RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 44, NUMBER 9 1 NOVEMBER 1991

Switching from a stable state to a periodic attractor in optical bistability

B. Macke, E. M. Pessina,* B. Ségard, and J. Zemmouri

Laboratoire de Spectroscopie Hertzienne, Université de Lille 1, 59655 Villeneuve d'Ascq CEDEX, France

(Received 1 August 1991)

The dynamics of a bistable optical system suddenly brought in a domain of multimode instability is studied both experimentally and numerically. The sine-wave self-pulsing is shown to start with an initial amplitude dramatically depending on the switching conditions.

PACS number(s): 42.65.−k, 33.80.−b, 42.50.−p

Instabilities in nonlinear optical systems are usually studied in the framework of linear stability analysis. The system being in an unstable steady state, these instabilities are evidenced by applying some noise or a small adiabatic perturbation (see, e.g., Fig. 3 of Ref. [1]). Quite different situations occur when a control parameter is subjected to a large and abrupt change. New attractors, inaccessible by adiabatic sweeping techniques, can be revealed [2,3] but, even in their absence, interesting dynamical effects are expected. Such an experimental situation is considered in this paper where the sudden change of the control parameter induces a transition from a fixed point to a periodic attractor, both adiabatically reachable. The self-pulsing is then triggered by the switching (not by the noise) and its establishment actually characterizes the system itself instead of its shortcomings (technical noise). The device under consideration is an optical bistable system subjected to an abrupt switching of the input power bringing it in a domain of multimode instability. The more interesting features occur when critical slowing down [4,5] and self-pulsing are observed simultaneously. The corresponding conditions are resumed in the diagram of Fig. 1. The bistable device is characterized by the existence of a supercritical Hopf bifurcation (H) [6] giving rise to a sine-wave instability. This bifurcation, located on the upper branch of the bistability cycle, occurs in the bistability domain. A stepwise pulse switches the input power from zero to a value P_1, slightly larger than the critical power P_c corresponding to the upper turning point A. The bi-stable system initially in a stable state of the lower branch of the hysteresis cycle then can jump to the unstable upper branch where self-pulsing will appear.

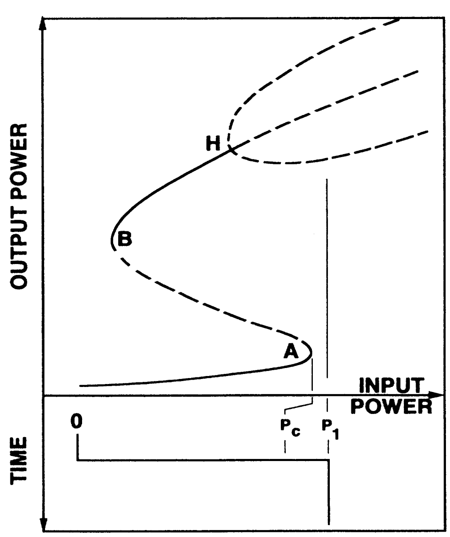

FIG. 1. Schematic drawing of the S-shaped curve of the bistable device locating the domain of instability and the corresponding Hopf bifurcation. Bottom: Input step pulse bringing the bistable system in the domain of instability.

Experiments were carried out at millimetric wavelength ($\lambda \cong 3.5$ mm) in a 182-m-long Fabry-Pérot cavity filled with a gas of hydrogen cyanide $HC^{15}N$ at low pressure (0.5–1 mTorr). The experimental setup is extensively described in a previous paper [7]. Briefly, the optical cavity is characterized by a free spectral range of 830 kHz and a modewidth of about 50 kHz (half width at half maximum) leading to a photon lifetime of 3.3 μs. The bistable is driven at a frequency close to that of the 0–1 rotational line of $HC^{15}N$ which behaves as a homogeneously broadened, two-level system in the experimental conditions. As usual, in the millimetric domain, the relaxation mechanism, mainly collisional, is characterized by a unique relaxation time in inverse ratio to the gas pressure ($T_1 \cong T_2 \cong 7$ μs at 1 mTorr). The input power delivered by a phase-locked high-power klystron (500 mW) is switched by a *p-i-n* diode modulator with a rise time of 10 ns, i.e., 3 orders of magnitude shorter than the time constant of the bistable components.

