}$$

we find the matrix representation of the interaction part of the Hamiltonian in the RWA to be

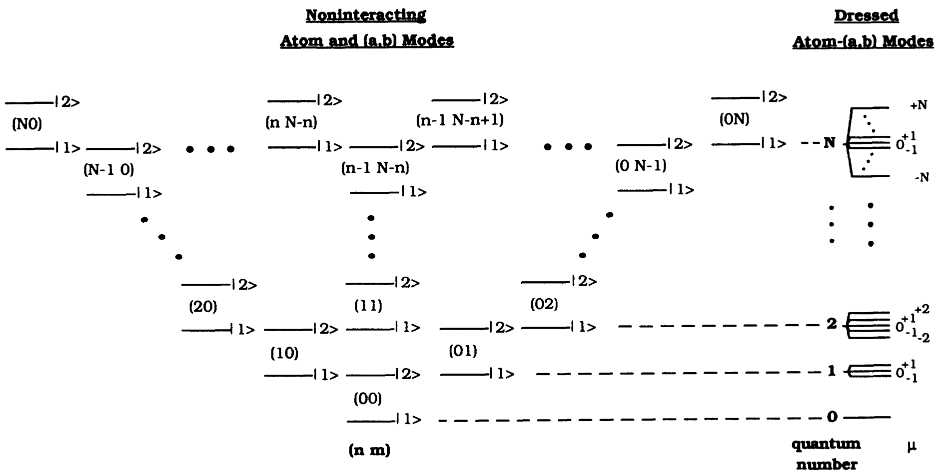

FIG. 3. Energy-level pictures for a two-level atom interacting with two quantized fields.

$$[H_{\text{int}}]=\begin{bmatrix} 0 & i\Omega^*_{aN} & & & & & & & \\ -i\Omega_{aN} & \Delta & & & & & & 0 & \\ & i\Omega^*_{b1} & \ddots & & & & & & \\ & & -i\Omega_{bN-n} & & & & & & \\ & & (N-n)\omega_{ba} & i\Omega^*_{an} & & & & & \\ & & -i\Omega_{an} & \Delta+(N-n)\omega_{ba} & & & & & \\ & & & i\Omega^*_{N-n+1} & \ddots & & & & \\ & & & & & i\Omega^*_{a1} & & & \\ & 0 & & & & \Delta+(N-1)\omega_{ba} & -i\Omega_{bN} & & \\ & & & & & i\Omega^*_{bN} & N\omega_{ba} & & \end{bmatrix}, \tag{41a}$$

where

$$\Delta=\omega_0-\omega_a\ , \tag{41b}$$

$$\omega_{ba}=\omega_b-\omega_a\ , \tag{41c}$$

and the Rabi frequencies are

$$\Omega_{\alpha n}\equiv\sqrt{n}\,g_\alpha d_\alpha,\quad \alpha=a,b\ . \tag{41d}$$

The eigenvalues $\lambda_{N\mu}$ associated with each eigenstate $|N(\mu)\rangle$ are obtained by solving Schrödinger's time-independent equation in the form

$$([H_{\text{int}}]-\lambda_{N\mu}[I])\overline{|N(\mu)\rangle}=0\ , \tag{42}$$

where $[I]$ is the identity matrix and the matrix representation of the eigenvectors is

$$\overline{|N(\mu)\rangle}=\begin{bmatrix} C_\mu(N,0,1) \\ C_\mu(N-1,0,2) \\ \vdots \\ C_\mu(n,N-n,1) \\ C_\mu(n-1,N-n,2) \\ C_\mu(n-1,N-n+1,1) \\ \vdots \\ C_\mu(1,N-1,1) \\ C_\mu(0,N-1,2) \\ C_\mu(0,N,1) \end{bmatrix}. \tag{43}$$

