Negative binomial states of the field-operator representation and production by state reduction in optical processes

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Properties of the radiation fields in negative binomial states are investigated. The density matrix of the field is related to the density matrix ρ_c of the chaotic field via $a^s \rho_c a^{\dagger s}$. Various quasiprobability distributions and the thermofield representation for negative binomial states of the field are derived. The production of negative binomial distribution in a number of nonlinear processes is demonstrated.

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I. INTRODUCTION

In classical probability theory, some of the discrete distributions that are commonly used are Poisson, geometric, binomial, and negative binomial. In quantum optics, some of these discrete distributions have played a very important role as far as the statistics of the radiation fields is concerned. For example, fields in coherent state (thermal state) are known [1] to lead to Poisson (geometric or Bose-Einstein) distribution for the number of photons. Binomial distribution was also introduced and has the interesting property in limiting cases: it corresponds to a field in either a Fock state or in a coherent state [2,3]. Negative binomial distribution for the photon fields has also been studied [4]. This has the attractive feature that in limiting cases it corresponds to fields in coherent and thermal states. Another discrete distribution, namely, logarithmic distribution [5] for photon numbers has been investigated with regard to the nonclassical character of the radiation fields. The logarithmic distribution is a special case of the negative binomial distribution with the term $n=0$ removed [6].

The negative binomial distribution (*nbd*) is defined by [7]

$$p_{nb}(n)=\begin{bmatrix} n+s \\ n \end{bmatrix}\beta^{s+1}(1-\beta)^n , \tag{1.1}$$

where

$$s\geq 0, \quad 0<\beta<1, \quad n=0,1,2,\ldots,\infty . \tag{1.2}$$

Note that the case $s=-1$ corresponds to the trivial distribution δ_{n0} and hence will not be considered. The distribution (1.1) has mean and variance given by

$$\langle n\rangle=(1+s)\left[\frac{1-\beta}{\beta}\right], \quad \langle n^2\rangle-\langle n\rangle^2=\frac{(1+s)(1-\beta)}{\beta^2} . \tag{1.3}$$

The parameter Q defined by

$$Q\equiv\frac{\langle n^2\rangle-\langle n\rangle^2}{\langle n\rangle}-1 , \tag{1.4}$$

is equal to

$$Q=\left[\frac{1}{\beta}-1\right] . \tag{1.5}$$

Note that Q signifies deviations from the Poisson distribution and is always *positive* since β lies between zero and one. Thus the fluctuations are super-Poissonian. In contrast the binomial distribution defined by

$$p_b(n)=\begin{bmatrix} N \\ n \end{bmatrix}\beta^n(1-\beta)^{N-n}; \quad n=0,1,2,\ldots,N \tag{1.6}$$

has the properties

$$\langle n\rangle=N\beta, \quad \langle n^2\rangle-\langle n\rangle^2=N\beta(1-\beta) , \quad Q=-\beta . \tag{1.7}$$

Recently the states of the field with distributions (1.1) and (1.6) have played an important role [2–5] in quantum optical problems. We can define the density matrix of the field as

$$\rho=\sum_n p_{nb}(n)|n\rangle\langle n| , \tag{1.8}$$

where $|n\rangle$ represents the Fock state of the field. We can also define a more general state by including off-diagonal elements as follows:

$$\rho=\sum_{n,m}[p_{nb}(n)p_{nb}(m)]^{1/2}e^{-i\Phi_{nm}}|n\rangle\langle m|, \quad \Phi_{nn}=0 , \tag{1.9}$$

where Φ_{nm} is the phase associated with the off-diagonal elements.

A number of papers have been devoted to the states like (1.8) and (1.9) and to the interaction of fields in such states with matter [3,4]. It is obviously of interest to find out how states like (1.8) and (1.9) can be *produced in practice*.

