Mass effects and one-particle detectors in quantum-measuring processes

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The measuring process of an object system and a detector are studied in the von Neumann model and one of its variants, in which the detector consists of a single particle. If the states of the particle after the interaction with the object system are described partly by coherent states, then the particle can play the role of a detector under certain conditions. The mass (or free Hamiltonian) of the detector does not affect the measuring process at all, while the mass of the object system plays an important role. In the von Neumann model, if the mass of the object system is quite large, the position measurement can be considered, and under certain conditions the statistical operator ρ tends to just its mixed-state part, i.e., quantum interference disappears. However, when the time after the interaction becomes very large, interference emerges again. On the other hand, if the mass of the object system is small, we can even consider the momentum measurement and show that ρ approaches its mixed-state part when the time becomes very large, i.e., ρ is equivalent to its mixed-state part. The momentum can also be measured almost completely in the variant model; the operator ρ becomes equivalent to its mixed-state part. The different quantum states distinguish the detectors from the object systems.

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I. INTRODUCTION

In quantum-measuring process, we always consider an interaction between an object system S and a detector A. Since we get information about the object system through the interaction, the detector must register each state of the system, and the states including the information about S must be almost stable. As a result, the detector may be a macroscopic body with many (N) degrees of freedom. These macroscopic detectors have many desirable characteristics as measuring apparatus when N is very large. In fact, on the basis of their macroscopic properties, many authors have discussed measuring processes [1–15], irreducibility, quantum macroscopic states and their superpositions [16,17], etc.

We have presented recently several models for the measuring process [18–22] with the help of generalized coherent (GC) states, and shown that the (macroscopic) detectors in these models have many desirable functions as measuring apparatus. The GC states in these models were generated by applying elements of the coherence groups SU(2) and SU(1,1) to a "base state." Using a group such as SU(1,1) we have given an explanation of why particle trajectories in the Wilson cloud chamber are straight and almost stable [19,20]. Also the notion of an equivalence class of quantum states, which was first discussed by Jauch [23], has been introduced into the above GC-state formalism [21], this notion being well suited to our approach.

The GC states have been developed by many authors [24–26] in order to find a link between quantum and classical theories; coherence groups and GC states are central in their treatments. In the large-N limit, the expectation value of any product of reasonable operators (called classical operators) approaches the product of the expectation values of each operator, which is called factorization [24]. This is a very important property of classical operators because the factorization relation shows that quantum fluctuations of these operators become irrelevant in this limit.

It should be noted here that some quantum theories with the $\hbar \rightarrow 0$ limit have the same mathematical structure as those with the large-N limit; the quantum theories approach the corresponding classical theories in the small-$\hbar$ limit [24]. Small $\hbar$ implies that it can be neglected in comparison to other quantities of the physical system under consideration.

In a previous paper [27], using the Heisenberg-Weyl (HW) group, we have applied the above approach to a quantum-measuring process with a one-particle detector, i.e., the detector consists of a single particle with a few (one or three, for example) degrees of freedom. It has been shown that the one-particle detector can behave macroscopically or classically under certain conditions; if the time t after the interaction between S and A is very long, then these conditions are satisfied. Differing from macroscopic detectors, the possibility of one particle acting as a quantum detector depends on its object system. All calculations have been done in the impulsive approximation, that is we have neglected the free Hamiltonians of S and A.

It is the purpose of this paper to investigate the roles played by the masses (or free Hamiltonian) of the object system and the one-particle detector. Thus we have to treat the free part as well as the interaction part. The models we will adopt are the von Neumann [28] and one of its variants. Using the canonical transformation, which has been developed in considering the squeezed states in quantum optics [29–34], it is shown that the mass of the one-particle detector does not play an important role, at least in these models. On the other hand, the mass of the object system is critical.

We will show that, in the case of the position measurement of S in the von Neumann model, the statistical operator ρ approaches its mixed-state part ρ_0 under certain conditions when the mass of S is very large. However, the difference between ρ and ρ_0 is very large as the time $t \to \infty$, which means that quantum interference appears again. Because of noncommutativity between the free Hamiltonian of S and the interaction, even the momentum measurement can be performed if the mass of S is quite small, although the von Neumann model has so far been considered adequate to measure only the position of S. In this case, the operator ρ approaches ρ_0 when the time t is very large. The momentum measurement can also be considered in the variant model. The calculation works out very nicely and the operator ρ becomes equivalent to its mixed-state part with respect to any observables of S and any polynomials in the position and momentum operators of A.

A short review of the HW group and the Glauber coherent states [25,27,35,36] will be given in Sec. II. The position measurement in the von Neumann model is discussed in Sec. III; the momentum measurement is investigated in Sec. IV. We also give the momentum measurement in the variant model in Sec. V. Finally, Sec. VI is devoted to conclusions.

II. GLAUBER COHERENT STATES

In our previous paper we considered the Glauber coherent states and their application to several measuring processes [27]. The classical limit we adopted these was $\hbar \to 0$, which implies that $\hbar$ can be negligible in comparison to other quantities of a physical system under consideration. In most measuring processes, the small-$\hbar$ limit corresponds to large time or large coherent parameter. For simplicity, we use the small-$\hbar$ limit in this section.

Several models have been investigated in the previous paper on the assumption that the free Hamiltonian of an object system S and a (one-particle) detector A is much smaller than their interaction (i.e., impulsive measurement). In order to treat the masses of S and A, we have to introduce their free Hamiltonians. Then we show that the mass of the object system, not of the detector, does play an essential role in the process.

Let us give a brief review of the Glauber coherent states [25,27,35,36]. Consider a single particle with three degrees of freedom; its position $\hat{Q}_i$ and momentum $\hat{P}_j$ operators satisfy the HW algebra $[\hat{Q}_i,\hat{P}_j]=i\hbar\delta_{ij}$ $(i,j=1,2,3)$. The theory is defined in a Hilbert space $\mathcal{H}$. Using annihilation and creation operators defined by

$$a_j=\left[\frac{s}{2\hbar}\right]^{1/2}\hat{Q}_j+\frac{i}{\sqrt{2\hbar s}}\hat{P}_j \ , \qquad a_j^\dagger=\left[\frac{s}{2\hbar}\right]^{1/2}\hat{Q}_j-\frac{i}{\sqrt{2\hbar s}}\hat{P}_j \ , \tag{2.1}$$

where s is a constant with dimension (mass)/(time), the basic commutation relation reduces to $[a_i,a_j^\dagger]=\delta_{ij}$.