This bistable device exhibits multimode instability which manifests itself by the appearance of sine-wave self-pulsing on the high transmission branch of its bistability cycle [7]. As indicated before, in all our experimental conditions, the Hopf bifurcation always occurs in the domain of bistability.

Figure 2 shows three typical records of the time evolution of the power transmitted by the bistable device in the conditions of Fig. 1. In all cases, the switching to the high transmission branch is followed by a strongly damped oscillation at low frequency (90 kHz) related to the relaxation oscillations of the bistable system. On recording [Fig. 2(a)], the sine-wave oscillations at a frequency of about 600 kHz appear almost immediately after the step pulse switching and superimpose themselves on the relaxation oscillations. On the contrary, in Fig. 2(b) and 2(c) the system jumps to the upper branch after a time delay related to the critical slowing down (critical delay) and the in-

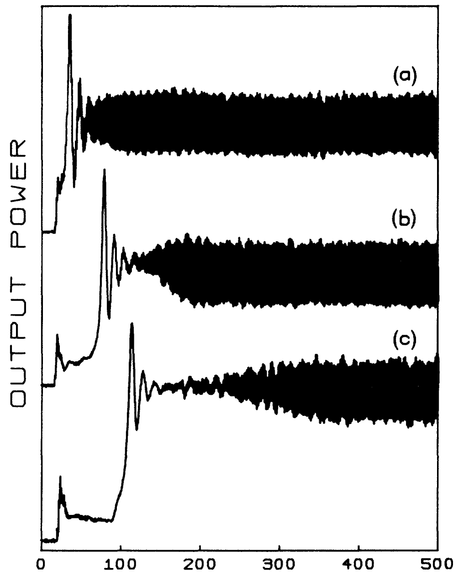

FIG. 2. Experimental time evolution of the power transmitted by the bistable device brought in its self-pulsing domain by an input power switching. The maximum input power corresponds to a Rabi frequency of about 1.3 MHz. Other parameters: (a) pressure $p \cong 0.6$ mTorr, molecular and cavity detuning $\Delta\nu = 314$ kHz; (b) $p \cong 0.5$ mTorr, $\Delta\nu = 315$ kHz; (c) $p \cong 0.7$, $\Delta\nu = 291$ kHz.

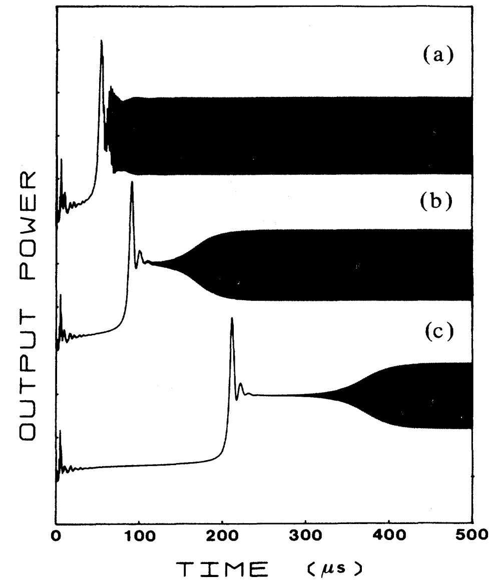

FIG. 3. Numerical results corresponding to the drawings of Fig. 2. Parameters: pressure $p = 0.6$ mTorr, molecular and cavity detuning $\Delta\nu = 314$ kHz, and Rabi frequency is (a) 1.378 MHz, (b) 1.338 MHz, (c) 1.309 MHz.

stability slowly grows and reaches its maximum amplitude at a time which increases with the critical delay.

