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Stabilization in superintense fields: A classical interpretation

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A classical approach is used to study the mechanism of the recently discovered suppression of ionization in superintense fields. We show that most features of this suppression occur in a classical context.

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In the last few years it has been demonstrated in numerical experiments that atoms can become stabilized under very strong laser pulses in the optical frequency regime [1–4]. This stabilization is characterized by a counterintuitive decrease of the ionization rate with increasing laser field strength. The possibility of stabilization for asymptotically high laser frequencies was first discussed more than 10 years ago by Gersten and Mittleman [5] and Gavrila and co-workers [6–9]. A few attempts have been made to specify the physical origin of stabilization. Jensen *et al.* [10] have tried to associate highly localized quantum-mechanical wave functions with the existence of unstable orbits in the chaotic phase space, and Mostowski and Eberly [11] have called attention to the importance of strong-field level degeneracy, and several groups have pointed to quantum interferences as a possible mechanism for stabilization [12–14]. Clearly there might be more than one "physical origin." In any event, our interest here is to show that many aspects of stabilization are essentially of classical nature [15]. We believe that our analysis aids in the understanding of the strong-field ionization process, and has some pictorial features not previously exhibited.

We consider a classical one-dimensional nonrelativistic atom model in which the electron moves in a soft-core Coulombic potential in the presence of an external electric field $\mathcal{E}(t)$ with frequency ω. The relevant Hamiltonian is

$$\tilde{H}=\frac{\tilde{p}^2}{2}-\frac{1}{(1+\tilde{x}^2)^{1/2}}-\tilde{x}\mathcal{E}(t)\sin\omega t\ , \tag{1}$$

where we are using atomic units ($e=m=1$). This model, which has been discussed thoroughly elsewhere [16], shares many features of the real hydrogen atom.

We begin our analysis by presenting several views of the result of integrating Newton's equation of motion for 5000 initially bound trajectories where the field $\mathcal{E}(t)$ has been smoothly turned on over 6 optical cycles and abruptly turned off after 50 optical cycles, where we have chosen $\omega=0.8$, a case well investigated for this atom quantum mechanically [1(b),1(c)]. Ionization will be associated with positive energy at field turn-off. Figure 1 shows the final fraction of ionized electrons as a function of various field amplitudes $\mathcal{E}_0$. The initial increase of the ionization rate with increasing $\mathcal{E}_0$ (weak-field regime) is evident in the figure. However, we find a reversed behavior after $\mathcal{E}_0$ has passed through the narrow strong-field regime around a critical value $\mathcal{E}_{\rm crit}$ (which is between 0.5 and 2 a.u. for our parameters, the same range obtained from a quantum-mechanical analysis using the same parameters [1(b), 1(c)]). The domain where $\mathcal{E}_0>\mathcal{E}_{\rm crit}$ is characterized by a counter intuitive decreasing ionization rate with increasing laser field, and so we can sensibly call it the stabilization regime. (To avoid misunderstanding, we point out explicitly that a decreasing ionization rate does not mean a zero ionization rate.)

Data of a different kind that lead to the same conclusion are shown in Fig. 2, where we show the average kinetic energy (in atomic units) of the ionized electrons after the laser pulse has been turned off. Note that the electrons do not acquire higher and higher kinetic energies as the field strength gets higher. In fact, the same field strength $\mathcal{E}_{\rm crit}$ for which the ionization rate takes its max-

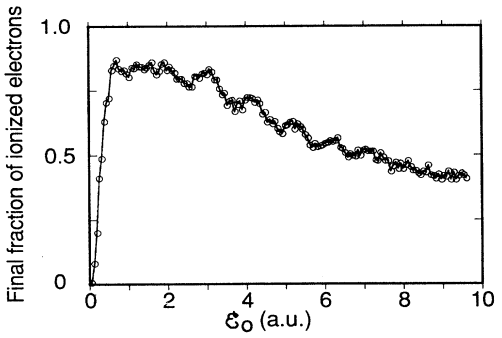

FIG. 1. Final fraction of electrons that are ionized (have positive energy) after an interaction time of 50 optical cycles as a function of $\mathcal{E}_0$. The laser frequency was $\omega=0.8$ a.u., and a 6-cycle smooth pulse turn-on was used.