The Glauber coherent states are given by

$$|\boldsymbol{\alpha}\rangle\equiv|\alpha_1,\alpha_2,\alpha_3\rangle=D(\boldsymbol{\alpha})|0\rangle \ , \tag{2.2}$$

where $\alpha_i\in\mathbb{C}$, $D(\boldsymbol{\alpha})\equiv D(\alpha_1)D(\alpha_2)D(\alpha_3)$, and $|0\rangle\equiv|0\rangle|0\rangle|0\rangle$ is a base state with $a_i|0\rangle=0$. For simplicity, we consider only one component $a_i\equiv a$ and $a_i^\dagger\equiv a^\dagger$. Each $D(\alpha)$ is defined by $D(\alpha)=e^{\alpha a^\dagger-\alpha^* a}$.

Let us introduce a representation of the HW group. Define [27]

$$D(\alpha,\beta)=e^{\alpha a^\dagger}e^{\beta}e^{\gamma a} \ , \tag{2.3}$$

where $\alpha,\beta,\gamma\in\mathbb{C}$, $\beta=\beta_1+i\beta_2$ $(\beta_i\in\mathbb{R})$. Setting $\beta_1=-\frac{1}{2}|\alpha|^2$ and $\gamma=-\alpha^*$, then we find $D(\alpha,\beta)=\exp(i\beta_2)D(\alpha)$, and

$$D(\alpha,\beta)D(\alpha',\beta')=D(\alpha'',\beta'') \tag{2.4}$$

with

$$\alpha''=\alpha+\alpha' \ , \quad \beta''=\beta+\beta'-\alpha^*\alpha' \ , \tag{2.5}$$

where $\beta_1''\equiv\mathrm{Re}(\beta'')=-|\alpha''|^2/2$. The 3×3 matrices

$$u(\alpha,\beta)=\begin{bmatrix}1&0&0\\ \alpha&1&0\\ \beta&-\alpha^*&1\end{bmatrix} \tag{2.6}$$

satisfies $u(\alpha,\beta)u(\alpha',\beta')=u(\alpha'',\beta'')$, where a'' and β'' are given by Eq. (2.5). Defining $D(\alpha,\beta)\equiv D(u(\alpha,\beta))$ we have a representation D of the HW group consisting of $u(\alpha,\beta)$. On the other hand, the matrix u is expressed by [27]

$$u(\alpha,\beta)=e^{\alpha\rho_-}e^{\beta\rho_3}e^{\gamma\rho_+} \ , \tag{2.7}$$

where

$$\rho_+=\begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix} , \quad \rho_-=\begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix} , \quad \rho_3=\begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix} . \tag{2.8}$$

Define $\rho_1=\rho_++\rho_-$ and $\rho_2=-i(\rho_+-\rho_-)$, then ρ_i $(i=1,2,3)$ construct the HW algebra. Using $D(\exp\rho_i)=\exp D(\rho_i)$ we find

$$D(\rho_+)=a \ , \quad D(\rho_-)=a^\dagger \ , \quad D(\rho_3)=1 \ . \tag{2.9}$$

The coherent states are obtained by applying elements of the HW group to a base state $|\mathrm{in}\rangle$ [25],

$$|\alpha,\beta\rangle=D(\alpha,\beta)|\mathrm{in}\rangle \ . \tag{2.10}$$

Note that $|\alpha,\beta\rangle=\exp(i\beta_2)|\alpha\rangle$ and $a|\mathrm{in}\rangle=0$. Different coherent states are not orthogonal; their overlaps are given by

$$\langle \alpha,\beta|\alpha',\beta'\rangle = \langle 0,0|D((u^{-1}(\alpha,\beta)u(\alpha',\beta'))|0,0\rangle$$
$$= e^{i(-\beta_2+\beta_2')}\exp[-\tfrac{1}{2}(|\alpha|^2+|\alpha'|^2-2\alpha^*\alpha')] , \tag{2.11a}$$

$$\langle \alpha|\alpha'\rangle = \exp[-\tfrac{1}{2}(|\alpha|^2+|\alpha'|^2-2\alpha^*\alpha')] , \tag{2.11b}$$

which is a well-known result.

Since any operator does not have a sensible macroscopic (classical) limit, we have to introduce a restricted set K of classical operators to control this limit [24]. In the present case, it follows from Eq. (2.11) that the internal products are nonzero: $\langle \alpha,\beta|\alpha',\beta'\rangle \neq 0$ for all $\alpha,\beta,\alpha',\beta' \in \mathbb{C}$. Thus the definition of a classical operator is as follows [21]: an operator $\hat{A}$ is called classical if $\lim_{\hbar\to 0}\langle \alpha,\beta|\hat{A}|\alpha'\beta'\rangle/\langle \alpha,\beta|\alpha'\beta'\rangle$ is finite for all $|\alpha,\beta\rangle, |\alpha',\beta'\rangle \in \mathcal{H}$. Recall that $\hbar$ can be negligible in comparison to other quantities of our physical system. Under this definition, all polynomials in $\hat{Q}$ and $\hat{P}$ are classical operators (see Ref. [24], p. 413).

Now consider a quantum-measuring process in which an object system S interacts with a detector A (one particle). After the interaction between S and A, the state of the total system may be written as

$$|\psi\rangle = \sum_n c_n |n\rangle|\alpha_n\rangle , \tag{2.12}$$

where $|n\rangle$ are states of S, $|\alpha_n\rangle$ coherent states of A and c_n coefficients. We will derive Eq. (2.12) in several simple models, and show that the condition of small $\hbar$ is satisfied when the time after the interaction between S and A becomes very large.

