Third-frequency-moment sum rule and long-wavelength plasmon dispersion in layered electron liquids

Kenneth I. Golden
Department of Computer Science and Electrical Engineering, The University of Vermont, Burlington, Vermont 05405

De-xin Lu
Department of Physics, Nanjing University, Nanjing 210008, The People's Republic of China

(Received 23 August 1991)

We derive the third-frequency-moment (ω^3) sum rule and exact high-frequency dielectric response function for the layered electron liquid in zero magnetic field. The latter is examined in the $kd \to 0$ and $kd \to \infty$ limits characteristic of bulk behavior and two-dimensional (2D) single-layer behavior, respectively (k is the 2D wave number, d is the distance between any two adjacent layers). We incorporate the correlational part of the ω^3 sum rule into a mean-field-theory description of the long-wavelength dispersion of the in-phase and out-of-phase plasmon modes in the presence of very strong correlations.

PACS number(s): 52.25.Mq, 52.35.Fp, 73.20.Mf

I. INTRODUCTION

A great deal of attention has been directed to the formulation and analysis of the third-frequency-moment (ω^3) sum rule for the purpose of assessing the behavior of strongly correlated Coulomb liquids at high frequencies and in the neighborhood of the plasma frequency. While this sum rule has been derived and extensively analyzed for three-dimensional (3D) [1–5] and two-dimensional (2D) [6–8] one-component plasma (OCP) configurations, its derivation and analysis for the layered OCP configuration has yet to be carried out. This is the first goal of the present paper.

The importance of the third-frequency-moment sum-rule coefficient lies in the fact that it is the lowest-order moment that exhibits correlational effects. In strong-coupling regimes characteristic of the crystalline or supercooled liquid phases of the OCP, it has been shown [9–12] that the correlational contributions to the dispersion of the 2D and 3D plasmon modes are identical to the correlational parts of their companion ω^3 sum rules. The infinite superlattice ($N_L \to \infty$), or layered OCP configuration, which is intermediate between the 2D and 3D OCP configurations, surely must exhibit this same feature—a feature that we will exploit in describing the plasmon dispersion of the layered OCP at strong coupling. This is the second goal of the present paper.

The plan of the paper is as follows. In Sec. II, starting from the fluctuation-dissipation relation for the layered OCP, we establish the third-frequency-moment sum rule and exact high-frequency expansion for the dielectric-response function in zero magnetic field. The latter is then examined in the $kd \to 0$ and $kd \to \infty$ limits characteristic of 3D bulk behavior and 2D single-layer behavior, respectively. In Sec. III, we incorporate the correlational part of the ω^3 sum rule into a mean-field-theory description of the dielectric-response function and we calculate the long-wavelength dispersion of the layered OCP plasmon modes in the presence of strong correlations. Conclusions are drawn in Sec. IV.

II. THIRD-FREQUENCY-MOMENT SUM RULE

The layered OCP model considered in this paper consists of a large number N_L of parallel plane layers a distance d apart. Each layer contains a 2D electron liquid with mean areal density $n_{2D} = N_e/S$ and a rigid inert neutralizing positive background. The introduction of a small external charge-density perturbation $\hat{\rho}(\mathbf{r}_{2D}, z, t)$ produces the induced charge-density response

$$\rho^{\text{ind}}(\mathbf{r}_{2D}, z, t) = -e \sum_{j=1}^{N_L} n_j^{\text{ind}}(\mathbf{r}_{2D}, t)\delta(z - z_j) , \tag{1}$$

where $n_j^{\text{ind}}(\mathbf{r}_{2D}, t)$ is the first-order density response (over and above n_{2D}) of the electrons in layer j; $z_j = jd$. Let $(\mathbf{k}, q)$ denote the wave-vector components conjugate to $(\mathbf{r}_{2D}, z)$, viz., $\mathbf{k}$ is a two-dimensional wave vector in the xy plane and q is the wave number perpendicular to the (xy) plane layers. We introduce the convenient superlattice dielectric-response function $\varepsilon(\mathbf{k}, q, \omega)$ through the constitutive relation [13]

$$\rho^{\text{ind}}(\mathbf{k}, q, \omega) = \left[\frac{1}{\varepsilon(\mathbf{k}, q, \omega)} - 1 \right] \hat{\rho}(\mathbf{k}, q, \omega) , \tag{2}$$

with the stipulation that the external charge-density perturbation, like the induced charge-density response, is to be confined to the lattice planes. Both quantities are therefore periodic in q for translations through any lattice number $Q = (2\pi/d \times \text{integer})$; as such, q is defined only in the first Brillouin zone, viz., $|q| \le \pi/d$.