We have found a very instructive operator representation of the negative binomial distribution. This representation enables one to discuss how such states can be produced by the process of state reduction which we discuss in Sec. III. We also discuss in Sec. IV other methods of producing negative binomial distribution using nondegenerate parametric amplifiers. In Sec. V we discuss a new class of density matrices for the radiation fields.

II. OPERATOR REPRESENTATION AND THE QUASIPROBABILITY DISTRIBUTION FOR THE NEGATIVE BINOMIAL DISTRIBUTION

We start from Eqs. (1.1) and (1.8)

$$\rho=\sum_{n=0}^{\infty}\frac{(n+s)!}{n!(s)!}\beta^{s+1}(1-\beta)^n|n\rangle\langle n| \; . \tag{2.1}$$

Noting that $a|n\rangle=\sqrt{n}\,|n-1\rangle;\langle n|a^{\dagger}=\sqrt{n}\,\langle n-1|$, we can rewrite (2.1) as

$$\begin{aligned}\rho&=\sum_{n=0}^{\infty}\frac{1}{s!}\beta^{s+1}(1-\beta)^n a^s|n+s\rangle\langle n+s|a^{\dagger s}\\&=\beta^{s+1}\frac{a^s(1-\beta)^{-s}}{s!}\sum_{n=0}^{\infty}(1-\beta)^{n+s}|n+s\rangle\langle n+s|a^{\dagger s}\\&=\beta^{s+1}\frac{(1-\beta)^{-s}}{s!}a^s\sum_{n=s}^{\infty}(1-\beta)^n|n\rangle\langle n|a^{\dagger s}\\&=\beta^{s+1}\frac{(1-\beta)^{-s}}{s!}a^s\sum_{n=0}^{\infty}(1-\beta)^n|n\rangle\langle n|a^{\dagger s}\\&=\beta^{s}\frac{(1-\beta)^{-s}}{s!}a^s\rho_c a^{\dagger s}=\frac{1}{s!n_c^s}a^s\rho_c a^{\dagger s}\;,\end{aligned}\tag{2.2}$$

where

TABLE I. Characteristics of commonly used photon number distributions and their quasiprobability distributions.

Sl. No.	Distribution	Density matrix ρ	$p(n)$	Mean	Variance
1	Poisson	$\|\alpha\rangle\langle\alpha\|$	$\frac{e^{-\bar{n}}n^{-n}}{n!}$	$\bar{n}$	$\bar{n}$
2	Bose-Einstein or geometric	$\left[\frac{\bar{n}}{1+\bar{n}}\right]^{a^{\dagger}a}\Big/(\bar{n}+1)$	$\left[\frac{\bar{n}}{1+\bar{n}}\right]^{n}\frac{1}{1+\bar{n}}$	$\bar{n}$	$\bar{n}^2+\bar{n}$
3	Binomial	$\rho=\sum_n p(n)\|n\rangle\langle n\|$	$\begin{bmatrix}N\\n\end{bmatrix}\beta^n(1-\beta)^{N-n}$	$N\beta$	$N\beta(1-\beta)$
4	Negative binomial	$\frac{1}{s!n_c^s}a^s\rho_c a^{\dagger s}$, $n_c=\frac{1-\beta}{\beta}$	$\begin{bmatrix}n+s\\n\end{bmatrix}\beta^{s+1}(1-\beta)^n$	$(s+1)\left[\frac{1}{\beta}-1\right]$	$(1+s)\frac{(1-\beta)}{\beta^2}$