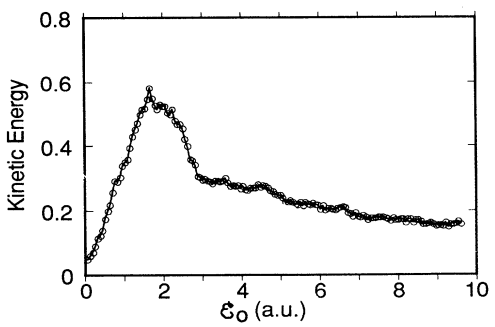

FIG. 2. Average kinetic energy (a.u.) of ionized electrons as a function of $\mathcal{E}_0$ (same laser pulse as Fig. 1).

imum value is the field strength imparting the maximum kinetic energy. At higher field strengths $\mathcal{E} > \mathcal{E}_{\text{crit}}$ the final kinetic energy is lower.

The main purpose of this paper is to show that the stabilization associated with Figs. 1 and 2 is consistent with straightforward classical behavior. Whether this stabilization can be identified with the stabilization observed in quantum-mechanical investigations cannot be rigorously proved, but they certainly share most if not all important characteristics. Our analysis is greatly simplified by making the Kramers-Henneberger (KH) transformation [17,18]

$$x = \tilde{x} - \alpha(t), \quad p = \tilde{p} - \dot{\alpha}(t) \tag{2}$$

to a different frame of reference, in which the dipole Hamiltonian may be written (exactly)

$$\mathcal{H} = \frac{p^2}{2} - \frac{1}{\{1 + [x + \alpha(t)]^2\}^{1/2}} . \tag{3}$$

Here $d\alpha/dt \equiv -(e/c)A(t)$, so α itself obeys the equation of motion for a free electron in the electric field $\mathcal{E} = -(1/c)dA/dt$ corresponding to vector potential $A(t)$:

$$\ddot{\alpha}(t) = \mathcal{E}(t)\sin\omega t . \tag{4a}$$

In the following we will assume that its solution is given by

$$\alpha(t) = -\alpha_0 \sin\omega t \tag{4b}$$

where

$$\alpha_0 = \frac{\mathcal{E}_0}{\omega^2} . \tag{5}$$

Formula (4b) gives the adiabatic solution, which is valid for times after the laser pulse has been smoothly turned on to a steady amplitude $\mathcal{E}_0$. The parameter α_0 plays a crucial role in all strong-field analyses based on the KH transformation [6–9].

The KH frame follows the oscillatory motion of a free electron in the absence of the atomic potential and has been shown [2] to be particularly useful in describing the dynamics when the external field $\mathcal{E}$ is comparable to or stronger than the field produced by the atomic core. In the framework of Hamiltonian $\tilde{H}$ the electron experiences both the atomic as well as the laser force, whereas in the KH frame the electron is driven by the atomic potential alone and the maximum force is independent of the laser field strength.

Now we will look at the time evolution of a single electron in the stabilization regime. We will find in the characteristics of a single "typical" trajectory several clear signs pointing to the reasons ionization becomes difficult in strong fields, and the mechanism by which ionization continues to occur, but at a much lowered rate. Figure 3 shows a trajectory in phase space which we have followed for 45 optical cycles. The electron's motion in the KH frame consists of three components that are present in the picture: drift, "wiggling," and "surfing." We see that after a brief turn-on interval the electron drifts steadily in one direction for many optical cycles. However, during this drift it is repeatedly passed by the

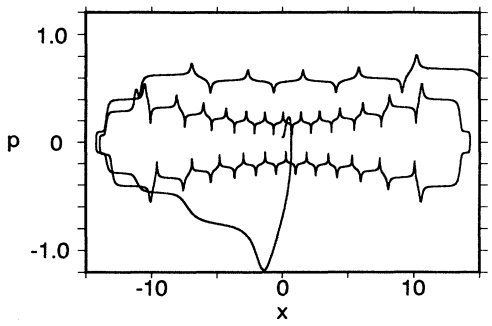

FIG. 3. A typical trapped classical orbit in phase space monitored for 45 optical cycles [$\omega = 0.8$, $\alpha_0 = 11.7$, 5-cycle turn-on, $x(0) = v(0) = 0$].

nucleus, which in the KH frame can be thought of as moving rapidly between the distant turning points at $x \approx \pm\alpha_0$. Each spike in the electron trajectory indicates an encounter between electron and nucleus in which the nucleus briefly pulls the electron toward it in one direction and then (after passing the electron) pulls it in the opposite direction, due to the attractive forces associated with the binding potential $V(x)$. However, because the nucleus is moving so rapidly (with a typical velocity $v \approx \omega\alpha_0$) the pulls occur very fast and the net changes in both electron position and momentum are very small.