We wish to calculate the $s = 1, 3$ frequency-moment sum-rule coefficients

$$\langle \omega^s \rangle(\mathbf{k}, q) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega\, \omega^s \text{Im}\hat{\chi}(\mathbf{k}, q, \omega) \tag{3}$$

in the exact high-frequency expansion

$$\text{Re}\hat{\chi}(\mathbf{k}, q, \omega) = -\frac{\langle \omega \rangle(\mathbf{k}, q)}{\omega^2} - \frac{\langle \omega^3 \rangle(\mathbf{k}, q)}{\omega^4} - \cdots \tag{4}$$

for the density-density response function $\hat{\chi}(\mathbf{k},q,\omega)$, which describes the linear response of the system to an external longitudinal field perturbation. The appropriate starting point for the calculation of the sum-rule coefficients is the fluctuation-dissipation relation

$$\mathrm{Im}\hat{\chi}(\mathbf{k},q,\omega)=\frac{n_{2D}}{2\hbar N_e N_L}\int_{-\infty}^{\infty}dt\, e^{i\omega t}\langle[n^{\dagger}_{\mathbf{k},q},n_{\mathbf{k},q}(t)]\rangle$$
$$=\frac{n_{2D}}{\hbar}\tanh\left[\frac{\hbar\omega}{2k_BT}\right]S(\mathbf{k},q,\omega)\ , \quad (5)$$

which can be derived from straightforward statistical-mechanical linear-response theory;

$$n_{\mathbf{k},q}=\sum_{i=1}^{N_e}\sum_{j=1}^{N_L}e^{-i\mathbf{k}\cdot\mathbf{x}_{i,j}}e^{-iqz_j} \quad (6)$$

is the Fourier transform of the local-density operator ($\mathbf{x}_{i,j}$ denotes the position of the particle i in layer j), the $\langle\ \rangle$ brackets refer to ensemble averaging over the equilibrium ensemble, and

$$S(\mathbf{k},q,\omega)=\frac{1}{2N_eN_L}\int_{-\infty}^{\infty}dt\, e^{i\omega t}\langle\{n^{\dagger}_{\mathbf{k},q},n_{\mathbf{k},q}(t)\}\rangle$$
$$-2\pi N_eN_L\delta_{\mathbf{k}}\delta_q\delta(\omega) \quad (7)$$

is the dynamical structure function.

The lowest order ($s=1$) f sum rule

$$\langle\omega\rangle(\mathbf{k},q)=-\frac{n_{2D}}{i\hbar N_eN_L}\langle[n^{\dagger}_{\mathbf{k},q},\dot{n}_{\mathbf{k},q}]\rangle=-\frac{n_{2D}k^2}{m} \quad (8)$$

readily results from substituting (6) into (4) and carrying out routine commutator calculations for the Hamiltonian and the local-density operator. The calculation of the ω^3-sum-rule ($s=3$) coefficient

$$\langle\omega^3\rangle(\mathbf{k},q)=\frac{n_{2D}}{i\hbar N_eN_L}\langle[n^{\dagger}_{\mathbf{k},q},\dddot{n}_{\mathbf{k},q}]\rangle\ , \quad (9)$$

however, is far more involved: repeated use of Heisenberg's equation followed by some lengthy commutator algebra yields

$$\langle\omega^3\rangle(\mathbf{k},q)=-\frac{n_{2D}k^2}{m}\left[\omega^2_{2D}(k)F(k,q)+\frac{3k^2}{m}\langle E^{2D}_{\mathrm{kin}}\rangle+\left[\frac{\hbar k^2}{2m}\right]^2+\omega^2_{2D}(k)D(\mathbf{k},q)\right]\ , \quad (10)$$

