RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 44, NUMBER 10 15 NOVEMBER 1991

Acceleration and focusing of electrons in two-dimensional nonlinear plasma wake fields

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(Received 10 June 1991)

A regime of the plasma wake-field accelerator (PWFA) is proposed, in which a high-intensity electron beam is used to excite extremely nonlinear, transverse motion-dominated plasma oscillations. Through computational analysis of the plasma electron motion and the associated wake fields, it is shown that if the beam is dense enough to eject nearly all of the plasma electrons from the beam channel then the short-range wake fields are of excellent quality for acceleration and focusing of electron beams. These results clear up many conceptual difficulties with the practical realization of a PWFA.

PACS number(s): 52.75.Di, 52.35.Mw, 52.40.Mj

The plasma wake-field accelerator (PWFA) [1], a scheme for achieving ultrahigh longitudinal electric fields in a beam-excited electron plasma wave, has been investigated vigorously in recent years [2–5]. The wave longitudinal fields can be used to accelerate particles to extremely high energy, with notable potential application to multi-TeV e^+e^- linear colliders. In the course of PWFA research, a related field has developed, in which the possibilities for powerful focusing of particle beams in plasmas are explored. This concept, termed a plasma lens, may also find use in a future linear collider, which because of the very high luminosity required by high-energy physics research, must collide extremely small beams at the interaction point, and thus demand final-focus lenses of the highest strength.

The physics of the PWFA in the linear regime, in which the unperturbed plasma electron density n_0 is much larger than the beam density n_b and the perturbed plasma density $n_1 \equiv n - n_0$, has been examined thoroughly, both theoretically and by experiment. The theoretical model developed predicts that a driving electron bunch loses energy by exciting a sinusoidal electron-plasma wave of frequency $\omega_p = (4\pi e^2 n_0/m_e)^{1/2}$ in its "wake." In addition, consideration of transverse effects leads to a model of beam focusing in which the beam's charge can be partly or completely neutralized by the plasma electrons, leaving the magnetic self-pinching forces of the beam uncompensated. By choosing certain driving and accelerating beam profiles, one can maximize the transformer ratio R (the ratio of peak accelerating field in the wake to the peak decelerating field felt by the driver), and minimize both the spread in accelerating fields and the nonlinear transverse forces acting on the accelerating beam. Most aspects of acceleration and focusing predicted by the linear model have been born out by experiment and computer simulation.

Another method for achieving high transformer ratios relies on the predictions of one-dimensional plasma wake-field theory [6,7]. In this case the transformer ratio is improved by forcing the physics of the wake-field supporting medium (the plasma electrons) to be strongly nonlinear, thus making the wake-field response quite different in the decelerating and accelerating phases of the wave. These nonlinear plasma waves have many intriguing aspects, including strong relativistic effects and limitations on wave amplitude due to thermal electron trapping and ion motion [8,9]. They should, however, be viewed as unrealistic for present laboratory use (though perhaps important in cosmic-ray acceleration processes) because of their one-dimensional nature. The only experiments performed so far which have accessed a mildly nonlinear regime [10] did so in a transverse motion-dominated regime where the rms radial beam size $\sigma_r < c/\omega_p = k_p^{-1}$, and no analytical theory is available.

The basic problems of the PWFA in the linear regime are underscored by examining the optimized driving beam profile—it has sharp edges in the radial distribution and must be a long upward longitudinal ramp (many times c/ω_p) followed by a short (less than c/ω_p) fall. Unfortunately, this transverse profile is quite unworkable, as the transverse forces on the driver are very nonlinear in radius and are also a strong function of longitudinal position in the beam. Both effects have negative implications for stable beam transport. The unfavorable longitudinal dependence of the wake fields is inherent in the Maxwell equations; if one defines the transverse wake field as $W_r \equiv q(E_r - H_\theta)$ and $W_z \equiv qE_z$ (here we use cgs units, and make the ultrarelativistic approximation $v_z = c$), under the steady-state assumption that the longitudinal and time dependence of the fields can be expressed as a function of $\xi = ct - z$, the wake-field components are related by the Panofsky-Wenzel theorem [11], which states that $\partial_\xi W_r = \partial_r W_z$. Since the decelerating field must be nearly constant inside the beam to maximize the transformer ratio [12], $W_r \sim \xi \partial_r W_z$ must be a linear function of ξ, if W_z has any r dependence. This is always the case in the linear regime, as the wake fields are dependent on charge density and current flow inside the beam. In the region within c/ω_p of the beam boundary, the plasma electrons act to electromagnetically shield the beam fields and there is some transverse dependence of W_z.