In the previous paper [21] we introduced an equivalence class which is defined as follows: two statistical operators ρ and ρ_0 are *equivalent* if

$$\mathrm{Tr}[(\rho-\rho_0)(\hat{O}\otimes\hat{A})] \to 0 \quad (\hbar\to 0), \tag{2.13}$$

where $\hat{O}$ is any observable of S and $\hat{A}$ any classical operator. Now set $\rho = |\psi\rangle\langle\psi|$ and let ρ_0 be its mixed-state part. Then we have

$$\mathrm{Tr}[(\rho-\rho_0)(O\otimes A)] = \sum_{n\neq m} c_n c_m^* \langle m|O|n\rangle\langle\alpha_m|A|\alpha_n\rangle , \tag{2.14}$$

Thus, if $\langle\alpha_m|\alpha_n\rangle|_{n\neq m} \to 0$ ($\hbar\to 0$), then Eq. (2.14)$\to 0$; ρ and ρ_0 are equivalent, which is written $\rho\sim\rho_0$. Although the two equivalent states in Jauch's treatment cannot be distinguished by measurement of any observables of a physical system [23], it is possible in principle with our approach, but more difficult as $\hbar\to 0$.

In our theory of "coherent measurement," it seems quite difficult to measure directly a continuous observable of S such as the position and momentum. Hence we have to change a state of S to a spectral-decomposed one using, for example, a Stern-Gerlach-type device [37–39]. Without this spectral decomposition we cannot prove at present that ρ is equivalent to its mixed-state part ρ_0. This needs further investigation.

III. POSITION MEASUREMENT IN THE VON NEUMANN MODEL

A. Impulsive measurement

As in Sec. II, the total system consists of the object system S and the detector A. The detector is one particle; at least one component (the z component, for example) of the detector state becomes a coherent state.

Let the total Hamiltonian be given by $H = H_S + H_A + H'$, where H_S and H_A are, respectively, free Hamiltonians of S and A, and H' their interaction [28],

$$H_S = \frac{\hat{\mathbf{p}}^2}{2m} , \quad H_A = \frac{\hat{\mathbf{P}}^2}{2M} , \quad H' = g\hat{\mathbf{q}}\cdot\hat{\mathbf{P}} , \tag{3.1}$$

where $\hat{\mathbf{p}}$ and $\hat{\mathbf{q}}$ are, respectively, the momentum and position operators of S, $\hat{\mathbf{P}}$ the momentum operator of A, m and M masses of S and A, respectively, and g a coupling constant. As is well known, in the impulsive approximation, $H_S, H_A \ll H'$, corresponding to, for example, very large masses of S and A.

Using a Stern-Gerlach-type device, a state of S is assumed to reduce to

$$|\varphi\rangle = c_1|\tilde{q}_{1z}\rangle|\varphi_1\rangle + c_2|\tilde{q}_{2z}\rangle|\varphi_2\rangle , \tag{3.2}$$

where c_i are constants, $|\varphi_i\rangle$ contain the spin, x, and y components of each wave packet, and $|\tilde{q}_{iz}\rangle$ are well-localized wave packets so that $|\tilde{q}_{iz}\rangle$ may almost be eigenvectors of $\hat{q}_z$: $\hat{q}_z|\tilde{q}_{iz}\rangle \simeq q_{iz}|\tilde{q}_{iz}\rangle$. That is, using the Stern-Gerlach-type device, we can always change a wave packet of S to a spectral-decomposed one in which the expectation value of $\hat{q}_z$ is much larger than its uncertainty. The initial state of the detector is to be $|\mathrm{in}\rangle \equiv |0,0,0\rangle \equiv |0\rangle|0\rangle|0\rangle$. Thus the initial state of S and A (at $t=0$) becomes $\Psi(0) = |\varphi\rangle|0\rangle|0\rangle|0\rangle$.

Using $H \simeq H'$, the state at time t can be obtained,

$$\begin{aligned}\Psi(t) &= e^{-(i/\hbar)Ht}\Psi(0)\\ &\simeq \sum_{i=1}^{2} c_i U_x U_y |\tilde{q}_{iz}\rangle|\varphi_i\rangle|0,0\rangle e^{-(i/\hbar)gtq_{iz}\hat{P}_z}|0\rangle\\ &= \sum_i c_i U_x U_y |\tilde{q}_{iz}\rangle|\varphi_i\rangle|0,0\rangle|gtq_{iz}\sqrt{s/2\hbar}\rangle\\ &\equiv \sum_i c_i U_x U_y |\tilde{q}_{iz},\varphi_i;0,0\rangle|\alpha_i\rangle ,\end{aligned} \tag{3.3}$$

where $U_i = \exp(-igt\hat{q}_i\hat{P}_i/\hbar)$. Since $|\alpha_i\rangle$ are coherent states, the state $\Psi(t)$ has the structure of "coherent correlation."

The statistical operator ρ corresponding to the state

$\Psi(t)$ is written as

$$\rho=\rho_0+(c_1^*c_2U_xU_y|\tilde{q}_{2z},\varphi_2;0,0\rangle \times\langle\tilde{q}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger\otimes|\alpha_2\rangle\langle\alpha_1|+\text{H.c.})\ , \quad (3.4)$$

where ρ_0 is the mixed-state part of ρ. Any classical operator can be written as

$$\hat{A}(\hat{Q}_i,\hat{P}_j)\equiv\hat{A}_{xy}(\hat{Q}_x,\hat{Q}_y,\hat{P}_x,\hat{P}_y)\hat{A}_z(\hat{Q}_z,\hat{P}_z)\ .$$

The difference between the expectation values of $\hat{O}\otimes\hat{A}$ ($\hat{A}\in K$) for ρ and ρ_0 is given by

$$\begin{aligned}\mathrm{Tr}[(\rho-\rho_0)(\hat{O}\otimes\hat{A})]&=c_1^*c_2\langle\tilde{q}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger\hat{O}\otimes\hat{A}_{xy}U_xU_y|\tilde{q}_{2z},\varphi_2;0,0\rangle\langle\alpha_1|\hat{A}_z|\alpha_2\rangle+\text{c.c.}\\&\simeq\text{const }\times c_1^*c_2\langle\tilde{q}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger\hat{O}\otimes\hat{A}_{xy}U_xU_y|\tilde{q}_{2z},\varphi_2;0,0\rangle\langle\alpha_1|\alpha_2\rangle+\text{c.c.}\\&=\text{const }\times c_1^*c_2\langle\tilde{q}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger\hat{O}\otimes\hat{A}_{xy}U_xU_y|\tilde{q}_{2z},\varphi_2;0,0\rangle\exp\left[-\frac{g^2(q_{1z}-q_{2z})^2st^2}{4\hbar}\right]+\text{c.c.}\\&\simeq 0\ ,\end{aligned}\quad (3.5)$$

when $\hbar\simeq 0$, which means physically $g^2(q_{1z}-q_{2z})^2st^2\gg 4\hbar$. We thus have $\rho\sim\rho_0$ under this condition. Note that q_{iz} is a constant of the order of the expectation value of $\hat{q}$ at $t=0$.