where $\omega_{2D}(k)=(2\pi n_{2D}e^2k/m)^{1/2}$ is the 2D plasma frequency, $F(k,q)=\sinh kd/(\cosh kd-\cos qd)$ is the superlattice form factor, $\langle E^{2D}_{\mathrm{kin}}\rangle$ is the expectation value of the (2D) kinetic energy per particle for an interacting system, and

$$D(\mathbf{k},q)=\frac{1}{N_eN_L}\sum_{\mathbf{k}'}\sum_{|q'|\le\pi/d}F(k',q')\frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k^3k'}\times[S(\mathbf{k}-\mathbf{k}',q-q')-S(\mathbf{k}',q')]$$
$$=\frac{2}{N_eN_Ld}\sum_{\mathbf{k}'}\sum_{q'}\frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k^3(k'^2+q'^2)}\times[S(\mathbf{k}-\mathbf{k}',q-q')-S(\mathbf{k}',q')] \quad (11)$$

is the correlational part with

$$S(\mathbf{k},q)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\, S(\mathbf{k},q,\omega)\ . \quad (12)$$

The high-frequency expression for the dielectric-response function

$$\varepsilon^{\mathrm{HF}}(\mathbf{k},q,\omega)=1-\frac{\omega^2_{2D}(k)}{\omega^2}F(k,q)-\frac{\omega^4_{2D}(k)}{\omega^4}F(k,q)\times\left[\frac{3}{2}ka\frac{\langle E^{2D}_{\mathrm{kin}}\rangle}{e^2/a}+\frac{\hbar^2k^3a^2}{8me^2}+D(\mathbf{k},q)\right] \quad (13)$$

follows from (4), (9), (10), and the relation

$$\frac{1}{\varepsilon(\mathbf{k},q,\omega)}=1+F(\beta,q)\Phi(k,\omega)\hat{\chi}(\mathbf{k},q,\omega)\ , \quad (14)$$

where $\beta=(k^2-\omega^2/c^2)^{1/2}$, $\Phi(k,\omega)=\phi_{2D}(k)\beta(k,\omega)/k$ is an effective potential that takes account of retardation, $\phi_{2D}(k)=2\pi e^2/k$ is the Fourier transform of the 2D Coulomb potential, and $a=(\pi n_{2D})^{-1/2}$ is the 2D Wigner-Seitz radius. In deriving (13), however, we have ignored the retardation effect in (14) [and in (18) below] by letting $\beta(k,\omega)\approx k$; this is a reasonable assumption in view of the fact that the displacement currents are almost always dominated by the electrostatic effects of nearby layers [13]. We note that our analysis is necessarily restricted to wave-number domains $k\gg\omega_p/c$; or, equivalently stated, the dimensionless wave number ka is stipulated to be much larger than the relativistically small quantity $(e^2/a)/mc^2$. This is a consequence of requiring $\omega_p<\omega$ for the convergence of the sum-rule expansion, while at the same time requiring $\omega\ll kc$ to guarantee that the displacement current will always be negligibly small.

We next consider the two limits $kd\to0$ and $kd\to\infty$ where the high-frequency expression (13) for the layered OCP dielectric-response function is expected to exhibit 3D bulk behavior and 2D single-layer behavior, respectively. The $kd\to0$ limit can be physically meaningful only if one sets d equal to the 2D Wigner-Seitz radius a and then lets kd and qd tend to zero at the same rate. In this limit, $F(k,q)=2k/(k^2+q^2)d$, and, insofar as correlational effects are concerned, the system is isotropic so that $S(\mathbf{k},q)=S(k_{3D})$; Eq. (13) therefore becomes

$$\epsilon^{\rm HF}(kd\to 0,qd\to 0,\omega)|_{d=a}$$
$$=1-\frac{\omega_{3D}^2}{\omega^2}\frac{k^2}{k_{3D}^2}-\frac{\omega_{3D}^4}{\omega^4}\frac{k^4}{k_{3D}^4}\left[\frac{\langle E_{\rm kin}^{2D}\rangle}{e^2/a_{3D}}+\left[1-\frac{q^2}{2k^2}\right]\frac{4}{45}\frac{(E_c)_{3D}}{e^2/a_{3D}}\right]\times(k_{3D}a_{3D})^2-\cdots\ , \quad (15)$$