The plasma lens has been studied seriously for use in linear colliders [13–16], originally under the assumption that $n_b \ll n_0$, and that the beam-rise length $k_p \sigma_z \gg 1$. This is a condition which ensures that the plasma electrons respond adiabatically to neutralize the charge of the beam. If the beam is also narrow ($k_p\sigma_r \ll 1$), then the plasma return current flows mainly outside the beam. The beam then is acted upon only by its own magnetic self-forces, and thus nonuniform beams suffer from focusing aberrations. For this reason it has been proposed to use an *underdense* plasma lens ($n_b > n_0$) to focus electrons. In this regime all of the plasma electrons are ejected from the beam channel, and an electron beam is focused by the uniform charge density of the ions. In this lens, not only is the focusing linear (free of geometric aberration), but its strength $K = 2\pi r_e n_0/\gamma$ is dependent only on the plasma density, *not* on the local beam density.

It is precisely this removal of the wake-field dependence on beam parameters that the scheme we now propose seeks to accomplish. We consider here cases where, like the underdense plasma lens, the beam is narrow ($k_p\sigma_r < 1$) and much denser than the plasma. In contrast to the plasma lens, for the PWFA it is desirable to excite large amplitude waves behind an impulselike beam and we, therefore, assume that the beam is fairly short, $k_p\sigma_z < 2$. In order to explore this regime quickly, we have used a computer code developed at Novosibirsk by Breizman and Chebotaev [17] which calculates the steady-state response (longitudinal and time dependence only a function of $\xi = ct - z$) of the plasma electron fluid, assuming cylindrical symmetry and stationary ions, to a specified beam excitation. The plasma response in this scenario can be described qualitatively as a fast rarefaction ($n \to 0$) of the beam volume due to the radial motion of the plasma electrons induced by the large transverse electric forces of the driving beam. Interior to the rarefaction (or *ion channel*) radius, an ion column of uniform charge density en_0 is left behind. The plasma electrons then are acted upon by the radial restoring force of the ion column and perform harmonic motion for approximately one period, after which the rarefaction front converges on the axis, producing a large density spike [18]. After this point the motion becomes nonlaminar, the wave is "broken" [19] and it becomes unsuitable for accelerating particles. A simple, one-dimensional nonrelativistic model for this motion can be developed if one assumes the plasma electrons move only radially and calculates the radial electric field by Gauss' law, using only the charge density interior to the fluid element at that value of ξ. The equation of motion for an electron-fluid element is then

$$\frac{d^2r}{d\xi^2} + \frac{k_p^2}{rn_0}\int_0^r [n_0 - n(\xi,r') - n_b(\xi,r')]r'dr' = 0 . \qquad (1)$$

If one examines the rarefaction front which develops in the beam's wake, then $n_b = n_0 = 0$ interior to the front and the force will be due only to the ions, in which case Eq. (1) predicts simple harmonic motion with wave number $k = k_p/\sqrt{2}$, or one half an oscillation in $k_p\xi \simeq 8.89$. This periodicity approximates well what is observed in the numerical studies, with some modifications arising from plasma-electron relativistic effects and from longitudinal motion. An illustration of the plasma response showing the formation of the predicted rarefaction is given in Fig. 1, which is the result of a fully electromagnetic two-dimensional particle-in-cell (PIC) simulation. Note that the electron motion is mainly laminar (validating our following analysis using the fluid model) until the wave breaks after one period, as predicted by our simple model.