It becomes more difficult to distinguish the two equivalent states ρ and ρ_0 as $\hbar\to 0$ or very large time t. Although we cannot derive the relation $\rho\sim\rho_0$ if $g^2(q_{1z}-q_{2z})^2st^2\lesssim 4\hbar$, we obtain $\rho\sim\rho_0$ again when the time t becomes very large. However, as will be seen in Sec. III B, t cannot become very large. In the case of detectors with many degrees of freedom, we can derive the equivalence relation independent of their object systems [18–22], while the possibility of one particle acting as a quantum detector depends on its object system.

B. General case

In order to clarify the roles of the free part H_S+H_A, especially the masses of S and A, we must treat the total Hamiltonian H. We will show that the position measurement of S can be performed approximately only if the object mass m is very large, i.e., in this case the statistical operator ρ approaches its mixed-state part ρ_0 under certain conditions. This also occurs at very small time, immediately after the beginning of the interaction. However, the final state in the large-time limit will recover the interferences, and $\rho\not\sim\rho_0$.

The initial state of S and A is the same as that in Sec. III A: $\Psi(0)=|\varphi\rangle|0\rangle|0\rangle|0\rangle$, where $|\varphi\rangle$ is given by Eq. (3.2).

To obtain the state of the total system $S+A$ at time t after the interaction, we must decompose the time evolution operator as follows:

$$\begin{aligned}e^{-(i/\hbar)Ht}&=e^{-(i/\hbar)H_St}e^{-(i/\hbar)\tilde{H}_At}e^{-(i/\hbar)(gt^2/2m)\hat{\mathbf{p}}\cdot\hat{\mathbf{P}}}e^{-(i/\hbar)gt\hat{\mathbf{q}}\cdot\hat{\mathbf{P}}}\\&\equiv e^{-(i/\hbar)H_St}U_xU_yU_z\ ,\end{aligned}\quad (3.6)$$

where

$$\begin{aligned}\tilde{H}_A&=\left[\frac{1}{2M}+\frac{g^2t^2}{6m}\right]\hat{\mathbf{P}}^2\ ,\\U_i&=e^{-(i/\hbar)(t/2M+g^2t^3/6m)\hat{P}_i^2}\\&\quad\times e^{-(i/\hbar)(gt^2/2m)\hat{p}_i\hat{P}_i}e^{-(i/\hbar)gt\hat{q}_i\hat{P}_i}\ .\end{aligned}\quad (3.7)$$

Suppose that $|\tilde{q}_{iz}\rangle$ are also localized in the momentum space, then we find

$$\begin{aligned}e^{-(i/\hbar)(gt^2/2m)\hat{p}_z\hat{P}_z}e^{-(i/\hbar)gt\hat{q}_z\hat{P}_z}|\tilde{q}_{iz}\rangle&\simeq\int dp_{iz}|p_{iz}\rangle\langle p_{iz}|\tilde{q}_{iz}\rangle e^{-(i/\hbar)[gtq_{iz}+gt^2p_{iz}/2m]\hat{P}_z}\\&\simeq\int_{V_i}dp_{iz}|p_{iz}\rangle\langle p_{iz}|\tilde{q}_{iz}\rangle e^{-(i/\hbar)[gtq_{iz}+gt^2p_{iz}/2m]\hat{P}_z}\\&\simeq\int_{V_i}dp_{iz}|p_{iz}\rangle\langle p_{iz}|\tilde{q}_{iz}\rangle e^{-(i/\hbar)[gtq_{iz}]\hat{P}_z}\\&\simeq|\tilde{q}_{iz}\rangle e^{-(i/\hbar)[gtq_{iz}]\hat{P}_z}\ ,\end{aligned}\quad (3.8)$$

when m is very large such that $m\gg p_{i0}t/2q_{iz}$, where V_i is a finite nonzero region of $\langle p_{iz}|\tilde{q}_{iz}\rangle$ and p_{i0} the maximal value of the region V_i. Note here that q_{iz} is the order of the expectation value of $\hat{q}$ at $t=0$, and that it is constant.

Using the above equations, the state at time t can be obtained,

$$\begin{aligned}\Psi(t)&=e^{-(i/\hbar)Ht}\Psi(0)\\&\simeq\sum_{i=1}^{2}c_ie^{-(i/\hbar)H_St}U_xU_y|\tilde{q}_{iz}\rangle|\varphi_i\rangle|0,0\rangle\\&\quad\times e^{-(i/\hbar)[t/2M+g^2t^3/6m]\hat{P}_z^2}e^{-(i/\hbar)gtq_{iz}\hat{P}_z}|0\rangle\ .\end{aligned}\quad (3.9)$$

Equation (3.9) contains the Glauber coherent states: $|gtq_{iz}\sqrt{s/2\hbar}\rangle$. Thus $\Psi(t)$ has "coherent-correlation" structure.

In order to treat the action of an operator such as $\exp[-(i/\hbar)(t/2M+g^2t^3/6m)\hat{P}_z^2]$, we can make use of techniques developed in squeezed states [29–34]. Let us introduce new operators given by the following canonical transformation:

$$b=S(\omega)a_zS^{-1}(\omega)\ ,\quad b^\dagger=S(\omega)a_z^\dagger S^{-1}(\omega)\ , \tag{3.10}$$

where $S(\omega)=\exp[-(i/\hbar)\omega\hat{P}_z^2]$ $(\omega\in\mathbb{R})$, which may also be written as

$$\begin{bmatrix} b \\ b^\dagger \end{bmatrix} = \begin{bmatrix} \mu & \nu \\ \nu^* & \mu^* \end{bmatrix} \begin{bmatrix} a_z \\ a_z^\dagger \end{bmatrix}, \tag{3.11}$$

where $\mu=1+i\omega s$ and $\nu=-i\omega s$ with $|\mu|^2-|\nu|^2=1$. Note that the transformation matrix in Eq. (3.11) belongs to SU(1,1) and $[b,b^\dagger]=1$. In a similar way to sqeezed states, we define a new state (which is not a squeezed state)