Substituting Eqs. (41a) and (43) into Eq. (42) results in the eigenvalue equation

$$-\lambda_{N\mu}(\cdots)-|\Omega_{aN}|^2[(\omega_{ba}-\lambda_{N\mu})(\cdots)-(|\Omega_{aN-1}|^2\{(2\omega_{ba}-\lambda_{N\mu})(\cdots)-[|\Omega_{aN-2}|^2((3\omega_{ba}-\lambda_{N\mu})(\cdots) \\ -\cdots-\{|\Omega_{a1}|^2[(N\omega_{ba}-\lambda_{N\mu})]\}\cdots)]\})]=0\ , \tag{44}$$

where the $(\cdots)$ refers to contributions that we need not decipher now. In a recent study of a three-level atom in which two transitions were pumped by a single field [23–25], the dressed eigenstate corresponding to the zero eigenvalue was of very special significance. In the present problem we have two fields interacting with a single transition. There is a striking similarity between these two problems, and perhaps the $\lambda_{N0}=0$ eigenstate will have a special distinction in the present problem also. Let us investigate that possibility. For $\lambda_{N0}=0$, Eq. (44) becomes

$$\omega_{ba}|\Omega_{aN}|^2((\cdots)-\{|\Omega_{aN-1}|^2[2(\cdots)-(|\Omega_{aN-2}|^2\{3(\cdots)-\cdots-(|\Omega_{a1}|^2[N])\cdots\})]\})=0 . \tag{45}$$

Obviously, if $\omega_b=\omega_a$ we will have an identity and $\lambda_{N\mu}=0$ will be an eigenvalue. Therefore, we take the frequencies of the two radiation modes to be equal. Then Eq. (42) gives

$$-\lambda_{N\mu}C_\mu(N,0,1)+i\Omega^*_{aN}C_\mu(N-1,0,2)=0 , \tag{46a}$$

$$-i\Omega_{aN}C_\mu(N,0,1)+(\Delta-\lambda_{N\mu})C_\mu(N-1,0,2) -i\Omega_{b1}C_\mu(N-1,1,1)=0 , \tag{46b}$$

$$-i\Omega^*_{b1}C_\mu(N-1,0,2)-\lambda_{N\mu}C_\mu(N-1,1,1) +i\Omega^*_{aN-1}C_\mu(N-2,1,2)=0 , \tag{46c}$$

$$\vdots$$

$$-i\Omega^*_{bN-n}C_\mu(n,N-n-1,2)-\lambda_{N\mu}C_\mu(n,N-n) +i\Omega^*_{an}C_\mu(n-1,N-n,2)=0 , \tag{46d}$$

$$-i\Omega_{an}C_\mu(n,N-n,1)+(\Delta-\lambda_{N\mu})C_\mu(n-1,N-n,2) -i\Omega_{bN-n+1}C_\mu(n-1,N-n+1,1)=0 , \tag{46e}$$

$$\vdots$$

$$-i\Omega^*_{bN-1}C_\mu(1,N-2,2)-\lambda_{N\mu}C_\mu(1,N-1,1) +i\Omega^*_{a1}C_\mu(0,N-1,2)=0 , \tag{46f}$$

$$-i\Omega_{a1}C_\mu(1,N-1,1)+(\Delta-\lambda_{N\mu})C_\mu(0,N-1,2) -i\Omega_{bN}C_\mu(0,N-1,1)=0 , \tag{46g}$$

$$-i\Omega^*_{bN}C_\mu(0,N-1,2)-\lambda_{N\mu}C_\mu(0,N,1)=0 . \tag{46h}$$

Setting $\lambda_{N\mu}=\lambda_{N0}=0$ in Eqs. (46) we find

$$C_0(n,m,2)=0 \tag{47}$$

for all n,m, and the $C_0(n,m,1)$ are nonzero.