where $k_{3D}^2=k^2+q^2$, $(E_c)_{3D}$ is the correlation energy per particle ($-\frac{1}{2}$, times the Madelung energy for the 3D OCP bcc crystal), $a_{3D}=(3/4\pi n_{3D})^{1/3}$ is the 3D Wigner-Seitz radius, and $\omega_{3D}^2=4\pi n_{3D}e^2/m$. Comparison with the exact high-frequency expression [5]

$$\varepsilon_{3D}^{\rm HF}(\mathbf{k}_{3D}\to 0,\omega)=1-\frac{\omega_{3D}^2}{\omega^2}-\frac{\omega_{3D}^4}{\omega^4}\left[\frac{2}{3}\frac{\langle E_{\rm kin}^{3D}\rangle}{e^2/a_{3D}}+\frac{4}{45}\frac{(E_c)_{3D}}{e^2/a_{3D}}\right]\times(k_{3D}a_{3D})^2-\cdots \quad (16)$$

for the 3D OCP dielectric-response function at long wavelengths reveals that, aside from the difference between $\langle E_{\rm kin}^{2D}\rangle$ and $(\frac{2}{3})\langle E_{\rm kin}^{3D}\rangle$ (which is quite marked at zero temperature, but disappears in the classical limit), and aside from the (k^2/k_{3D}^2) and $(1-q^2/2k^2)$ factors, (15) replicates (16). The unavoidable (k^2/k_{3D}^2) factor also shows up in the random-phase approximation (RPA) and is well understood [13].

In the $kd\to\infty$ limit, $F(k,q)=1$, $S(\mathbf{k}',q')=S(k')$, and

$$\sum_{|q'|\le\pi/d}F(k',q')=N_L\ .$$

In this limit, the exact 2D OCP $\varepsilon^{\rm HF}(\mathbf{k},\omega)$ for the single uncoupled layer [6–8] is rigorously recovered from (13).

III. LONG-WAVELENGTH PLASMON DISPERSION

We turn now to the calculation of plasmon dispersion in the strongly coupled layered OCP: the primary goal here is to determine how the mode structure is affected by strong particle correlations. Their importance in superlattice structures is especially underscored by the recent prediction [14] that the critical $r_s=a/a_0$ value (a_0 is the Bohr radius) marking the occurrence of quantum Wigner crystallization in a single isolated layer can be significantly decreased in a superlattice by the Coulomb interaction between layers.

The plasmon frequency of the layered electron liquid is calculated from the dispersion relation

$$\varepsilon(\mathbf{k},q,\omega(\mathbf{k},q))=0\ . \quad (17)$$

In constructing $\varepsilon(\mathbf{k},q,\omega)$, we are guided by the observation [9–12] that the correlational contribution to the dispersion of the OCP is identical to the correlational part $D(\mathbf{k},q)$ of its companion ω^3 sum rule at long wavelengths. A correct description of long-wavelength plasmon dispersion in the presence of strong correlations can therefore be realized by incorporating $D(\mathbf{k},q)$ into our construction of $\varepsilon(\mathbf{k},q,\omega)$. This can be accomplished within the framework of the mean-field-theory formalism presented below.

It is convenient to work in terms of the screened density-density response function $\chi_{SC}(\mathbf{k},q,\omega)$

$$\varepsilon(\mathbf{k},q,\omega)=1-F(\mathbf{k},q)\phi_{2D}(k)\chi_{SC}(\mathbf{k},q,\omega)\ , \quad (18)$$

which portrays the linear response to the total (external plus induced polarization) field perturbation. At long wavelengths and in the absence of collisions, this response function can be well approximated by the hydrodynamic RPA formula

$$\chi_{SC}^0(\mathbf{k},q,\omega)=\frac{n_{2D}k^2}{m\omega^2-3k^2\langle E_{\rm kin}^{2D}\rangle_0}\ , \quad (19)$$

