Theory of ions emitted from a plasma by relativistic self-focusing of laser beams

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Fast ions are emitted from the focus of a high-intensity laser beam irradiating a plasma. The self-focusing of the laser beam is caused by the dependence of the index of refraction on the relativistic mass of the electrons, which again depends on the electric-field strength. The ions are accelerated out of the laser focus due to the combined action of nonlinear forces and double layers. Results for the maximal energy and angular distribution of the ions are presented and compared with experimental data.

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Self-focusing of electromagnetic waves in a medium can arise if the index of refraction depends on the electric-field strength. In this case the effective radius of the laser beam decreases along the beam direction until it reaches a value of about a half wavelength of the incoming light. The theory of self-focusing was treated, for example, in the articles of Akhmanov, Sukhorukov, and Khokhlov [1], Svelto [2], Spatschek [3], and Hora [4].

If a very intense laser beam (e.g., 10^{14} W/cm^2 for a Nd-glass laser with $\lambda = 1.06$ μm) penetrates a plasma, relativistic self-focusing of the beam arises because of the relativistic velocity of the electrons which enters the relativistic mass of the electrons in the plasma frequency [5]

$$\omega_p^2 = \frac{4\pi e^2 n_e}{\gamma m_e} . \tag{1}$$

Here, m_e is the rest mass of the electron and n_e the electron density. The relativistic Lorentz factor γ depends on the electric-field strength E [5]. For example, the factor γ is given for a circularly polarized wave of frequency ω as

$$\gamma = \left[1 + \left[\frac{e}{m_e \omega c} \right]^2 E^2 \right]^{1/2} . \tag{2}$$

The electric field of the very intense laser beam can be calculated by solving the nonlinear wave equation $\Delta \mathbf{E} + k^2 \mathbf{E} = 0$, derived from the Maxwell equations in certain approximations, where $k^2 = k^2(E) = n^2\omega^2/c^2$ and n is the refractive index of a noncollisional plasma depending on the plasma frequency given in Eq. (1):

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} . \tag{3}$$

Analytical calculations of the self-focusing of a laser field penetrating a plasma were carried out by Häuser, Scheid, and Hora [6]. In these calculations we neglected the effects of collisions. Collisions were only found to have an influence near the low-intensity threshold and when the electron density is very close to the critical density [4,5]. Since we assume that the threshold has been exceeded in the experiments considered in this Brief Report, we simply neglect the collisional implications in our treatment.

An important consequence of relativistic self-focusing is the increase of the energy density of the laser field up to extremely high values in the region around the focal point. That means that nonlinear forces and double layers are developing which drive electrons and ions out of the focus region [4,7].

The ions emitted from the focus can be accelerated up to energies of 10 MeV/nucleon depending on the laser intensity [8,9]. Therefore this effect can in principle be applied for the acceleration of particles by intense laser beams and has already been discussed under the name "laser focus accelerator" by Sessler [10], Hora *et al.* [11], and Clark *et al.* [12]. Laser-target experiments are reviewed by Gitomer *et al.* [13].

In this Brief Report we describe the theory of ions emitted out of the region of the laser focus. We derive their energy and angular distribution. The theory is proved by comparing the calculated energies and angular distributions with energies and angular distributions of fast ions observed in several laser-target experiments [14–19].

High-energy ions are accelerated from the laser focus. Two main effects are important for the acceleration. The first effect is the nonlinear force; the second effect is the appearance of double layers, which are caused by the violation of the charge neutrality in the plasma.

Nonlinear forces arise if a laser field acts on the electrons with a strong gradient of the field strength. The electrons follow the laser field and carry out a fast quivering motion. If the electric-field strength varies along the paths of the electrons, the electrons are drawn with a drift velocity out of the region of the highest field strength. The nonlinear force and its effects were discussed, for example, by Askar'yan [20], Kane and Hora [21], and Hora [4]. For circularly polarized light the nonlinear force is obtained as

$$\mathbf{F}_{\rm NL} = -\tfrac{1}{2} \frac{e^2}{m_e \gamma \omega^2} \nabla E^2 = -\nabla(m_e c^2 \gamma) . \tag{4}$$

Obviously the electrons are accelerated out of the region of the laser focus, because the electric-field strength reaches its maximal value at the focus. Now let us consider the effect of the electrons on the ions. Since the ions have larger masses, their quivering motion is negligible and, therefore, a nonlinear force is not acting on the ions. While the electrons are emitted out of the laser focus, the ions remain. Therefore a double layer develops around the laser focus. As a consequence of the violation of the charge neutrality a strong electric field arises between the electrons and ions which accelerates the ions and deaccelerates the electrons. In total a drift motion of the electrons and ions out of the region of the laser focus can be observed.