Sl. No.	Q parameter $\frac{\langle n^2\rangle-\langle n\rangle^2}{\langle n\rangle}-1$	$P(\alpha)$	$Q(\alpha)$	$\Phi(\alpha)$
1	0	$\delta^{(2)}(\alpha-\alpha_0)$	$\frac{1}{\pi}e^{-\|\alpha-\alpha_0\|^2}$	$\frac{2}{\pi}e^{-2\|\alpha-\alpha_0\|^2}$
2	$\bar{n}$	$\frac{1}{\pi\bar{n}}e^{-\|\alpha\|^2/\bar{n}}$	$\frac{1}{\pi(\bar{n}+1)}e^{-\|\alpha\|^2/(\bar{n}+1)}$	$\frac{1}{\pi(\bar{n}+\frac{1}{2})}e^{-\|\alpha\|^2/(\bar{n}+\frac{1}{2})}$
3	$-\beta$	$\frac{1}{\beta}P\left[\frac{\alpha}{\sqrt{\beta}}\right]$ $P(\alpha)$: P function for the Fock state $\|N\rangle$	$\frac{(1-\beta)^N}{\pi}e^{-\|\alpha\|^2}\times L_N\left[\frac{-\beta}{1-\beta}\|\alpha\|^2\right]$	$\frac{2(1-2\beta)^N}{\pi}e^{-2\|\alpha\|^2}\times L_N\left[\frac{-4\beta}{1-2\beta}\|\alpha\|^2\right]$
4	$\left[\frac{1}{\beta}-1\right]$	$\frac{\|\alpha\|^{2s}}{\pi n_c^{s+1}s!}e^{-\|\alpha\|^2/n_c}$	$\frac{e^{-\|\alpha\|^2/(n_c+1)}}{\pi(n_c+1)^{s+1}}\times L_s\left[-\frac{n_c}{n_c+1}\|\alpha\|^2\right]$	$\frac{e^{-2\|\alpha\|^2}}{\pi 2^s(n_c+\frac{1}{2})^{s+1}}\times {}_1F_1\left[s+1;1;\frac{2\|\alpha\|^2 n_c}{n_c+\frac{1}{2}}\right]$

$$\rho_c=\sum_{n=0}^{\infty}\beta(1-\beta)^n|n\rangle\langle n|,\quad n_c=\left[\frac{1}{\beta}-1\right]. \tag{2.3}$$

Here ρ_c represents the chaotic state of the field with mean number given by $(1/\beta-1)\equiv n_c$. We thus find the very interesting relation [Eq. (2.2)] between the negative binomial and thermal state of the field. The moments of the photon number operator are given by

$$\langle a^{\dagger p}a^p\rangle=\frac{\beta^s(1-\beta)^{-s}}{s!}\mathrm{Tr}[\rho_c(a^\dagger)^{p+s}(a)^{p+s}],$$
$$=\frac{(p+s)!n_c^p}{s!}. \tag{2.4}$$

The quasiprobabilities associated with the negative binomial distribution can be obtained from the representation (2.2). For example, the P function associated with ρ is found as follows—we start with the P representation for the density matrix ρ_c for the chaotic field [1,8]

$$\rho_c=\int|\alpha\rangle\langle\alpha|d^2\alpha\frac{1}{\pi n_c}e^{-|\alpha|^2/n_c}, \tag{2.5}$$

and hence

$$a^s\rho_c a^{\dagger s}=\int|\alpha|^{2s}\frac{1}{\pi n_c}e^{-|\alpha|^2/n_c}|\alpha\rangle\langle\alpha|d^2\alpha. \tag{2.6}$$

Therefore the P function for negative binomial states of the field is

$$P(\alpha)=\frac{1}{\pi(n_c)^{s+1}}\frac{|\alpha|^{2s}}{s!}e^{-|\alpha|^2/n_c}. \tag{2.7}$$

Note that (2.7) implies that the distribution of the intensity I which is proportional to $|\alpha|^2$ is the gamma distribution $I^s e^{-I}/\Gamma(s+1)$. The Q function defined by [9]

$$Q(\alpha)=\langle\alpha|\rho|\alpha\rangle, \tag{2.8}$$

can be obtained by noting that

$$a|\alpha\rangle=\alpha|\alpha\rangle,\quad a^\dagger|\alpha\rangle=e^{-1/2|\alpha|^2}\frac{\partial}{\partial\alpha}e^{1/2|\alpha|^2}|\alpha\rangle. \tag{2.9}$$