By definition, the dressed states are the stationary states of the Hamiltonian [Eq. (38)] describing the atom, the two radiation modes, and their interaction. Therefore, if nothing else is allowed to interact with this system, these states will never change in time. If we now introduce the vacuum field modes which have been ignored until now, the dressed atom will spontaneously radiate. This resonance fluorescence will be described by a cascade of population down the quantum ladder of dressed-state groups. The nature of the dipole coupling is such as to allow transitions only from N to $N-1$, with no spontaneous transitions within each subgroup. The transition rates between neighboring dressed groups are just the Einstein A coefficients calculated by Fermi's golden rule to be

$$\gamma_{\mu\nu}=\frac{4\bar{\omega}^3_{\mu\nu}}{3c^3}|\mathbf{D}_{\nu\mu}|^2 , \tag{48}$$

where $\bar{\omega}_{\mu\nu}=E_{N\mu}-E_{N-1\nu}$ and the transition dipole moment between dressed states $|N(\mu)\rangle$ and $|N-1(\nu)\rangle$ is

$$\mathbf{D}_{\nu\mu}=\langle N-1(\nu)|\hat{\mathbf{d}}|N(\mu)\rangle =\mathbf{d}\sum_{n=1}^{N}C^*_\nu(n,N-n-1,1)C_\mu(n,N-n-1,2) . \tag{49}$$

Now, recalling Eq. (47) we see that

$$\gamma_{0\nu}=0 \tag{50}$$

for all N and ν, while $\gamma_{\mu0}$ is nonzero. What this means is that if the frequencies of our two field modes are equal, over time all population will decay into the set of dressed states labeled by $|N(0)\rangle$, and will not be able to leave. Once this happens, there will be no further atomic dynamics, i.e., the fluorescence will cease.

In the three-level atom study [23–25] a similar result was discovered. In that case, the dressed-state dipole moment went to zero because of a cancellation of two terms. There was a quantum interference between the two transitions. Intuitively what happened was the single pump mode forced the two atomic transitions to emit 180° out of phase with each other so that they destructively interfered. Hence, there was no resonance fluorescence. In the present problem, each individual term in the dipole sum is zero. This occurs because the cavity mode is forced to develop 180° out of phase with the pumping mode. When these two fields impinge on the atom they destructively interfere before interacting with the atom. In the dressed-atom picture the population becomes trapped in a manifold of nonradiating states.

V. DRESSED-ATOM APPROACH IN THE (A,B)-MODE REPRESENTATION

For completeness, we may now discuss the dressed-state formalism as it applies to the (A,B)-mode description of Sec. III. From the transformation defined by Eq. (18) and using $\omega_a=\omega_b=\omega_0$, the Hamiltonian (38) in the (A,B) mode becomes

$$\hat{H}=\omega_0(\hat{A}^\dagger\hat{A}+\hat{B}^\dagger\hat{B}+\tfrac{1}{2}\hat{\sigma}_z)+gd(\hat{A}^\dagger\hat{\sigma}_--\hat{A}\hat{\sigma}_+) . \tag{51}$$

As in Sec. III, we have a Jaynes-Cummings interaction between the atom and the A mode, and a noninteracting B mode. Analogous to Eq. (39), we may write the dressed states in terms of Jaynes-Cummings states for the atom–A-mode interaction and Fock states for the B

mode as

$$|N(\mu)\rangle = |N(\pm n)\rangle = |n(\pm)\rangle_A |N-n\rangle_B \ , \tag{52}$$

where from Eqs. (13) we have

$$|n(\pm)\rangle_A = \frac{1}{\sqrt{2}}(|n\rangle_A |1\rangle \mp i |n-1\rangle_A |2\rangle) \tag{53}$$

with $n=1,2,\ldots,N$, and a ground state of $|0(-)\rangle_A = |g\rangle_A = |0\rangle_A |1\rangle \equiv |0_A,1\rangle$ [26]. The μth sublevel of the Nth multiplet is detuned from $N\omega_0$ by $\pm g\sqrt{n}$.