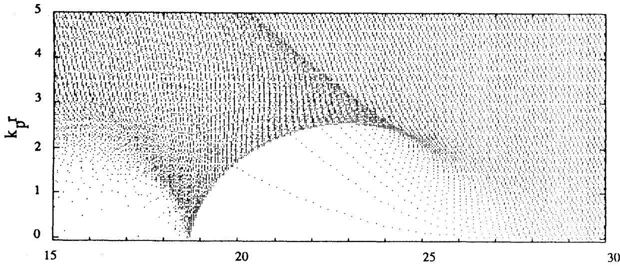

FIG. 1. Plot of plasma-electron positions from a PIC simulation of wake fields with $n_b/n_0 = 4$, $k_p\sigma_z = 1.8$, and $k_p\sigma_r = 0.6$ at the point where the beam center $k_p\xi_0 = 25.5$. Note that for the first oscillation a rarefied cavity forms, and that the motion is nearly laminar. After the first oscillation the wave breaks and the motion becomes very nonlaminar. Beam travels to right in this simulation.

While the plasma-electron motion in this regime of the beam-plasma interaction is fascinating, the form of the wake fields is of primary interest. Some illustrative examples have been obtained from numerical solution of the fluid equations. Figure 2(a) shows the longitudinal dependence of the on-axis longitudinal wake field $W_z(\xi,0)$

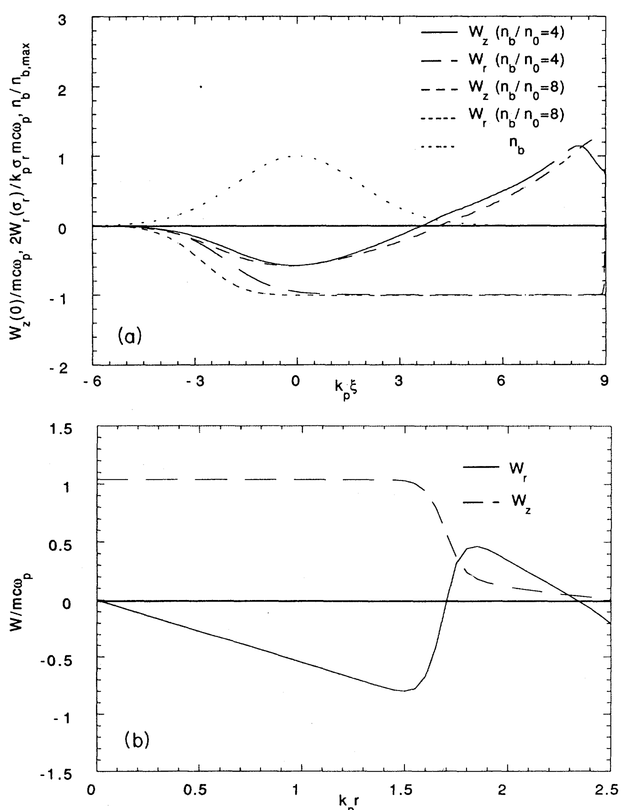

FIG. 2. (a) Comparison of longitudinal and transverse wake fields for two different peak beam densities, $n_b/n_0 = 4$ and 8 with corresponding $k_p\sigma_r = 0.6$ and 0.424, and keeping $k_p\sigma_z = 1.8$. (b) Radial profile of the plasma wake fields, shown at longitudinal position $k_p(\xi - \xi_0) = -6.8$, for the case $n_b/n_0 = 4$ ($k_p\sigma_r = 0.6$).

and of the off-axis transverse wake field $W_r(\xi,\sigma_r)$, for two cases with equal charge per bunch, where $k_p\sigma_z=1.8$ (the beam current is taken to have a Gaussian profile in both r and ξ), and $k_p\sigma_r=0.6$ and 0.424, corresponding to two different peak beam densities $n_b/n_0=4$ and 8. Note that the longitudinal field is changed little by the narrowing of the beam; the transverse wake field is almost identical in the two cases and, in addition, is *independent* of ξ. In this plot $W_r(\xi,\sigma_r)$ is normalized to $(k_p\sigma_r/2)m_e c\omega_p$, which is the value obtained assuming $n=0$, and the resultant transverse force, due only to the ions, is therefore linear in r with strength $K\equiv -W_r/rp_zc=2\pi r_e n_0/\gamma$.