Calculations show that

$$Q(\alpha)=\frac{1}{\pi(n_c+1)(n_c)^s s!}e^{-|\alpha|^2}\frac{\partial^{2s}}{\partial\alpha^s\partial\alpha^{*s}}e^{|\alpha|^2}e^{-|\alpha|^2/(n_c+1)}. \tag{2.10}$$

The derivatives in (2.10) can be expressed in terms of Laguerre polynomials [10]

$$\frac{\partial^{2m}}{\partial\alpha^m\partial\alpha^{*m}}e^{-\gamma|\alpha|^2}=(-\gamma)^m m!e^{-\gamma|\alpha|^2}L_m(\gamma|\alpha|^2)m!, \tag{2.11}$$

and hence

$$Q(\alpha)=\frac{1}{\pi(n_c+1)^{s+1}}e^{-|\alpha|^2/(n_c+1)}L_s\left[-\frac{n_c}{n_c+1}|\alpha|^2\right]. \tag{2.12}$$

In order to obtain the Wigner function for negative binomial states we use the relation [11] between the Wigner function $\Phi(\alpha,\alpha^*)$ and the P function

$$\Phi(\alpha,\alpha^*)=\frac{2}{\pi}\int P(\alpha_0,\alpha_0^*)\exp(-2|\alpha-\alpha_0|^2)d^2\alpha_0. \tag{2.13}$$

On substituting (2.7) in (2.13) we get

$$\Phi(\alpha,\alpha^*)=\frac{e^{-2|\alpha|^2}}{\pi(2)^s(n_c+\frac{1}{2})^{s+1}}\sum_{n=0}^{\infty}\left[\frac{2|\alpha|^2 n_c}{n_c+\frac{1}{2}}\right]^n\frac{(n+s)!}{n!n!s!}, \tag{2.14}$$

which can be expressed in terms of the degenerate hypergeometric function [12] ${}_1F_1$

$$\Phi(\alpha,\alpha^*)=\frac{e^{-2|\alpha|^2}}{\pi(2)^s(n_c+\frac{1}{2})^{s+1}}\,{}_1F_1\left[s+1;1;\frac{2|\alpha|^2 n_c}{n_c+\frac{1}{2}}\right], \tag{2.15}$$

$${}_1F_1(a;b;\gamma)=1+\frac{a}{b}\frac{\gamma}{1!}+\frac{a(a+1)}{b(b+1)}\frac{\gamma^2}{2!}+\frac{a(a+1)(a+2)}{b(b+1)(b+2)}\frac{\gamma^3}{3!}+\cdots. \tag{2.16}$$

These quasiprobabilities are useful in the study of amplification and attenuation of a field in the negative binomial state. In Table I we compare important properties of different photon number distributions (Poisson, Bose-Einstein, binomial, and negative binomial). We also present different quasiprobabilities such as the P function, Q function, and the Wigner function associated with the corresponding states of the field.

III. PRODUCTION OF NEGATIVE BINOMIAL STATES BY STATE REDUCTION

We next consider how negative binomial states can be produced in optical processes. We will show the utility of the state reduction methods. Consider, for example, the process of m-photon absorption from a thermal beam of photons. The m-photon absorption can be described by the effective Hamiltonian

$$H_{\mathrm{eff}}=\hbar(ga^m S^\dagger+\mathrm{H.c.}), \tag{3.1}$$

where g is the m-photon matrix element. $S^\dagger$ is the operator which accounts for the excitation of the atom with the absorption of m photons. To lowest order in the coupling constant, the wave function of the system at time t is

$$|\psi(t)\rangle=|\psi_R\rangle|g\rangle-i(gta^m S^\dagger+\mathrm{H.c.})|\psi_R\rangle|g\rangle, \tag{3.2}$$

where $|\psi_R\rangle$ ($|g\rangle$) is the initial state of the field (atom). Suppose at time t the atom is measured to be in the excited state $|e\rangle\equiv S^\dagger|g\rangle$, then the state of the field to order g^2t^2 is reduced to

$$\rho_{\mathrm{field}}\propto a^m|\psi_R\rangle\langle\psi_R|a^{\dagger m}. \tag{3.3}$$