$$|\alpha;\mu\nu\rangle=D(\alpha)S(\mu\nu)|0\rangle=S(\mu\nu)D(\beta)|0\rangle\ , \tag{3.12}$$

where $S(\mu\nu)\equiv S(\omega)$ and $\beta=\mu\alpha+\nu\alpha^*$. The matrix elements with respect to such states are given by

$$\langle\alpha;\mu\nu|\gamma;\rho\sigma\rangle=\frac{1}{\sqrt{h}}\exp\left[-\tfrac{1}{2}|\beta|^2-\tfrac{1}{2}|\delta|^2+\frac{1}{h}\beta^*\delta-\frac{f}{2h}\beta^{*2}+\frac{f^*}{2h}\delta^2\right], \tag{3.13a}$$

$$\langle\alpha;\mu\nu|B(b,b^\dagger)|\gamma;\rho\sigma\rangle=B\left[\frac{\beta}{2}+\frac{\partial}{\partial\beta^*},\beta^*\right]\langle\alpha;\mu\nu|\gamma;\rho\sigma\rangle\ , \tag{3.13b}$$

$$\langle\alpha;\mu\nu|\gamma;\mu\nu\rangle=\langle\beta|\delta\rangle=\exp(-\tfrac{1}{2}|\beta|^2-\tfrac{1}{2}|\delta|^2+\beta^*\delta)\ , \tag{3.13c}$$

where $\delta=\rho\gamma+\sigma\gamma^*$, $h=\mu^*\rho-\nu^*\sigma$, $f=\mu\sigma-\nu\rho$, and B is a polynomial in b and $b^\dagger$. The proof of Eq. (3.13) is made in a similar way to Yuen [30].

Using the above new states, the final state $\Psi(t)$ becomes

$$\Psi(t)\simeq\sum_i c_i e^{-(i/\hbar)H_St}U_xU_y|\tilde{q}_{iz}\rangle|\varphi_i\rangle|0,0\rangle|\alpha_i;\mu,\nu\rangle$$
$$\equiv\sum_i c_i e^{-(i/\hbar)H_St}U_xU_y|\tilde{q}_{iz},\varphi_i;0,0\rangle|\alpha_i;\mu\nu\rangle\ , \tag{3.14}$$

where

$$\omega=\frac{t}{2M}+\frac{g^2t^3}{6m}\ ,\quad \alpha_i=gtq_{iz}\sqrt{s/2\hbar}\ . \tag{3.15}$$

In deriving Eq. (3.14), we have used the equality $S(\omega_1)D(\alpha_i)=D(\alpha_i)S(\omega_1)$ or $\alpha_i=\beta_i$.

The statistical operator ρ corresponding to the state $\Psi(t)$ is written as

$$\rho=\rho_0+[c_1^*c_2e^{-(i/\hbar)H_St}U_xU_y|\tilde{q}_{2z},\varphi_2;0,0\rangle$$
$$\times\langle\tilde{q}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger e^{(i/\hbar)H_St}$$
$$\otimes|\alpha_2;\mu,\nu\rangle\langle\alpha_1;\mu,\nu|+\text{H.c.}]\ , \tag{3.16}$$

where ρ_0 is the mixed-state part of ρ. Any classical operator can be written as

$$\hat{A}(\hat{Q}_i,\hat{P}_j)\equiv\hat{A}_{xy}(\hat{Q}_x\hat{Q}_y,\hat{P}_x,\hat{P}_y)A_z(\hat{Q}_z,\hat{P}_z)\ .$$

Assuming that $\hat{A}_z(\hat{Q}_z,\hat{P}_z)\equiv B(b,b^\dagger)$ $[b\equiv(1+i\omega_1s)a_z-i\omega_1sa_z^\dagger]$, we find

$$\mathrm{Tr}[(\rho-\rho_0)(\hat{O}\otimes\hat{A})]=c_1^*c_2\langle\tilde{q}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger e^{(i/\hbar)H_St}\hat{O}\otimes\hat{A}_{xy}e^{-(i/\hbar)H_St}U_xU_y|\tilde{q}_{2z},\varphi_2;0,0\rangle$$
$$\times\langle\alpha_1;\mu,\nu|B(b,b^\dagger)|\alpha_2;\mu,\nu\rangle+\text{c.c.} \tag{3.17}$$

Recall that $B(b,b^\dagger)$ is a polynomial in b and $b^\dagger$. Then, using Eq. (3.13), we get

$$\langle\alpha_1;\mu,\nu|B(b,b^\dagger)|\alpha_2;\mu,\nu\rangle$$
$$=B\left[\frac{\beta_1}{2}+\frac{\partial}{\partial\beta_1^*},\beta_1^*\right]\langle\alpha_1;\mu,\nu|\alpha_2;\mu,\nu\rangle$$
$$=F(\alpha_1,\alpha_2)\exp\left[-\frac{g^2t^2s(q_{1z}-q_{2z})^2}{4\hbar}\right], \tag{3.18}$$

where $F(\alpha_1,\alpha_2)$ is a polynomial in α_i.

It follows from Eqs. (3.8), (3.17), and (3.18) that the statistical operator ρ approaches its mixed-state part ρ_0 if the mass m of S is very large such that

$$m\gg\frac{p_{i0}t}{2q_{iz}}\ ,\quad g^2(q_{1z}-q_{2z})^2st^2\gg4\hbar\ , \tag{3.19}$$

which gives the condition for the time

$$\frac{2mq_{iz}}{p_{i0}}\gg t\gg\left[\frac{4\hbar}{g^2s(q_{1z}-q_{2z})^2}\right]^{1/2}. \tag{3.20}$$

The free Hamiltonian H_A of the detector has nothing to do with this approximation. The first condition in Eq. (3.19) is requisite for the position measurement, and the second one is needed for $\rho\simeq\rho_0$. It should be noted that the second condition is the one for impulsive approximation, for which the condition $H_A\ll H'$ is not needed. We cannot have $t\to\infty$ because of the first condition. Hence we arrive at our conclusion: the statistical operator ρ is not equivalent to its mixed-state part ρ_0. We cannot measure the position of S using a one-particle detector in the von Neumann model. This is because the free Hamiltonian H_S of S disturbs the position measurement.