In order to describe this motion in detail, we assume that the ions are fully ionized. This is a good approximation, especially for light ions ($Z \lesssim 20$).

In experiments also ions are detected with less ionization. These ions are not considered in our present model since we are mainly interested in strong anisotropies of the ion emission which mainly result from the fully ionized ions. Less ionization leads to more isotropic angular distributions and would exceed the possibilities of our present model.

The experiments mentioned were carried out with laser beams in linear polarization. But the ion energies were not found to depend on the angle around the laser beam axis. Therefore we are permitted to describe the laser beam by a circularly polarized field in the same way as we have done in Ref. [6]. This assumption simplifies the further calculations. Using the nonlinear force (4) we can write the following equations of motion for the ions:

$$m_i \frac{d}{dt}\mathbf{v}_i = Z\mathbf{F}_{\rm NL} = -\nabla V , \tag{5}$$

with the potential of the ions

$$V = Zm_e c^2(\gamma - 1) , \tag{6}$$

which is time independent in the case of a circularly polarized laser wave. Assuming that the kinetic energy of the ions near the focus is negligible at time $t=0$ in comparison to their potential energy, we find that the kinetic energy of the ions after their emission from the laser focus is given by

$$T(t \to \infty) = V(\mathbf{r}(t=0)) . \tag{7}$$

Here, $\mathbf{r}(t=0)$ is the starting point of an individual ion.

In several experiments with very intense laser irradiation [14–19] the maximal kinetic energies of the emitted ions were measured. For comparison with these data we have calculated the largest possible value of the potential in Eq. (6), i.e., the largest value $\gamma_{\rm max}$ of γ. Obviously, the value of $\gamma_{\rm max}$ is obtained for the field strength in the center of the laser focus and given by

$$\gamma_{\rm max}^2 = 1 + \left[\frac{eE_{\rm max}}{m_e \omega c} \frac{R_0}{R_f} \right]^2 = 1 + \frac{P}{P_{\rm rel}} . \tag{8}$$

Here, $E_{\rm max}$ is the maximal field strength of the laser beam entering the plasma with an initial radius R_0 and a radius R_f in the focus region. R_f is assumed to be a half wavelength of the incoming beam [4]: $R_f = \lambda/2$. P is the laser power, defined as $P = \pi R_0^2 I_0$ with the initial intensity I_0, and the relativistic laser power $P_{\rm rel}$ (in W) is

$$P_{\rm rel} = \pi R_f^2 I_{\rm rel} = m_e^2 \pi^2 c^5/(4e^2) = 2.15 \times 10^{10} . \tag{9}$$

$I_{\rm rel}$ is the so-called relativistic intensity and is given by $I_{\rm rel} = (m_e \omega)^2 c^3/(4\pi e^2)$. Combining Eqs. (7) and (8) we obtain the maximal energy of the ions as a function of the laser power and the charge number only:

$$T_{\rm max} = Zm_e c^2 \left[\left[1 + \frac{P}{P_{\rm rel}} \right]^{1/2} - 1 \right] . \tag{10}$$

$T_{\rm max}$ is independent of the wavelength, which means that the maximal ion energy does not depend on the type of laser.

Figure 1 shows $T_{\rm max}$ as a function of P for $Z=1$, 38, and 92. It is possible to compare these results with measured values of several experiments [14–19] which are presented in Table I. We see that Eq. (10) for $T_{\rm max}$ is correct to within a factor of 2 to 3. We mention that collisions of the ions with the surrounding low-temperature plasma can modify the energy distribution and can decrease the maximum kinetic energy of the ions. These effects were neglected in our model. They play a certain role if we compare calculated results with those of time-of-flight measurements. However, in the experiment of Basov *et al.* [17], the ion energies were measured around the focus before essential secondary interactions happened. This was done with a sophisticated technique of the measurement of x-ray Doppler shifts.

Also the direction of emission of the ions can be calculated by using Eq. (5) and the known laser field. This direction is important for the concept of a laser focus accelerator. If the ions are emitted in a preferential direction, it is possible to apply relativistic self-focusing of laser beams to the acceleration of ions in a new type of heavy ion accelerator [10–12].