This is apart from a normalization constant. Thus if the initial state of the radiation field is a thermal state ρ_c, then (3.3) goes over to

$$\rho_{\text{field}} \propto a^m \rho_c a^{\dagger m} . \tag{3.4}$$

Thus the negative binomial state of the field can be generated by *state reduction* [13,14] *in the process of m-photon absorption from a thermal beam.*

Alternatively one can consider the following micromaserlike [15] situation—consider a cavity at a *finite, but low temperature.* The initial state of the field is a thermal state. Consider now the passage of a beam of *well-separated* two-level atoms in ground state through the cavity such that *at a given* time *only one atom is in the cavity,* i.e., the transit time through the cavity must be small compared to the time separation between atoms. The interaction is given by (3.1) with $m=1$ and the evolution for short times is given by (3.2) with $|\psi_R\rangle\langle\psi_R|$ to be replaced by the density matrix ρ_c for a thermal field. Assume that the atom spends short enough time (τ_{transit}) so that the perturbative result (3.2) is applicable with $m=1$. Suppose that the exiting atom is found to be in the excited state, then by the process of state reduction the state of the field is reduced to (3.4) with $m=1$. By repeating this process with m successive atoms, one can reduce the state of the field to (2.2) with $s=m$. The above two methods are based on the assumption that the atoms spend only a short time in the interaction region so that $g\tau_{\text{transit}} \ll 1$.

IV. PRODUCTION OF NEGATIVE BINOMIAL STATES USING PARAMETRIC AMPLIFICATION

We next show that the negative binomial states of the field can be produced in the process of parametric amplification by suitably choosing initial conditions. To see this we consider SU(1,1) coherent states. Perelemov [16] has considered in detail the coherent states associated with the group SU(1,1) defined by the generators $K_\pm, K_3$ with commutation relations

$$[K_-,K_+]=2K_3, \quad [K_3,K_+]=K_+, \quad [K_3,K_-]=-K_- . \tag{4.1}$$

The SU(1,1) coherent states are defined by

$$|\zeta\rangle=(1-|\zeta|^2)^k \sum_{n=0}^{\infty} \left[\frac{\Gamma(n+2k)}{\Gamma(n+1)\Gamma(2k)} \right]^{1/2} \zeta^n |k,n+k\rangle , \tag{4.2}$$

where $|k,n+k\rangle$ are the eigenstates of K_3 and the Casimir operator $C=K_3^2-\frac{1}{2}(K_+K_-+KAK_+)$ with eigenvalues $k+n$ and $k(k-1)$, respectively. The allowed values of k are $\frac{1}{2},1,\frac{3}{2},2,\ldots$. The diagonal elements have a distribution given by

$$p_n=(1-|\zeta|^2)^{2k}\frac{\Gamma(n+2k)}{\Gamma(n+1)\Gamma(2k)}|\zeta|^{2n} , \tag{4.3}$$

which is the same as the negative binomial distribution (1.1) with $s=2k-1$. The SU(1,1) algebra can be realized [17] in terms of the two modes a,b of the field, i.e., $K_+=a^\dagger b^\dagger$, $K_-=ab$, $K_3=\frac{1}{2}(a^\dagger a+b^\dagger b+1)$, $C=-\frac{1}{4}+\frac{1}{4}(a^\dagger a-b^\dagger b)^2$. Then in terms of the Fock states $|n,m\rangle$ of a two-mode radiation field the parameter s becomes equal to $(m-n)$ and (4.2) can be written as

$$|\zeta\rangle=(1-|\zeta|^2)^{(1+s)/2} \times \sum_{n=0}^{\infty} \left[\frac{\Gamma(n+s+1)}{\Gamma(n+1)\Gamma(s+1)} \right]^{1/2} \zeta^n |n+s,n\rangle , \tag{4.4}$$

which can also be expressed as

$$|\zeta\rangle=\exp[\gamma(a^\dagger b^\dagger-ab)]\frac{(a^\dagger)^s}{\sqrt{s!}}|0,0\rangle, \quad \zeta=\tanh\gamma . \tag{4.5}$$