In Fig. 4 we graphically represent these atom—(A,B)-mode dressed states. In Fig. 4(a) we depict the "noninteracting" and "dressed" atom–A-mode states and the B-mode states separately. Note that the atom–A-mode pictures are identical to the atom-field picture in Fig. 2, while the noninteracting B mode is merely a Fock energy ladder. In Fig. 4(b) we combine these two pictures to obtain the same dressed-state picture as we did in Fig. 3 using the (a,b) representation. This new interpretation lends insight into the origin of the central, unshifted states labeled $|N(\mu=0)\rangle$, into which we found that everything flows. In every instance, these states are the ground state of a Jaynes-Cummings ladder for a particular value of $|m\rangle_B$. That is, when spontaneous emission is allowed to proceed normally, the atom–A-mode interaction results in a cascade down each of the Jaynes-Cummings ladders to the ground state of the atom–A-mode dressed states $(|n(\pm)\rangle_A \rightarrow |g\rangle_A = |0_A,1\rangle)$, with the B mode unchanging $(|m\rangle_B \rightarrow |m\rangle_B)$. The net result is that the ground state of the atom–A-mode, B-mode system is a linear combination over m of the $|0_A, m_B, 1\rangle$ states. From Eq. (52) we may write these "Nth-level ground states" as

$$|N(0)\rangle = |g\rangle_A |N_B\rangle = |0_A, N_B, 1\rangle \ . \tag{54}$$

Transforming these states to the physical (a,b)-mode representation using the inverse of Eq. (23) we write

$$|0_A,N_B,1\rangle = \hat{\mathcal{U}}^\dagger(\theta)|0_a,N_b,1\rangle \tag{55a}$$

$$= \hat{\mathcal{U}}^\dagger(\theta)\frac{(\hat{b}^\dagger)^N}{\sqrt{N!}}|0_a,0_b,1\rangle \tag{55b}$$

$$= \hat{\mathcal{U}}^\dagger(\theta)\frac{(\hat{b}^\dagger)^N}{\sqrt{N!}}\hat{\mathcal{U}}(\theta)|0_a,0_b,1\rangle \ , \tag{55c}$$

where we have used the fact that the vacuum is rotationally invariant to write $|0_a,0_b,1\rangle = \hat{\mathcal{U}}(\theta)|0_a,0_b,1\rangle$. Now, from Eqs. (18a), (18b), and (21b) we find

$$\hat{\mathcal{U}}^\dagger(\theta)\hat{b}^\dagger\hat{\mathcal{U}}(\theta) = \hat{b}^\dagger\cos\theta - \hat{a}^\dagger\sin\theta \ , \tag{56}$$

from which we may write

$$|N(0)\rangle = \frac{(\hat{b}^\dagger\cos\theta - \hat{a}^\dagger\sin\theta)^N}{\sqrt{N!}}|0_a,0_b,1\rangle \ . \tag{57}$$