We found that the potential around the region of the laser focus can be approximated by

$$V = V_0 - Ax^2 - Br^2 . \tag{11}$$

The constants V_0, A, and B depend on the laser type and plasma density and were obtained by Häuser [22]. The coordinate x describes the direction of the laser beam. The coordinate r is the radial coordinate perpendicular to this direction: $r = (y^2+z^2)^{1/2}$. For the circularly polarized laser wave we assume cylindrical symmetry of V around the beam direction in Eq. (11). Depending on the ratio A/B, these ellipsoids have an oblate, spherical, or prolate shape with respect to the symmetry axis. Since the force and, therefore, the trajectories of the ions stand perpendicularly to the equipotential surfaces, we conclude that for $A/B > 1$ (oblate ellipsoids) the ions are emitted mainly in the positive and negative x direction, and for $A/B < 1$ (prolate ellipsoids) they are emitted mainly perpendicularly to the x direction, that means perpendicularly to the laser beam. For $A=B$ they leave the focus region isotropically.

For intensities $I > 0.5 I_{\rm rel}$ the ratio A/B is smaller than

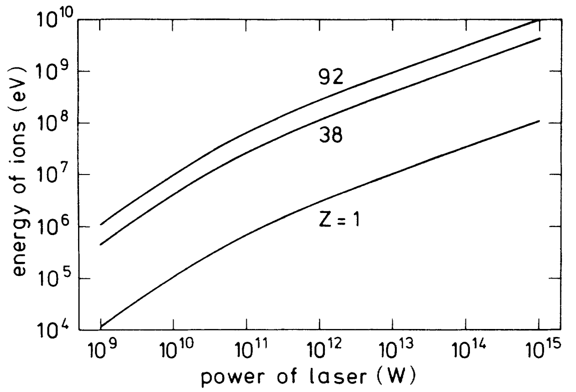

FIG. 1. The maximal energy of the emitted ions as a function of the laser power for the charge numbers of ions $Z=1$, 38, and 92.

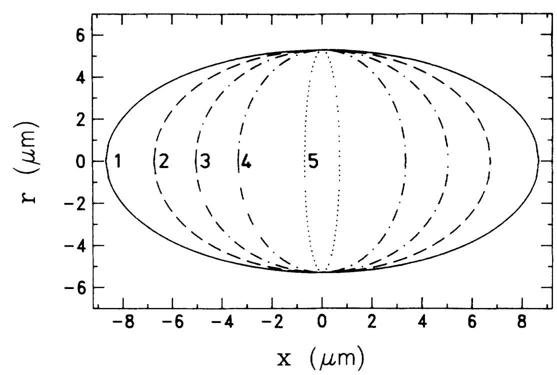

FIG. 2. Equipotential surfaces of the potential (6) near the focus for various electron densities. For the calculation we assumed a CO_2 laser ($\lambda=10.6$ μm) with an initial beam radius of $R_0=51$ μm and an intensity 2×10^{14} W cm^{-2} corresponding to a laser power of $P=1.6\times10^{10}$ W. These numbers were realized in the experiment of Ehler [16]. The numbers 1–5 on the curves indicate the electron densities $n_e=5.0$, 6.2, 7.5, 8.7, and 9.9×10^{18} cm^{-3}, respectively. For $n_e=7.2\times10^{18}$ cm^{-3} the equipotential surface is a spherical one, yielding isotropic emission of ions.

unity and the ions are mainly emitted in the direction perpendicular to the beam axis [22]. Therefore, if the intensity of the incoming beam reaches about half of the relativistic intensity, the emission of the ions is in principle unfavorable for a laser focus accelerator.

In the case of $I \ll I_{rel}$ we obtain for the ratio

$$A/B=\frac{P}{2P_{rel}}(n_{ec}/n_e-1)^{-1} \tag{12}$$

with $n_{ec}=m_0\omega^2/4\pi e^2$ as the critical electron density. According to Eq. (12) and the condition $n_e/n_{ec}<1$, all ratios of A/B are possible. Then the angular distribution of the ions is only a function of the electron density and laser power.

Figure 2 shows the equipotential surface around the laser focus for various densities in the case of a CO_2 laser with a wavelength of $\lambda=10.6$ μm. The initial radius of the laser beam is 51 μm and its intensity $I=2\times10^{14}$ W cm^{-2}, which corresponds to a laser power of $P=1.6\times10^{10}$ W. These values are taken from the experiment of Ehler [16]. For an electron density of 0.72×10^{19} cm^{-3} we obtain an isotropic emission of ions. For larger densities the emission is mainly parallel and antiparallel to the beam axis and for lower densities perpendicular. In the experiment of Ehler [16] the ions get emitted along the beam direction. Though the electron density was not measured in this experiment, we conclude on the basis of our theory that the electron density can be assumed to be between $n_e=0.72\times10^{19}$ cm^{-3} and $n_e=n_{ec}=10^{19}$ cm^{-3}, since in between these electron densities the ions are emitted along the beam direction.