Thus the states $|\zeta\rangle$ are essentially negative binomial states of a two-mode radiation field in which the probability of finding n signal photons obeys the negative binomial distribution. Note that $i(a^\dagger b^\dagger-ab)$ is the interaction Hamiltonian for parametric amplification and hence a parametric amplifier *can produce the state* $|\zeta\rangle$ *provided the input to the amplifier is such that the difference between the idler and signal photons is s.*

We next consider an alternative scheme which also uses the parametric interaction between two modes. This follows from considerations based on the thermofield representation [18–20] of the density matrix (2.2). In thermo-field-dynamics [18,19] one associates with a density matrix ρ a state vector $|\psi\rangle^{(\alpha)}(0<\alpha<1)$ in an extended Hilbert space. One doubles the degrees of freedom. The association is such that the expectation values are identical, i.e.,

$$\langle A\rangle=\mathrm{Tr}\rho A=\langle\psi^{(1-\alpha)}|A|\psi^{(\alpha)}\rangle . \tag{4.6}$$

Here A is the observable in the original Hilbert space. If originally we consider the density matrix associated with a single mode a of the field, then the state vector $|\psi\rangle$ involves two modes, say, a and b. Thus the chaotic field ρ_c with the average number of photons n_c can be represented by the state vector

$$|\psi_c^{(\alpha)}\rangle=\frac{(1-f)^\alpha}{(1-f^{2\alpha})^{1/2}}\exp[\theta(a^\dagger b^\dagger-ab)]|0,0\rangle , \tag{4.7}$$

where $|0,0\rangle$ is the vacuum state of the two-mode field and where

$$\tanh\theta=\left[\frac{n_c}{1+n_c}\right]^\alpha \equiv f^\alpha , \tag{4.8}$$

$$|\psi^{(0)}\rangle=\sum_n |n,n\rangle . \tag{4.9}$$

Note that in (4.6) A will be a function of the operators a and $a^\dagger$. Thus it is clear from (4.6) that

$$\rho=\mathrm{Tr}_b|\psi^{(\alpha)}\rangle\langle\psi^{(1-\alpha)}| . \tag{4.10}$$

The choice of α depends on the system under consideration. For dissipative systems it appears that one has to choose [19] $\alpha=1$. Since the negative binomial state is generated from the chaotic field [Eq. (2.2)] it is clear that the state vector for $\alpha=1$ can be written in the form

$$|\psi^{(1)}\rangle_{nbd}=\left[\frac{1-f}{1+f}\right]^{1/2}\frac{a^s b^s}{s!n_c^s}\exp[\theta(a^\dagger b^\dagger-ab)]|0,0\rangle \ . \tag{4.11}$$

It is interesting to note that the thermofield representation of the negative binomial state can be obtained from Schumaker-Caves squeezed state [17]—we have to annihilate s pairs of a and b photons out of the Schumaker-Caves squeezed state. The representation (4.11) is to be contrasted from the state (4.4). In the state (4.4), the number of photons in the mode b is distributed according to negative binomial distribution and the difference in the number in the two modes is held fixed; whereas in the representations (4.10) and (4.11) the two modes appear symmetrically and the *number of pairs has a negative binomial distribution.*