IV. MOMENTUM MEASUREMENT IN THE VON NEUMANN MODEL

So far the von Neumann model [the Hamiltonian of which is given by Eq. (3.1)] has been used to measure the position of the object system S because the interaction with the detector A contains position operator $\hat{\mathbf{q}}$ of S. However under the condition of very small object mass m, we can discuss the momentum measurement and show that the statistical operator ρ approaches its mixed-state part ρ_0 as the time t becomes very large, i.e., we have $\rho \sim \rho_0$. This is because $[H_S, H'] \neq 0$. We will elaborate this in this section.

In a similar way to Sec. III, a state of S may reduce to

$$|\varphi\rangle = c_1 |\tilde{p}_{1z}\rangle |\varphi_1\rangle + c_2 |\tilde{p}_{2z}\rangle |\varphi_2\rangle \ , \tag{4.1}$$

where $|\tilde{p}_{iz}\rangle$ are well-localized wave packets in the momentum space so that $|\tilde{p}_{iz}\rangle$ may almost be eigenvectors of $\hat{p}_z$: $\hat{p}_z|\tilde{p}_{iz}\rangle \simeq p_{iz}|\tilde{p}_{iz}\rangle$. Using a Stern-Gerlach-type device, we can always change a wave packet of S to a spectral-decomposed one in which the expectation value of $\hat{p}_z$ is much larger than its uncertainty. The initial state of the detector is to be $|\text{in}\rangle \equiv |0,0,0\rangle \equiv |0\rangle|0\rangle|0\rangle$. Thus the initial state of $S+A$ (at $t=0$) becomes $\Psi(0) = |\varphi\rangle|0\rangle|0\rangle|0\rangle$.

The decomposition of the time evolution operator now becomes

$$\begin{aligned} e^{-(i/\hbar)Ht} &= e^{-(i/\hbar)H_S t} e^{-(i/\hbar)\tilde{H}_A t} e^{-(i\hbar)gt\hat{\mathbf{q}}\cdot\hat{\mathbf{P}}} \\ &\quad \times e^{-(i/\hbar)(gt^2/2m)\hat{\mathbf{p}}\cdot\hat{\mathbf{P}}} \\ &\equiv e^{-(i/\hbar)H_S t} U_x U_y U_z \ , \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} \tilde{H}_A &= \left[\frac{1}{2M} - \frac{g^2t^2}{3m} \right] \hat{\mathbf{P}}^2 \ , \\ U_i &= e^{-(i/\hbar)[t/2M - g^2t^3/3m]\hat{P}_i^2} e^{-(i/\hbar)gt\hat{q}_i\hat{P}_i} \\ &\quad \times e^{-(i/\hbar)(gt^2/2m)\hat{p}_i\hat{P}_i} \ . \end{aligned} \tag{4.3}$$

Suppose that $|\tilde{p}_{iz}\rangle$ is localized in the configuration space, then we find

$$\begin{aligned} e^{-(i/\hbar)gt\hat{q}_z\hat{P}_z} e^{-(i/\hbar)(gt^2/2m)\hat{p}_z\hat{P}_z} |\tilde{p}_{iz}\rangle &\simeq \int dq_{iz} |q_{iz}\rangle\langle q_{iz}|\tilde{p}_{iz}\rangle e^{-(i/\hbar)(gtq_{iz}+gt^2p_{iz}/2m)\hat{P}_z} \\ &\simeq \int_{V_i} dq_{iz} |q_{iz}\rangle\langle q_{iz}|\tilde{p}_{iz}\rangle e^{-(i/\hbar)(gtq_{iz}+gt^2p_{iz}/2m)\hat{P}_z} \\ &\simeq \int_{V_i} dq_{iz} |q_{iz}\rangle\langle q_{iz}|\tilde{p}_{iz}\rangle e^{-(i/\hbar)(gt^2p_{iz}/2m)\hat{P}_z} \\ &\simeq |\tilde{p}_{iz}\rangle e^{-(i/\hbar)(gt^2p_{iz}/2m)\hat{P}_z} \ , \end{aligned} \tag{4.4}$$

when m is very small such that $m < < p_{iz}t/2q_{i0}$, where V_i is a finite nonzero region of $\langle q_{iz}|\tilde{p}_{iz}\rangle$. q_{i0} is the maximal value of the region V_i, which is constant

The state at time t is given by

$$\begin{aligned} \Psi(t) &= e^{-(i/\hbar)Ht}\Psi(0) \\ &\simeq \sum_{i=1}^{2} c_i e^{-(i/\hbar)H_S t} U_x U_y |\tilde{p}_{iz}\rangle|\varphi_i\rangle|0,0\rangle e^{-(i/\hbar)(t/2M - g^2t^3/4m)\hat{P}_z^2} e^{-(i/\hbar)(gt^2p_{iz}/2m)\hat{P}_z}|0\rangle \ . \end{aligned} \tag{4.5}$$

The right-hand side of Eq. (4.5) contains the Glauber coherent states: $|(gt^2p_{iz}/2m)\sqrt{s/2\hbar}\rangle$. Here let us introduce new operators b, $b^\dagger$ and new states $|\alpha;\mu\nu\rangle$ given by Eqs. (3.10)–(3.12) to treat the action of $S(\omega) = \exp[-(i/\hbar)\omega\hat{P}_z^2]$. Then the state $\Psi(t)$ is written in terms of the new states

$$\begin{aligned} \Psi(t) &\simeq \sum_i c_i e^{-(i/\hbar)H_S t} U_x U_y |\tilde{p}_{iz}\rangle|\varphi_i\rangle|0,0\rangle|\alpha_i;\mu,\nu\rangle \\ &\equiv \sum_i c_i e^{-(i/\hbar)H_S t} U_x U_y |\tilde{p}_{iz},\varphi_i;0,0\rangle|\alpha_i;\mu,\nu\rangle \ , \end{aligned} \tag{4.6}$$

where

$$\omega = \frac{t}{2M} - \frac{g^2t^3}{3m} \ , \quad \alpha_i = \frac{gt^2p_{iz}}{2m}\sqrt{s/2\hbar} \ . \tag{4.7}$$