Numerical simulations were performed in order to analyze these behaviors and, in particular, to precise the influence of the dynamics in the vicinity of the turning point responsible for the critical slowing down. The Fabry-Pérot cavity is modeled by an equivalent ring cavity of double length and the evolution of the bistable device is described by the well-known Bloch-Maxwell equations in the plane-wave approximation with the boundary corresponding to the ring cavity [8]. Figure 3 displays the time dependence of the transmitted power obtained for input powers P_I larger than but closer and closer [recordings 3(a)–3(c)] to the critical power P_c. The similarity with the experimental pictures is remarkable. A detailed study of expanded views of the bistable dynamics following the relaxation oscillations shows that the instability exponentially increases at least before the saturation of the corresponding amplifying process. Whatever the input power P_I, the corresponding time constant remains approximately constant ($25\ \mu s \pm 10\%$). In fact, this time constant characterizes the unstable state of the stationary solution as shown by numerical simulations performed when the bistable system initially located at this unstable state is subjected to a small perturbation. So, the instability amplitude reaches its half maximum after a time duration (here after growing duration) which only depends on its initial value. The growing duration of the oscillations initially increases with the critical delay and becomes practically constant when this delay is larger than 120 μs (i.e., about $10T_2$;$T_2 = 12.7\ \mu s$). In this later case the growing duration is comparable to that obtained when the starting of the instability, from the unstable state, is only induced by numerical fluctuations due to the finite resolution of the computer. For long critical delays, the noise then seems to be mainly responsible for the appearance of the self-pulsing regime. Note that all these behaviors are also observed on all the variables describing the system (field, polarization, and population).

This dynamics is rather surprising since after the critical delay the bistable system is far from any equilibrium. The self-pulsing is then expected to accompany the transition to the upper branch as on recording 3(a). On the contrary, the system appears to be stabilized on the upper unstable branch before the destabilization process occurs. All works as if the dynamical eigenvariables involved in the self-pulsing are decoupled from those involved in the transition from the lower branch of the bistability curve to the upper one. This peculiar behavior is illustrated by using a representation in the phase space. Figure 4 gives the projection of the orbit in the plane defined by the real and imaginary parts of the intracavity field. The spirals corresponding to the relaxation oscillations and to the self-pulsing actually appear as located in different subspaces. The self-pulsing regime is then expected to be triggered not by the transition from the lower branch to the upper one but by a transient generated by the propagation of the initial switching in the cavity at a frequency equal to the free spectral range (830 kHz), actually close to that of the self-pulsing. The amplitude of such a transient falls down with time and any delay, such as that due to the critical slowing down, significantly reduces its efficiency to excite the self-pulsing. For very long critical delays this amplitude vanishes and the instability start from noise fluctuations. The amplitude of the transient can also be lowered by lengthening the switching rise time. Dynamics compa-

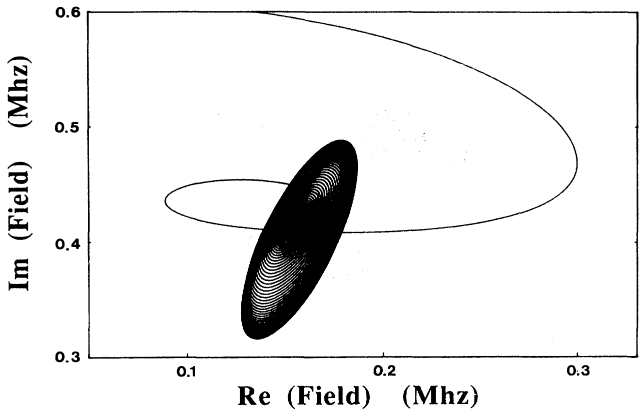

FIG. 4. Projection on a plane of the orbit in the phase space corresponding to Fig. 3(c). Re (Field) and Im (Field) are the components of the output field, respectively, in-phase and in phase quadrature with respect to the input field. The field is measured by the corresponding Rabi frequency.