V. A GENERAL CLASS OF THE FIELD DENSITY MATRICES

We close this paper by considering a general class of the states of the field. The relation (2.2) suggests that it would be interesting to study a class of states generated from a given density matrix $\tilde{\rho}$ by

$$\rho=\mathcal{N}a^s\tilde{\rho}a^{\dagger s} \ , \tag{5.1}$$

where $\mathcal{N}$ is the normalization constant determined by $\mathrm{Tr}\rho=1, \mathrm{Tr}\tilde{\rho}=1$. Note that $\rho=\tilde{\rho}$ if $\tilde{\rho}$ is chosen as a coherent state, i.e., $\tilde{\rho}=|\alpha\rangle\langle\alpha|$. The density matrix (1.9) is also a special case of (5.1). The relations between the quasiprobabilities associated with ρ and $\tilde{\rho}$ can be obtained. Calculations show that different quasiprobabilities and some of the lower-order moments are related by

$$P(\alpha)=\mathcal{N}|\alpha|^{2s}\tilde{P}(\alpha) \ , \tag{5.2}$$

$$Q(\alpha)=\mathcal{N}e^{-|\alpha|^2}\frac{\partial^{2s}}{\partial\alpha^s\partial\alpha^{*s}}e^{|\alpha|^2}\tilde{Q}(\alpha) \ , \tag{5.3}$$

$$\Phi(\alpha)=\exp\left[-\frac{1}{2}\frac{\partial^2}{\partial\alpha\partial\alpha^*}\right]Q(\alpha) \ , \tag{5.4}$$

$$\langle a\rangle=\langle a^{\dagger s}a^{s+1}\rangle_{\tilde{\rho}} \ , \tag{5.5}$$

$$\langle a^\dagger a\rangle=\langle a^{\dagger s+1}a^{s+1}\rangle_{\tilde{\rho}} \ , \tag{5.6}$$

where

$$\langle G\rangle=\mathrm{Tr}(\rho G), \quad \langle G\rangle_{\tilde{\rho}}=\mathrm{Tr}(\tilde{\rho}G) \ . \tag{5.7}$$

We also note that the state (5.1) can be obtained by state reduction methods if we start initially with a field in the state $\tilde{\rho}$.

A large number of the states of the field are represented by density matrices $\tilde{\rho}$ of the form

$$\tilde{\rho}=[\tfrac{1}{4}(e^{2\varphi}-1)]^{-1/2}\exp\left[-2e^{-\varphi}\cosh^{-1}(\coth\varphi)\left[\mu(a-\alpha_0)^2+\mu^*(a^\dagger-\alpha_0^*)^2+\tau(a^\dagger-\alpha_0^*)(a-\alpha_0)+\frac{\tau}{2}\right]\right] \ ,$$
$$e^{2\varphi}=4(\tau^2-4|\mu|^2) \ , \tag{5.8}$$

where the parameters are related to the lower-order moments of a and $a^\dagger$

$$\langle a\rangle=\alpha_0, \quad \langle a^2\rangle=-2\mu^*+\alpha_0^2, \quad \langle a^\dagger a\rangle=\tau-1+|\alpha_0|^2 \ . \tag{5.9}$$

The state (5.8) includes as special cases coherent states, thermal states, superposition of thermal and coherent states, two photon coherent states, and squeezed states for dissipative systems. The Q function for the state (5.8) is Gaussian [21]

$$Q(\alpha,\alpha^*)=\frac{1}{\pi(\tau_0^2-4|\mu|^2)^{1/2}}\exp\left[-\frac{\mu(\alpha-\alpha_0)^2+\mu^*(\alpha^*-\alpha_0^*)^2+\tau_0|\alpha-\alpha_0|^2}{(\tau_0^2-4|\mu|^2)}\right], \quad \tau_0=\tau+\tfrac{1}{2} \tag{5.10}$$

and thus $Q(\alpha)$ associated with (5.1) is easily obtained. The Gaussian character of (5.10) is quite useful in the calculation of mean values like (5.6) and (5.7).

In conclusion we have given a compact operator representation for the density matrix associated with the negative binomial distribution. We have also presented different methods which can produce negative binomial states of the field.

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