where, at zero temperature $\langle E_{\rm kin}^{2D}\rangle_0=\pi n_{2D}\hbar^2/2m$ is the kinetic energy per particle of a noninteracting 2D system; in the classical limit, $\langle E_{\rm kin}^{2D}\rangle_0=k_BT$. Equation (19) is an adaptation of Fetter's [13] hydrodynamic description of a layered electron fluid with collisional damping to a description without collisional damping. The adaptation amounts to reformulating the Ref. [13] collisional adiabatic energy equation for the hydrodynamic pressure into a collisionless adiabatic energy equation: the heat-capacity ratio $C_p/C_V=2$ quoted in Ref. [13] reflects the fact that, in the presence of particle collisions, adiabatic compressions of plasma waves are necessarily two dimensional; however, in the absence of collisions, the adiabatic compressions are essentially one dimensional [15], whence the appearance of the numerical coefficient $C_p/C_V=3$ in the denominator of (19) and in the RPA third-frequency-moment sum rule

$$\langle\omega^3\rangle(\mathbf{k},q)|_{\rm RPA}=-\frac{n_{2D}k^2}{m}\left[\omega_{2D}^2(k)F(k,q)+\frac{3k^2}{m}\langle E_{\rm kin}^{2D}\rangle_0+\left[\frac{\hbar k^2}{2m}\right]^2\right]\ , \quad (20)$$

which (19) satisfies.

To account for correlational effects, we introduce the *static local field correction* $G(\mathbf{k},q)$ through the phenomenological mean-field formula [5]

$$\chi_{SC}(\mathbf{k},q,\omega)=\frac{\chi_{SC}^0(\mathbf{k},q,\omega)}{1+F(k,q)\phi_{2D}(k)\chi_{SC}^0(\mathbf{k},q,\omega)G(\mathbf{k},q)}\ . \quad (21)$$

Satisfaction of the ω^3 sum rule (10) can then be guaranteed by setting [5,12]

$$F(k,q)G(\mathbf{k},q)=-D(\mathbf{k},q)+\frac{3}{2}\left[\frac{\langle E_{\rm kin}^{2D}\rangle_0-\langle E_{\rm kin}^{2D}\rangle}{e^2/a}\right]ka\ . \quad (22)$$

Here, we wish to reiterate our rationale for embedding the correlational part, $D(\mathbf{k},q)$, of the ω^3-sum-rule coefficient into the mean-field-theory formula (20): at very strong coupling, the correlational contributions to the long-wavelength dispersion of the 3D and 2D OCP plasmon modes are known to be identical to the correlational part of the ω^3 sum rule. We therefore argue that this must also be an inherent feature of the intermediate layered OCP configuration thereby motivating the ansatz (22).

Now, the plasmon modes of the layered OCP are spread into a band with each mode labeled by a q value. The frequency band is bounded from above and below by the $q=0$ *in-phase* (bulklike) and $q=\pm\pi/d$ *out-of-phase* (acousticlike modes), respectively. In describing the long-wavelength dispersion of the plasmon band in the presence of strong correlations, it suffices here to direct our attention to these two boundary modes. From Eqs. (17)–(19), (21), and (22), one readily obtains the following plasmon frequencies.

Bulk plasmon:

$$\omega^2(\mathbf{k}\to 0,\ q=0)=\omega_{3D}^2\left[1+\frac{3}{4}\frac{\langle E_{\rm kin}^{2D}\rangle}{e^2/a}k^2ad+\frac{1}{2}k^2d^2I_-\right], \tag{23}$$