Since $[\hat{a},\hat{b}]=0$ we can use the binomial theorem to write

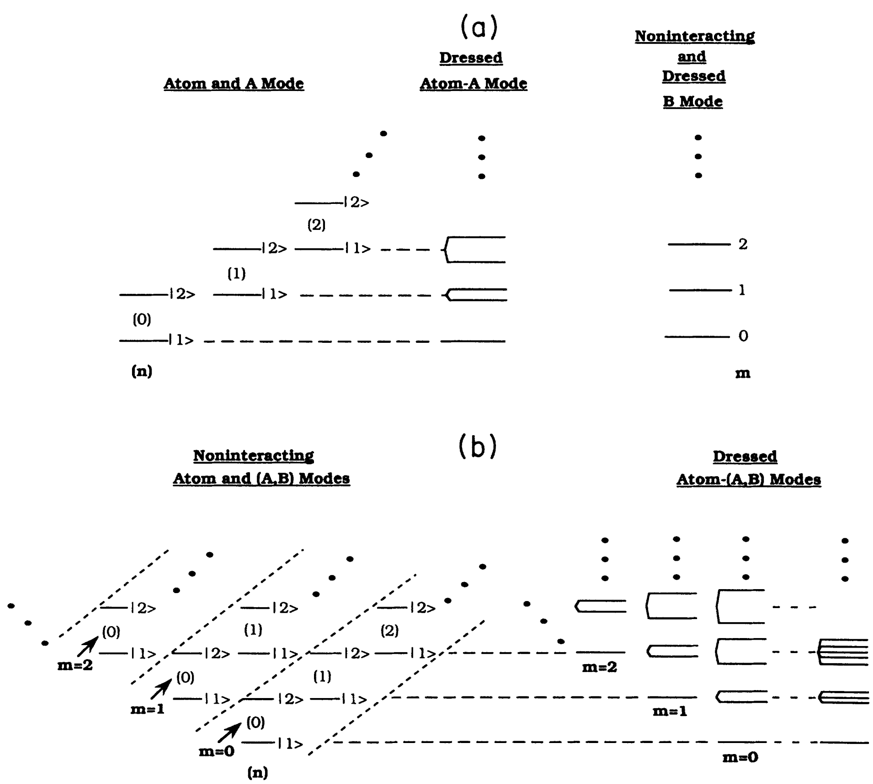

FIG. 4. Energy-level pictures for a two-level atom interacting with two quantized fields represented by the (A,B) modes described in the paper. In (a) the atom–A-mode system is shown separately from the B-mode system. In (b) the two systems are shown together to obtain the picture given in Fig. 3.

$$|N(0)\rangle=\frac{1}{\sqrt{N!}}\sum_{k=0}^{N}(-\hat{a}^{\dagger}\sin\theta)^k(\hat{b}^{\dagger}\cos\theta)^{N-k}\frac{N!}{k!(N-k)!}|0_a,0_b,1\rangle \tag{58a}$$

$$=\sum_{k=0}^{N}\left[\frac{N!}{k!(N-k)!}\right]^{1/2}(-\sin\theta)^k(\cos\theta)^{N-k}|k_a,(N-k)_b,1\rangle\ . \tag{58b}$$

A simple calculation shows that with the initial state given as $|\psi(0)\rangle=|0_a,\beta_b,1\rangle$ (as in Sec. III), the initial population is distributed over these $|N(0)\rangle$ states as

$$|\langle\psi(0)|N(0)\rangle|^2=e^{-|\beta\sin\theta|^2}\left[\frac{(|\beta\cos\theta|^2)^N e^{-|\beta\cos\theta|^2}}{N!}\right], \tag{59}$$

such that the initial probability for the distribution across these levels is

$$P_0=\sum_N|\langle\psi(0)|N(0)\rangle|^2=e^{-|\beta\sin\theta|^2}\ . \tag{60}$$

A probability of $1-P_0$ is then distributed among all of the other states. As the system evolves toward steady state, $|\psi(0)\rangle\to|\psi_{SS}\rangle=|0_A,(\beta\cos\theta)_B,1\rangle$ [see Eq. (32)]. The steady-state distribution is then given by [using Eq. (54) for $|N(0)\rangle$]

$$|\langle\psi_{SS}|N(0)\rangle|^2=\frac{(|\beta\cos\theta|^2)^N e^{-|\beta\cos\theta|^2}}{N!}, \tag{61}$$

so that the population is spread coherently across these "Nth-level ground states" with a mean excitation level of $\bar{N}=|\beta\cos\theta|^2$.

VI. SUMMARY AND CONCLUSIONS

When a classical field interacts with an atom in free space (ordinary resonance fluorescence), the ensuing transitions occur between pairs of neighboring doublets in the Jaynes-Cummings ladder of eigenstates. In a steady state, as long as there is a driving field, the free-space atom will fluoresce.

We have shown that when the atom is placed inside a lossless cavity, the effects are very different. The *total* field that acts at the site of the atom is now the sum of the intracavity field plus the driving field. For a coherent driving field, the atom-cavity system adjusts itself so that in steady state the two fields are both coherent and 180° out of phase; they therefore cancel at the site of the atom and the atom ceases to fluoresce.