These insignificant encounters occur twice every optical cycle, and after relatively many of them the electron reaches one of the turning points in the nuclear motion. For the first time, the effect of the pulling force can be significant, for two reasons. First, the nucleus is traveling slowly near its turning point, and the attractive pull can be relatively long in duration, imparting an appreciable amount of momentum. Second, the nucleus must change direction at its turning point and if the electron is in the right position the pair of near-canceling nuclear pulls, to the right and then to the left, can become a pair of constructively additive leftward pulls (at the right turning point). The trajectory in the figure shows that because of this second feature the electron momentum undergoes a sign reversal.

We want to point out that this reversal *must occur* each time the electron reaches a turning point of the nuclear motion. Suppose that it does not occur. Then the electron continues to drift and drifts slightly beyond the turning point. On the next return of the nucleus to that turning point, the nucleus will not be able to overtake the electron but will have to turn short of the electron's position. In such a case the electron *must* feel a pair of pulls in the same direction, causing it to begin a drift back to the other turning point. Each time the electron acquires a bit more momentum it will reach the next turning point with a somewhat higher velocity and can then drift with the nucleus a somewhat longer distance. It is during this co-drifting stage near a turning point, while the electron is effectively "surfing" on the trailing slope of the nuclear binding potential, that the main momentum buildup of the electron occurs. This can lead quickly to sufficiently high drift velocity to permit permanent escape, i.e., ionization. Figure 4 below shows additional evidence that clearly sup-

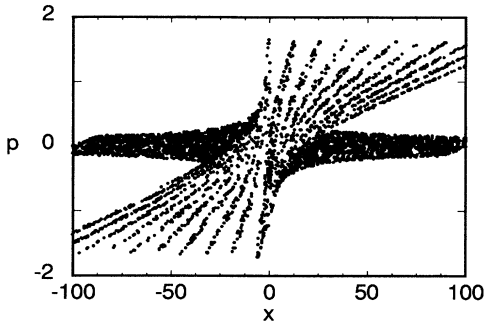

FIG. 4. Final distributions of the trajectories in phase space after 10 optical cycles. The initial trajectories were distributed on the negative energy surface with constant density ($\omega=0.8$, $\alpha_0=2$).

ports this picture of ionization.

Of course we must also point out that the aspects of ionization explained by this analysis of the "typical" trajectory in Fig. 3 are also completely consistent with the existence of stabilization. That is, for a stronger laser field the probability of ionization is lower. The reason is obvious. For a larger $\mathcal{E}_0$ there is a larger α_0 and the spatial distance the electron has to cover in drifting slowly between the two turning points becomes longer and the electron spends more time in the "ionization-free" region far inside the turning points. One must keep in mind that in the high-frequency regime the optical period is only a very small fraction of the long time needed by the electron to move from $-\alpha_0$ to $+\alpha_0$.

We present in Fig. 4 the final distribution of electrons after 10 optical cycles. The ionized electrons form rays in the lower and upper momentum plane, whose slope is given by the inverse of the total time which has passed after the electrons have been ionized. Figure 4 suggests that ionization occurs mainly with a single kick, which is delivered to any electron happening to be near the critical ("magic") point in phase space [19]. The figure shows that electrons are ejected from the atom within very brief time intervals only twice per optical period. One may interpret this as the consequence of very effective surfing. Note in addition that the escape route is one sided in both positive and negative directions. That is, an electron receives sufficient momentum to escape only after reversing its momentum and beginning to travel back toward the opposite turning point. Thus an escaping electron always drifts all the way back across the further turning point before leaving the atom altogether.