The radial dependence of the wake fields is explicitly shown in Fig. 2(b), which plots W_r and W_z, as a function of r, a distance $k_p\xi=6.8$ behind the driving beam center, and accelerating phase of the wave excited in the $n_b/n_0=4$ case. It is apparent from inspection that W_r is linear in r out to more than two times σ_r of the drive beam. In short, the wake field provides linear focusing with a strength independent of ξ over a long region of accelerating phases. In addition, over the same volume the accelerating wake field is independent of r, which is a very desirable property, in that the energy gained by the accelerating beam electrons is independent of transverse position. Thus these waves are capable of stable, linear transport and uniform acceleration of electron beams. In addition, the driving beam will be transported with a minimum of transverse-profile deformation, as the ion-derived focusing is both uniform and linear over much of the length of the beam.

The conclusions drawn from the fluid code are mainly supported by the PIC simulations. Some differences are apparent in the final region of the ion channel, where the electrons have nearly completed their first oscillation, and their orbits begin to cross. While the fluid code shows a spike in the accelerating field in this region, in the PIC simulations the spike is enhanced and the peak accelerating field is larger (by a factor of 2 in the case shown). The electrons feel more longitudinal acceleration as they approach the axis, resulting in a shortening of the wave period in the PIC simulations. The anomalous steepening of the wake field near peak has in fact been observed in the nonlinear measurements of Ref. 10. This portion of the wave is not of overriding importance, however, because the beam loading should be done far forward of wave breaking, where the ion channel can be wide compared to the loading beam.

The excellent qualities of the accelerating and transverse wake fields inside of the rarefaction front can be understood by returning to the Panofsky-Wenzel theorem. As in the case of the underdense plasma lens, the transverse wake field saturates at a constant level once all the electrons have been ejected from the beam channel. Thus $\partial_\xi W_r=0$, W_z can have no radial dependence, and the desirable focusing properties imply a constant accelerating field profile. From another point of view, the transverse magnetic wake-field interior to the rarefaction front is analogous to the azimuthally symmetric TM electromagnetic mode of a disk-loaded traveling-wave ($v_\phi=c$) linear-accelerator structure. In such a mode W_z is independent of r but there is no focusing for ultrarelativistic particles, as $E_r=H_\theta$. In our case the electromagnetic contribution to the focusing likewise vanishes in the absence of both plasma-electron charge and current density in the beam interior and the focusing arises from superposition of the ions' electrostatic fields.

The longitudinal form of the accelerating wake field is dependent on both the drive beam and on self-consistent beam-loading effects. Figure 3 shows an example of the effects of accelerating beam loading on the wake fields, with the drive beam as in the $n_b/n_0=4$ case, and a loading beam with $n_b/n_0=2$, $k_p\sigma_r=0.6$, and $k_p\sigma_z=0.45$. Note that the variation in ξ of the accelerating field is diminished in this case and that the loading beam produces no change in W_r, as expected. Not illustrated is the related finding that W_z is also independent of r out to $r=3\sigma_r$ (the channel radius) at the peak of the loading beam. In contrast to the linear regime [20], this result holds true regardless of the width of the loading beam, as long as it is smaller than the channel radius.

Using a symmetric beam (the rise length σ_- and the fall length σ_+ are equal), it can be seen from Fig. 2 that the transformer ratio $R\simeq 2$, which is a familiar result from the linear regime [12]. This can be improved, as in the linear regime, by using a long adiabatic rise $k_p\sigma_-\gg 1$, which feeds energy into the wave slowly, followed by a short fall $k_p\sigma_+<1$ which sets free the plasma oscillation with its associated large longitudinal wake [2]. This phenomenon is shown in Fig. 4, with parameters $k_p\sigma_-=5$, $k_p\sigma_+=1$, $k_p\sigma_r=0.5$, and $n_b/n_0=4$. This case shows a large transformer ratio $R=5.6$ (compared to $R=\pi k_p\sigma_-/2=7.85$ for the equivalent linear case) and also illustrates that most of the driving-beam particles lie within the region of linear ion focusing. It should be noted here that the useful length of the beam ramp will be limited by either the onset of ion motion (σ_- is limited, assuming singly an ionized species, by $k_p\sigma_-\ll[\pi n_0 M_i/(n_b-n_0)m_e]^{1/2}$, which is about 43 for H^+ ions and $n_b/n_0=4$) or the electron hose instability [21].