Acoustic plasmon:

$$\omega^2(\mathbf{k}\to 0,\ |q|=\pi/d)\approx\frac{k^2e^2d}{ma^2}(1+2I_+)\ , \tag{24}$$

where

$$I_\pm\equiv\frac{1}{kd}D\left[\mathbf{k}\to 0,\ |q|=\frac{\pi}{2d}\pm\frac{\pi}{2d}\right]$$
$$=\frac{1}{N_eN_L}\sum_{\mathbf{k}'}\sum_{|q'|\le\pi/d}F_\pm(k',q')[S(\mathbf{k}',q')-1]\left[\frac{5}{6k'd}+\tfrac{11}{16}[\coth k'd-F_\pm(k',q')]+\tfrac{3}{8}k'd[F_\pm^2(k',q')-\tfrac{3}{2}F_\pm^1(k',q')\coth k'd+\tfrac{1}{2}]\right], \tag{25}$$

$$F_\pm(k',q')=\frac{\sinh k'd}{\cosh k'd\pm\cos q'd}\ .$$

For the acoustic plasmon, we note that the long-wavelength hypothesis $k^2\langle E_{\rm kin}^{2D}\rangle/m\ll\omega^2$ indeed remains intact since it is tantamount to the strong Coulomb coupling condition $\langle E_{\rm kin}^{2D}\rangle\ll(e^2/a)(d/a)$ that prevails for the layered OCP under consideration in this paper. At the same time, this requires that the kinetic-energy term be dropped from (24) as we have done.

Accurate calculations of $D(\mathbf{k},q)$ and $I_\pm$ are contingent on the availability of static structure-function data for the layered OCP. While Monte Carlo (MC) simulations provide these data for the 2D and 3D OCP in the classical [16] and quantum [17] domains, MC structure-function data for the layered OCP have yet to be generated.

The appearance of the layered OCP correlation energy per particle

$$\frac{(E_c)_{\rm LOCP}}{e^2/a}=\frac{1}{N_eN_L}\sum_{\mathbf{k}}\sum_{|q|\le\pi/d}F_-(k,q)\frac{1}{ka}[S(\mathbf{k},q)-1] \tag{26}$$

in the expression (25) for I_- is hardly surprising, especially in view of the fact that the correlational contribution to the 2D and 3D OCP counterparts of (23) is a pure correlation energy density [9,10] term [see, e.g., (29) below]. As an aside, we note that I_- can also be written as

$$I_-=\frac{2}{N_eN_Ld^2}\sum_{\mathbf{k}'}\sum_{q'}\frac{q'^2}{(k'^2+q'^2)^2}\left[1-\frac{3}{2}\frac{k'^2}{k'^2+q'^2}\right]\times[S(\mathbf{k}',q')-1]\ , \tag{27}$$

where [as in the second version of (11)] the q' summation extends from $-\infty$ to $+\infty$. From the point of view of passing to the $d=a$ bulk limit, the form (27) is more transparent than (25): in this limit, we recall that $S(\mathbf{k}',q')=S(k_{3D}')$ whence (27) and (23) become

$$I_-=\frac{8}{45}\frac{(E_c)_{3D}}{e^2/a_{3D}}\left[\frac{a_{3D}}{a}\right]^2 \tag{28}$$

and

$$\omega(\mathbf{k}\to 0,\ q=0)=\omega_{3d}\left[1+\frac{1}{2}\frac{\langle E_{\rm kin}^{2D}\rangle}{e^2/a_{3D}}k^2a_{3D}^2+\frac{2}{45}\frac{(E_c)_{3D}}{e^2/a_{3D}}k^2a_{3D}^2\right]. \tag{29}$$

Equation (29) without the kinetic-energy term is identical to the Ref. [10], Eq. (8), long-wavelength plasmon dispersion formula (with $q=0$) corresponding to the quasiharmonic Wigner lattice along the [1,0,0] direction.

Equations (23) and (24) describe long-wavelength

($kd \to 0$) plasmon dispersion in the strongly coupled layered OCP. Our preliminary estimates indicate that I_- and I_+ are negative and, as expected, overwhelm their kinetic-energy counterparts at high coupling. We note that the correlational correction to the $q=0$ plasmon dispersion (like its kinetic-energy counterpart) is $O(k^2)$, as it should be for the bulk mode. As to the $|q|=\pi/d$ plasmon, when $d/a \gg 1$, the contribution

$$I_+ \approx \frac{5}{16}\frac{a}{d}\frac{(E_c)_{2D}}{e^2/a} \tag{30}$$

is small and affects the acoustic velocity only negligibly. For order unity values of d/a, however, it appears that the acoustic velocity should be substantially diminished by the presence of strong correlations.