The statistical operator ρ corresponding to the state $\Psi(t)$ is written as

$$\rho = \rho_0 + (c_1^* c_2 e^{-(i/\hbar)H_S t} U_x U_y |\tilde{p}_{2z},\varphi_2;0,0\rangle\langle\tilde{p}_{1z},\varphi_1;0,0| U_y^\dagger U_x^\dagger e^{(i/\hbar)H_S t} \otimes |\alpha_2;\mu,\nu\rangle\langle\alpha_1;\mu,\nu] + \text{H.c.}) \ , \tag{4.8}$$

where ρ_0 is the mixed-state part of ρ. In this case, Eq. (3.17) now becomes

$$\mathrm{Tr}[(\rho-\rho_0)(\hat{O}\otimes\hat{A})]=c_1^*c_2\langle\tilde{p}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger e^{(i/\hbar)H_S t}\hat{O}\otimes\hat{A}_{xy}e^{-(i/\hbar)H_S t}U_x U_y|\tilde{p}_{2z},\varphi_2;0,0\rangle \times\langle\alpha_1;\mu,\nu|B(b,b^\dagger)|\alpha_2;\mu,\nu\rangle+\mathrm{c.c}\ . \tag{4.9}$$

Using Eq. (3.13), we get

$$\langle\alpha_1;\mu,\nu|B(b,b^\dagger)|\alpha_2;\mu,\nu\rangle =B\left[\frac{\beta_1}{2}+\frac{\partial}{\partial\beta_1^*},\beta_1^*\right]\langle\alpha_1;\mu,\nu|\alpha_2;\mu,\nu\rangle =F(\alpha_1,\alpha_2)\exp\left[-\frac{g^2t^4(p_{1z}-p_{2z})^2s}{16m^2\hbar}\right], \tag{4.10}$$

where $F(\alpha_1,\alpha_2)$ is a polynomial in α_i.

Equations (4.4), (4.9), and (4.10) show that the statistical operator ρ is very close to ρ_0 if the mass of S is quite small such that

$$m\ll\frac{p_{iz}t}{2q_{i0}},\quad m^2\ll\frac{g^2t^4s(p_{1z}-p_{2z})^2}{16\hbar}\ . \tag{4.11}$$

That is, we can assert that in the situation (4.11), the momentum of S can be measured in the von Neumann model. The first condition in Eq. (4.11) is needed for the momentum measurement, while the second one gives us $\rho\simeq\rho_0$. Since the two conditions are satisfied for the large-time limit, ρ is equivalent to its mixed part. Although the von Neumann model was presented to measure the position of S, it is performed very approximately. On the other hand, the momentum of S is measured nicely. The mass of the detector M does not contribute to the condition (4.11).

V. MOMENTUM MEASUREMENT IN THE VARIANT VON NEUMANN MODEL

In the von Neumann model investigated in the previous sections, the mass M of the one-particle detector is unimportant in the measuring process. As easily seen from Eq. (3.1), the free Hamiltonian of the detector H_A commutes with the interaction H', and as a result the detector mass M does not influence the process. We thus consider in this section a simple variant model with $[H_A,H']\neq 0$, and investigate the effect of the mass of the detector.

The total Hamiltonian we now consider is $H=H_S+H_A+H'$, where H_S and H_A are as defined previously, and the interaction H' between S and A is given by

$$H'=g\hat{\mathbf{p}}\cdot\hat{\mathbf{Q}}\ , \tag{5.1}$$

which commutes with H_S, not H_A. The initial state of the total system $S+A$ is the same as that in Sec. IV: $\Psi(0)=|\varphi\rangle|0\rangle|0\rangle|0\rangle$, where $|\varphi\rangle$ is given by Eq. (4.1). To obtain the final state we must decompose the time evolution operator as follows:

$$e^{-(i/\hbar)Ht}=e^{-(i/\hbar)\tilde{H}_S t}e^{-(i/\hbar)H_A t}e^{-(i/\hbar)[(gt^2/2M)\hat{\mathbf{p}}\cdot\hat{\mathbf{P}}+gt\hat{\mathbf{p}}\cdot\hat{\mathbf{Q}}]} \equiv e^{-(i/\hbar)\tilde{H}_S t}U_xU_yU_z\ , \tag{5.2}$$

where

$$\tilde{H}_S=\left[\frac{1}{2m}-\frac{g^2t^2}{12M}\right]\hat{\mathbf{p}}^2\ ,\quad U_i=e^{-(i/\hbar)(t/2M)\hat{P}_i^2}e^{-(i/\hbar)[(gt^2/2M)\hat{p}_i\hat{P}_i+gt\hat{p}_i\hat{Q}_i]}\ . \tag{5.3}$$

Using the above decomposition, the state at time t is given by

$$\Psi(t)=e^{-(i/\hbar)Ht}\Psi(0) \simeq\sum_{i=1}^{2}c_ie^{-(i/\hbar)\tilde{H}_S t}U_xU_y|\tilde{p}_{iz}\rangle|\varphi_i\rangle|0,0\rangle e^{-(i/\hbar)(t/2M)\hat{P}_z^2}e^{-(i/\hbar)[(gt^2p_{iz}/2M)\hat{P}_z+gtp_{iz}\hat{Q}_z]}|0\rangle\ , \tag{5.4}$$

which contains the Glauber coherent states:

$$e^{-(i/\hbar)[(gt^2p_{iz}/2M)\hat{P}_z+gtp_{iz}\hat{Q}_z]}|0\rangle=|gtp_{iz}(ts/2M-i)/\sqrt{2\hbar s}\,\rangle\ . \tag{5.5}$$

Using the operators $b,b^\dagger$ and the states $|\alpha;\mu\nu\rangle$ defined by Eqs. (3.10)–(3.12), the state $\Psi(t)$ is written in the form

$$\Psi(t)\simeq\sum_i c_ie^{-(i/\hbar)\tilde{H}_S t}U_xU_y|\tilde{p}_{iz}\rangle|\varphi_i\rangle|0,0\rangle|\alpha_i;\mu,\nu\rangle \equiv\sum_i c_ie^{-(i/\hbar)\tilde{H}_S t}U_xU_y|\tilde{p}_{iz},\varphi_i;0,0\rangle|\alpha_i;\mu,\nu\rangle\ , \tag{5.6}$$

where

$$\omega=\frac{t}{2M}\ ,\quad \alpha_i=-\frac{gtp_{iz}}{\sqrt{2\hbar s}}\left[\frac{ts}{2M}+i\right]. \tag{5.7}$$

In deriving Eqs. (5.6) and (5.7), we have used $S(\mu\nu)D(\beta_i)=D(\alpha_i)S(\mu\nu)$ with $\beta_i=\mu\alpha_i+\nu\alpha_i^*$.