The scheme we have proposed here for accelerating electrons has an additional advantage, in that the plasma density can be orders-of-magnitude smaller than in the linear regime. This means that the operating wavelength can be longer and the focusing strength, as well as the

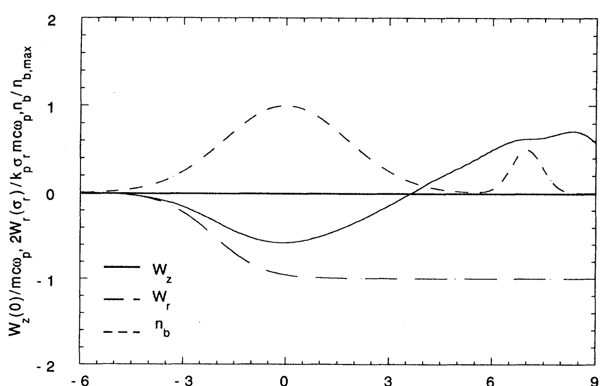

FIG. 3. Beam loading in nonlinear plasma wake fields. Note that the transverse wake field is unaffected by the accelerating beam and that the wave period is slightly lengthened.

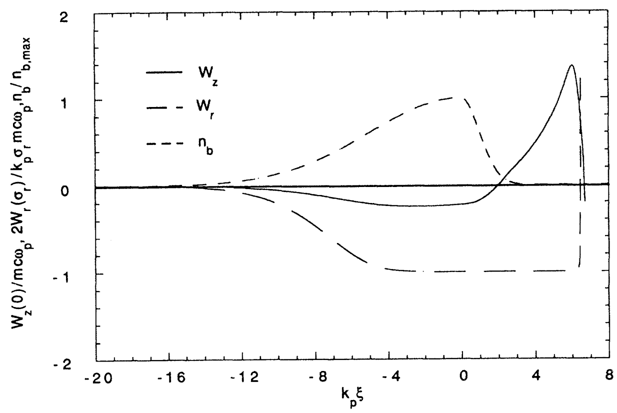

FIG. 4. Enhanced transformer ratio in nonlinear PWFA, using an asymmetric drive beam. In this example $k_p\sigma_- = 5$, $k_p\sigma_+ = 1$, $k_p\sigma_r = 0.5$, and $n_b/n_0 = 4$. The transformer ratio is $R = 5.6$.

multiple scattering due to beam-ion collisions [22], can be kept acceptably small, while not sacrificing the accelerating gradient. As an example, we take $n_0 = 10^{14}$ cm^{-3}, which gives accelerating gradients for the $n_b/n_0 = 4$ case in Fig. 2 that are well in excess of 1 GeV/m. The focusing strength is $K = 177/\gamma$ in this case, which for a matched 100 GeV beam of normalized emittance 10^{-6} rad m gives a betatron wavelength of 20 m and a beam size of 4 μm. The synchrotron-radiation effects from the focusing force in this example are negligible. One major failing of this nonlinear scheme is that it does not work well for positrons, as the wake fields throughout the rarefied regions are strongly defocusing for positrons, enough so that it is not feasible to balance the transverse forces with external optics. Use of a pure electron plasma confined by a strong magnetic field [23] may help overcome these difficulties. In addition to e^+e^- physics, one can also entertain other applications of this scheme in particle physics which use ultrahigh-energy electrons: *ep* linear accelerator-on-ring colliders and applications which convert the high-energy electrons to photons for use in either γ onto fixed-target (for example, to make a B factory [24]) or $\gamma\gamma$ colliders.

Further theoretical investigation of the detailed issues involved with this regime of the PWFA—parametric dependences of the accelerating fields, transformer ratios, and instabilities—will be undertaken in the future, using PIC simulation and fluid codes. In terms of future experiments, higher peak-current electron beams should be available at ULCA and at the Argonne Wake-field Accelerator facility [25], for which the design parameters of the beam match the examples in Fig. 2 with $n_0 = 10^{14}$ cm^{-3}. It should be possible soon to experimentally test the physics of this regime of the PWFA.

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