To gain further insight into why the fields destructively interfere in steady state at the atom, we quantized the driving field and looked at the problem from a dressed-state point of view. In addition to providing insight into the turning off of the resonance fluorescence, this step also proved that the result is valid in both classical and quantum-mechanical pictures. In order to connect the two pictures, we regained the classical field result by taking the limit of a large number of photons in the driving field with a vanishing coupling of the atom to this field. In the "physical" (a,b)-mode picture, we see that the eigenstates of the combined atom-field system, the dressed states, form a $2N+1$ multiplet for N excitations in the system [Fig. 3(b)]. The central eigenstate, of energy $N\omega_0$, is a linear combination of states which have the atom in its ground state, i.e., no excitation in the atom. Population flows into these central eigenstates and becomes trapped there. In steady state the population is distributed across these central eigenstates to form a coherent state of amplitude $-\mathcal{E}/g$ within the cavity. Since the atom is in the ground state, it cannot radiate and so essentially decouples from the fields.

In this two-quantized-mode dressed-state picture, we also considered the problem from the point of view of composite modes A and B, formed by linear combinations of the "physical" modes a and b. From this vantage point the origin of the central eigenstates became apparent. The two-mode problem broke up quite naturally into an A mode that interacts with the atom through the usual Jaynes-Cummings interaction, and a B mode that is decoupled from both the A mode and the atom (as long as the fields are on resonance). Thus for each B-mode photon in the system, determined from the initial field distribution, there is a standard Jaynes-Cummings ladder of doublets. For the Nth B-mode photon, the Jaynes-Cummings ground state of the A-mode–atom system forms the central eigenstate of the (a,b) description [see Fig. 4(b)], i.e., a kind of "Nth-level ground state." The initial state $|0_a,\beta_b,-\rangle$, the atom-cavity subsystem in its "bare vacuum" with the pump in a coherent state, is distributed across all of the (A,B) eigenstates. Again, a population flows into the "Nth-level ground states" and becomes trapped, leading to the eventual decoupling of the atom from the fields.

If we had chosen the driving field to be in some continuous distribution ($\mathcal{P}$) of coherent states, then $\tilde{\rho}(0)$ could have been represented by an integral over $\mathcal{P}(\beta)$ times the right-hand side of Eq. (28). In this case, the complete suppression with $\kappa=0$ and the partial suppression with $\kappa\neq0$ of the resonance fluorescence described in this paper would still hold exactly, however the steady-state cavity field would have become an integral over $\mathcal{P}(\beta)$ times the right-hand side of Eq. (34a). If we allow the driving field to have a stochastically varying phase, then the results presented above would hold when the time constants for the atom–driving-field interaction $(1/\gamma$ and $1/g_b$ or $1/\mathcal{E}d)$ are each much less than the time constant associated with the stochastic phase.

If there are nodes within the cavity and the atom is placed at one of these nodes, it will not couple to the cavity $(g_a\to0)$. Since we have assumed a nonzero cavity-coupling constant throughout this paper, the solution $\mathcal{E}/g_a$ is not appropriate here. For zero coupling constant, we must go back to the original equations, prior to any division by g_a. In this case we will, of course, get the free-space result [see Eq. (6)].

The suppression of fluorescence from an atom in a cavity is known in the context of absorptive optical bistability and cavity-enhanced spontaneous emission ($\gamma, g \ll \kappa$). We have described the cavity-induced suppression of fluorescence in the limit $\gamma, g \gg \kappa \rightarrow 0$, where the suppression holds for arbitrary strengths of the driving field. If the driving field is taken to be classical or a coherent state, the cavity field is *analytically* found to be a coherent state whose amplitude is equal to that of the driving field, but shifted 180° in phase. The effect presented here is another interesting example of the difference between the radiative properties of an atom in free space and an atom in a cavity.

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