In order to get the angular distribution of the ions, we solved Eq. (5). Figure 3 shows the calculated angular distribution of the ions in comparison with experimental data of Basov *et al.* [17]. In this experiment the plasma consisted of phosphorus ions and electrons with a measured density of $n_e=2\times10^{20}$ cm^{-3}. A Nd-glass laser

TABLE I. Measured and calculated maximal kinetic energies of ions with a charge number Z emitted from plasmas irradiated by CO_2 and Nd-glass lasers. The calculated energies are obtained with Eq. (10).

Reference	Laser	Power (W)	Z	T_{max}^{expt}	T_{max}^{calc}
[14]	CO_2	4×10^9	1	150 keV	46 keV
[15]	CO_2	5.65×10^9	1	60 keV	63 keV
[16]	CO_2	1.66×10^{10}	1	140 keV	170 keV
[17]	Nd	1.2×10^{10}	15	4 MeV	1.9 MeV
[18]	Nd	2.5×10^{10}	79	5.6 MeV	19 MeV
[19]	??	1.7×10^{10}	8	980 keV	1.4 MeV

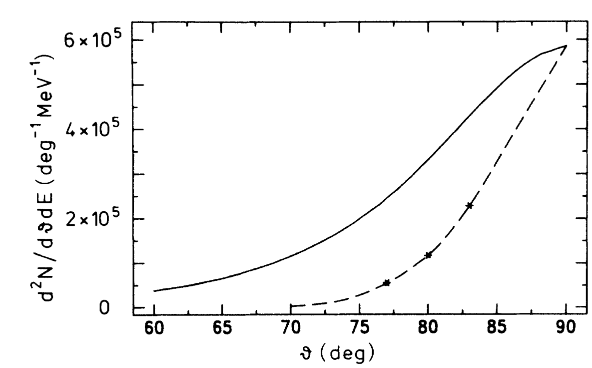

FIG. 3. Calculated and measured double-differential angular distribution of P ions emitted from the focus region of a Nd-glass laser ($\lambda=1.06$ μm). The experiment was carried out by Basov *et al.* [17] with a laser beam of an intensity 6×10^{14} W cm^{-2} and a radius $R_0=25$ μm. The experimental data, shown by stars, are connected by a dashed curve and normalized to the calculated solid curve. They correspond to ions with an energy of 2 MeV.

with $\lambda = 1.06$ μm was used with an initial intensity of 6×10^{14} W cm^{-2}; the radius of the beam was $R_0 = 25$ μm. The figure shows ions emitted with an energy of 2 MeV, which is half of the measured maximal energy of 4 MeV. Only three experimental points shown as stars were obtained by measuring photons emitted from the P ions. They are connected by a dashed curve calculated with a polynomial approach by Basov *et al.* [17]. The maximum of the experimental curve is chosen the same as that of the theoretical curve, since the experiment yielded no absolute values of the angular distribution. The measured total number of about 10^8 fast ions is roughly in agreement with the number of 2×10^7 ions obtained in our calculation.

One can see from Fig. 3 that our model gives the correct main direction of emission of ions which is perpendicular to the axis of the laser beam. The experimental curve is steeper than the calculated one, indicating that the laser focus is probably more extended in beam direction.

As far as experimental data are available, the calculated results agree satisfactorily with the experimental ones. Also two other experiments were published with measured angular distributions, namely, those of Church, Martin, and Pepin [14] and Ehler [16]. But since in these cases the electron density was not measured, we cannot compare our theory with these data. In both experiments angular distributions of ions were found with the main direction along the laser axis.

The conditions in the experiment of Basov *et al.* [17] are just at the threshold of relativistic self-focusing. Near the threshold a transition from the ordinary ponderomotive self-focusing [23] to the relativistic self-focusing happens. The relativistic self-focusing is a fast process and starts immediately after switching the laser on, whereas the ponderomotive self-focusing arises later in time when the electrons are moved out of the laser beam. So the ponderomotive self-focusing does not hinder the relativistic self-focusing, but allows an increase of the self-focusing effects. A detailed analytical study of both types of self-focusing effects near the threshold will be carried out next.

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