IV. CONCLUSIONS

Starting from the superlattice fluctuation-dissipation relation [Eq. (5)] (derived by us from straightforward statistical-mechanical linear-response theory), we have established the third-frequency-moment sum rule [Eq. (10)] and exact high-frequency dielectric-response function [Eq. (13)] for the layered electron OCP. Equations (5), (10), and (13) are valid in both the quantum and classical domains and they are valid for $a \le d \le \infty$ and for arbitrary values of the k and q wave numbers.

In the $d=a$, $kd \to 0$, $qd \to 0$ limit, characteristic of bulk behavior, the correlational part of the high-frequency dielectric-response-function expression (13) for the layered OCP comes remarkably close to replicating its 3D counterpart in (16) at long wavelengths; the two become identical when $q=0$. In the opposite limit $kd \to \infty$, (13) exactly reproduces its 2D single-layer dielectric-response-function counterpart.

We have embedded the correlational part of the third-frequency-moment sum rule into a standard mean-field-theory formula for the dielectric-response function and we have calculated the dispersion of the $q=0$ and $|q|=\pi/d$ plasmon modes. The resulting plasmon frequency formulas (23) and (24) should provide a reliable description of long-wavelength plasmon dispersion in the strongly coupled layered OCP.

The $O(k^2)$ correlational correction that appears in the $q=0$ bulk plasmon contains the expected correlation energy density contribution. Our preliminary estimates indicate that, similarly, to what occurs in strongly coupled 2D and 3D OCP's, this correction is negative and overwhelms its $O(k^2)$ kinetic-energy counterpart. As to the $|q|=\pi/d$ plasmon, it appears that for order unity values of the d/a ratio, the correlational correction could substantially reduce the acoustic velocity in the strong-coupling domain.

ACKNOWLEDGMENTS

This work has been partially supported by the National Science Foundation, Grant No. ECS-8713628. This research was initiated when D.L. was visiting the University of Vermont (UVM). He wishes to thank UVM for its hospitality.

[1] R. D. Puff, Phys. Rev. **137**, 406A (1965).
[2] K. N. Pathak and P. Vashishta, Phys. Rev. B **7**, 3649 (1973).
[3] S. Ichimaru and T. Tange, Phys. Rev. Lett. **32**, 102 (1974).
[4] S. Ichimaru, Rev. Mod. Phys. **54**, 1017 (1982).
[5] N. Iwamoto, E. Krotschek, and D. Pines, Phys. Rev. B **29**, 3936 (1984).
[6] A. Czachor, A. Holas, S. R. Sharma, and K. S. Singwi, Phys. Rev. B **25**, 2144 (1982).
[7] R. P. Sharma, H. B. Singh, and K. N. Pathak, Solid State Commun. **42**, 823 (1982).
[8] De-xin Lu and K. I. Golden, Phys. Rev. A **28**, 976 (1983).
[9] K. I. Golden and De-xin Lu, Phys. Rev. A **31**, 1763 (1985).
[10] K. I. Golden, Phys. Lett. **112A**, 397 (1985).
[11] G. Kalman and K. I. Golden, Phys. Rev. A **41**, 5516 (1990).
[12] K. I. Golden, G. Kalman, and Ph. Wyns, Phys. Rev. A **41**, 6940 (1990).
[13] A. L. Fetter, Ann. Phys. (N.Y.) **88**, 1 (1974).
[14] L. Swierkowski, D. Neilson, and J. Szymanski, Phys. Rev. Lett. **67**, 240 (1991).
[15] See, e.g., D. R. Nicholson, *Introduction to Plasma Theory* (Wiley, New York, 1983), p. 135.
[16] J.-P. Hansen, Phys. Rev. A **8**, 3096 (1973); **8**, 3110 (1973); H. Totsuji, *ibid.* **17**, 399 (1978); R. C. Gann, S. Chakravarty, and G. V. Chester, Phys. Rev. B **20**, 326 (1979).
[17] D. M. Ceperley and B. Adler, Phys. Rev. Lett. **45**, 566 (1980); S. Tanatar and D. M. Ceperley, Phys. Rev. B **39**, 5005 (1989).