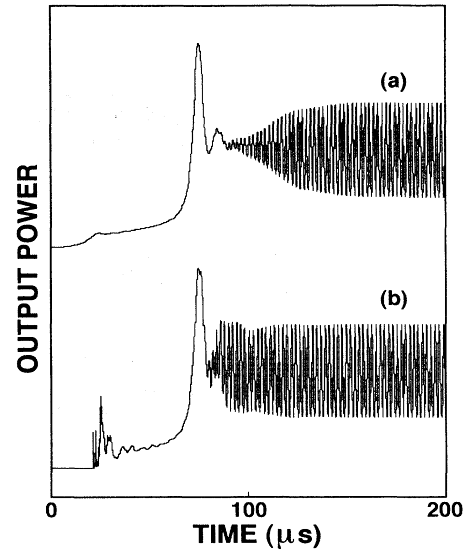

FIG. 5. Starting of the self-pulsing from the input power switching (a) finite-time switching (rise time=20 ms), (b) instantaneous switching. Same parameters as in Fig. 3 except for the input Rabi frequency (a) 1.38 MHz, (b) 1.363 MHz.

rable to that of recordings 3(a) and 3(b) are indeed obtained in this case. This result is illustrated in Fig. 5, where the time evolution of the transmitted power calculated with a 20-μs rise time [5(a)] is compared to that obtained with an instantaneous switching [5(b)]. In 5(a), the finite rise time of the switching obviously introduces an extra delay in the transition to the upper branch. This has been compensated by slightly adjusting the power used in the calculation of 5(b), so that the transition occurs at the same time for both. As expected the vanishing of the transient generated by the input power switching entails a strong reduction of the initial amplitude of the self-pulsing. This confirms the above-mentioned analysis.

In conclusion, the switching of a bistable device from a stable state to a multimode self-pulsing domain has been investigated. In the presence of critical slowing down, this system seems to be stabilized in its instable state before the appearance of the instability. A phenomenological model is proposed to describe the rising of this instability, but a detailed interpretation of the whole dynamics is an open challenge.

This work has been partially supported by the Commission of the European Communities under Contracts No. ST2J-0187F and No. SC1000237. Laboratoire de Spectroscopie Hertzienne is Unité de Recherche associée au Centre National de la Recherche Scientifique and is also supported by the Région Nord/Pas-de-Calais. The numerical code used to solve the Bloch-Maxwell equations is an adaptation of a code developed by D. K. Bandy and L. M. Narducci.

*Permanent address: Dipartimento di Fisica dell' Universita degli Studi, Via Celoria 16, Milano, Italy.

[1] Y. Silberberg and I. Bar Joseph, Phys. Rev. Lett. **48**, 1541 (1982).

[2] Cun Zheng Ning and Hermann Haken, Phys. Rev. A **41**, 6577 (1990).

[3] C. Z. Ning and H. Haken, in *OSA Proceedings on Nonlinear Dynamics in Optical Systems*, edited by N. B. Abraham, E. Garmire, and P. Mandel (Optical Society of America, Washington, DC, 1991), Vol. 7, pp. 297–301.

[4] R. Bonifacio and L. A. Lugiato, Opt. Commun. **19**, 172 (1976); Phys. Rev. A **18**, 1129 (1978).

[5] G. Grynberg, F. Biraben, and E. Giacobino, Appl. Phys. B **26**, 155 (1981).

[6] See, e.g., J. M. T. Thompson and H. B. Stewart, *Nonlinear Dynamics and Chaos* (Wiley, Chischester, 1986), pp. 115–117.

[7] B. Ségard, B. Macke, L. A. Lugiato, F. Pratti, and M. Brambilla, Phys. Rev. A **39**, 703 (1989).

[8] L. A. Lugiato, in *Theory of Optical Bistability*, Progress in Optics Vol. XXI, edited by E. Wolf (North-Holland, Amsterdam, 1984), pp. 69–216.