Finally, we would like to present a rough estimate for the critical-field strength at which the stabilization regime is entered. Basically the electron can surf effectively on the maximum slope of the potential only if its acceleration due to the gradient of the potential is approximately equal to the acceleration due to the laser ($\approx \mathcal{E}_0$). Therefore, the ionization rate takes its largest value if the laser field just matches the atomic field. It should be noted that this frequency-independent estimate is not in contradiction to the well-known fact that the ionization rate generally decreases with increasing frequency.

In summary, we find that the strong and superstrong field regime are determined by two qualitatively different types of phase-space trajectories. They differ by the length of the average (surfing) time the electron can stay close to the maximum slope of the (traveling) atomic potential on its way from one turning point to the other. We point out that stabilization is due to the decrease of this time when the field strength is increased. The longer the electron can experience binding (surfing) the more probable it is that it becomes ionized. We also discovered that the kinetic energy of the ionized electron decreases with increasing field strength in the stabilization regime and this is fully consistent with our classical picture of the ionization mechanism.

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[1] (a) Q. Su, J. H. Eberly, and J. Javanainen, Phys. Rev. Lett. **64**, 862 (1990); Q. Su and J. H. Eberly, J. Opt. Soc. Am. B 7, 564 (1990); (b) Phys. Rev. A **43**, 2474 (1991); (c) Q. Su, Ph.D. thesis, University of Rochester, 1991.

[2] J. H. Eberly and Q. Su, in *Multiphoton Processes,* edited by G. Mainfray and P. Agostini (CEA Press, Paris, 1991), p. 81.

[3] K. C. Kulander, K. J. Schafer, and J. L. Krause, in *Multiphoton Processes* edited by G. Mainfray and P. Agostini (CEA Press, Paris, 1991), p. 119; K. C. Kulander, K. J. Schafer, and J. L. Krause, Phys. Rev. Lett. **66**, 2601 (1991).

[4] J. H. Eberly, R. Grobe, C. K. Law, and Q. Su, in *Atoms in Intense Radiation Fields,* edited by M. Gavrila (Academic, Orlando, in press).

[5] J. I. Gersten and M. H. Mittleman, J. Phys. B **9**, 2561 (1976).

[6] M. Gavrila and J. Z. Kaminski, Phys. Rev. Lett. **52**, 613 (1984).

[7] M. Pont, N. R. Walet, M. Gavrila, and C. W. McCurdy, Phys. Rev. Lett. **61**, 939 (1988).

[8] M. Pont and M. Gavrila, Phys. Rev. Lett. **65**, 2362 (1990).

[9] M. Pont, N. R. Walet, and M. Gavrila, Phys. Rev. A **41**, 477 (1990).

[10] R. V. Jensen, M. M. Sanders, M. Saraceno, and B. Sundaram, Phys. Rev. Lett. **63**, 2771 (1989); R. V. Jensen and B. Sundaram, Phys. Rev. Lett. **65**, 1964 (1990).

[11] J. Mostowski and J. H. Eberly, J. Opt. Soc. Am. B **8**, 1212 (1991).

[12] M. V. Fedorov and A. M. Movsesian, J. Opt. Soc. Am. B **6**, 928 (1989).

[13] J. Parker and C. R. Stroud, Jr., Phys. Rev. A **41**, 1602 (1990).

[14] K. Burnett, P. L. Knight, B. R. M. Piraux, and V. C. Reed, Phys. Rev. Lett. **66**, 301 (1991).

[15] J. Grochmalicki, M. Lewenstein, and K. Rzazewski, Phys. Rev. Lett. **66**, 1038 (1991).

[16] J. Javanainen, J. H. Eberly, and Q. Su, Phys. Rev. A **38**, 3430 (1988); J. H. Eberly, *ibid.* **42**, 5750 (1990).

[17] W. C. Henneberger, Phys. Rev. Lett. **21**, 838 (1968).

[18] H. A. Kramers, *Collected Scientific Papers* (North-Holland, Amsterdam, 1956), p. 262; W. C. Henneberger, Phys. Rev. Lett. **21**, 838 (1968); see also C. K. Choi, W. C. Henneberger, and F. C. Sanders, Phys. Rev. A **9**, 1895 (1974).

[19] As a further check we have monitored the initial positions in phase space of those electrons which have been ionized within one optical cycle. We found that almost all ionized electrons stem from small regions around the potential's turning points at α_0.