The statistical operator ρ corresponding to the state $\Psi(t)$ is written as

$$\rho=\rho_0+[c_1^*c_2 e^{-(i/\hbar)\tilde{H}_S t}U_x U_y|\tilde{p}_{2z},\varphi_2;0,0\rangle\langle\tilde{p}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger e^{(i/\hbar)\tilde{H}_S t}\otimes|\alpha_2;\mu,\nu\rangle\langle\alpha_1;\mu,\nu|+\text{H.c.}] , \tag{5.8}$$

where ρ_0 is the mixed-state part of ρ. Equation (3.17) in this case becomes

$$\begin{aligned}\text{Tr}[(\rho-\rho_0)(\hat{O}\otimes\hat{A})]=c_1^*c_2\langle\tilde{p}_{1z},\varphi_1;0,0|U_y^\dagger U_x^\dagger e^{(i/\hbar)\tilde{H}_S t}\hat{O}\otimes\hat{A}_{xy}e^{-(i/\hbar)\tilde{H}_S t}U_x U_y|\tilde{p}_{2z},\varphi_2;0,0\rangle \\ \times\langle\alpha_1;\mu,\nu|B(b,b^\dagger)|\alpha_2;\mu,\nu\rangle+\text{c.c} .\end{aligned} \tag{5.9}$$

Using

$$\langle\alpha_1;\mu,\nu|B(b,b^\dagger)|\alpha_2;\mu,\nu\rangle=B\left[\frac{\beta_1}{2}+\frac{\partial}{\partial\beta_1^*},\beta_1^*\right]\langle\beta_1|\beta_2\rangle$$
$$=F(\beta_1^*,\beta_2)\langle\beta_1|\beta_2\rangle , \tag{5.10a}$$

$$\langle\beta_1|\beta_2\rangle=\exp\left[-\frac{g^2t^2(p_{1z}-p_{2z})^2}{4\hbar s}\frac{4M^2+s^2t^2}{4M^2}\right] , \tag{5.10b}$$

where $F(\beta_1^*,\beta_2)$ is a polynomial in β_1^* and β_2, the statistical operator ρ is equivalent to its mixed-state part ρ_0 when

$$\frac{g^2t^2(p_{1z}-p_{2z})^2}{4\hbar s}\frac{4M^2+t^2s^2}{4M^2}\gg 1 . \tag{5.11}$$

Differing from the position measurement in the von Neumann model, this condition can be satisfied for all values of mass M of the detector except zero. In the large-M limit ($2M\gg ts$), the above condition reduces to $g^2t^2(p_{1z}-p_{2z})^2\gg 4\hbar s$, which is the condition for $\rho\sim\rho_0$ in the impulsive measurement. In the small-M limit (or large-t limit) the condition (5.11) reduces to

$$\frac{g^2t^2(p_{1z}-p_{2z})^2}{4\hbar s}\frac{t^2s^2}{4M^2}\gg 1 . \tag{5.12}$$

Thus we have $\rho\sim\rho_0$. Also in this model, the mass of the detector does not play an essential role in the measuring process.

VI. CONCLUSIONS

Using the von Neumann and its variant models, we have investigated the mass effects. The detector A we have considered consists of a single particle with three degrees of freedom, which we have called a one-particle detector.

The von Neumann model with the Hamiltonian given by Eq. (3.1) has been used for the position measurement of an object system. In the impulsive approximation, $H_S,H_A\gg H'$, the position measurement was neatly performed, that is, we can prove the statistical operator ρ is equivalent to its mixed-state part ρ_0 with respect to any object observable $\hat{O}$ and any polynomial $\hat{A}$ in detector operators $\hat{Q}_i$ and $\hat{P}_j$. The condition for the impulsive approximation is a very large mass satisfying Eq. (3.20). The mass M of the detector need not be very large; it does not affect the measuring process. This is the physical meaning of the impulsive approximation.

As discussed in Sec. III B, we cannot prove $\rho\sim\rho_0$ in the general case of the position measurement. However, under the conditions given by Eq. (3.20), the statistical operator ρ is quite close to its mixed-state part. Since this condition breaks down as the time $t\to\infty$, the appearance of the quantum interference between two coherent states depends on the time. Consequently, strictly speaking, the object position can only be measured very approximately in the von Neumann model. This results from the noncommutativity between H_S and H'; the free part H_S disturbs the position measurement. Note that the position operator $\hat{\mathbf{q}}$ is not a quantum-nondemolition (QND) observable because $[\hat{q}_i,H_S]\neq 0$ [40–43].

Next we have studied the momentum measurement in the von Neumann model, which can be performed almost completely if the mass m of S is quite small, satisfying two conditions (4.11). Under these conditions, the statistical operator ρ approaches ρ_0. Note here that the time t satisfying the conditions (4.11) can become very large (or $t\to\infty$). The possibility of the momentum measurement in the von Neumann model also comes from noncommutativity between H_S and H'; the free part H_S again disturbs the measurement. The momentum operator is not a QND observable, and the interaction H' is not back-action-evading (BAE) type [40–43]. Nevertheless, the operator ρ can be very close to ρ_0 under the condition (4.11), and get $\rho\sim\rho_0$.

In the last model with the interaction $H'=g\hat{\mathbf{p}}\cdot\hat{\mathbf{Q}}$, the momentum measurement can be performed almost completely. We have shown in Sec. V that the operator ρ is equivalent to ρ_0. Although the detector Hamiltonian H_A does not commute with the interaction H' ($[H_A,H']\neq 0$), it does not disturb the measuring process at all. Since $[\hat{p}_i,H'],[\hat{p}_i,H_S]=0$, the interaction H' is a BAE type and $\hat{\mathbf{p}}$ a QND observable.

The above four cases we have investigated show that the mass of the detector M is not important in the measuring process. Hence we can consider (especially in the second and the fourth models) the paradoxical case where the object mass m is very large whereas the detector mass M is quite small; in this case also, the essentials of the situation remain unchanged. Moreover, if each state of an object system S is partly described by a "coherentlike" state, then there may be a possibility of a model without a detector.

The quantum-measuring process is said to be irreversible, a derivation of this being possible for macroscopic bodies. In the case of a one-particle detector, is it possible to derive the irreversibility of the measuring process? Such a derivation may result from considerations of quantum chaos [44], for example.

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