RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 44, NUMBER 6 15 SEPTEMBER 1991

Superradiant evolution of radiation pulses in a free-electron laser

R. Bonifacio, N. Piovella, and B. W. J. McNeil
Università di Milano, via Celoria 16, 20133 Milano, Italy
(Received 8 February 1991)

We demonstrate analytically and numerically superradiant spiking behavior in the leading and trailing regions of a radiation pulse propagating within a long electron pulse in a single-pass, high-gain free-electron laser (FEL). A single superradiant spike is observed when the radiation pulse is shorter than a cooperation length L_c. We show this work may be relevant to the understanding of the spiking behavior in the FEL oscillator, and to possible spiking mechanisms in a perturbed steady-state amplifier.

I. INTRODUCTION

The steady-state (SS) theory of the high-gain free-electron laser (FEL) amplifier [1] neglects the relative slippage of the electron and radiation pulse envelopes. The evolution of the radiation field and the electrons may be described by a one-dimensional (1D) system of coupled, first-order, ordinary differential equations (see Ref. [1] for details of the form of the "universally scaled" equations). Within this zero-slippage approximation the radiation intensity scales as $n_e^{4/3}$ (n_e is the electron-beam density) and saturates with a dimensionless intensity of $|A|^2_{\rm sat}\sim 1.4$ at resonance [see Eq. (1)].

Pulse effects due to the slippage have been described numerically [2] and analytically [3,4], using a set of partial differential equations. These have been solved with initial small bunching and/or initial radiation field, in order to simulate respectively the shot noise and the spontaneous emission, distributed over all the electron pulse, as in the self-amplified spontaneous-emission (SASE) regime [1].

The investigations have predicted the existence of superradiant phenomena where the radiation intensities emitted by the electron pulse scale as n_e^2. The superradiant phenomena may be subdivided into two catagories: *weak* and *strong* superradiance [2]. The former results when the electron pulse length is less than a *cooperation length* L_c [see Eq. (1)], the latter when the electron pulse is much longer than L_c. Strong superradiant phenomena are seen to occur in the trailing slippage region of the electron pulse with radiation intensities much greater than the SS saturated value.

From the results of the linear theory [3-5], a nonexponential growth of the superradiant intensity, as $|A|^2 \propto \exp(gz^{2/3})$, has been predicted until the first peak of emission. We proved [4] that the peak intensity scales as n_e^2 and that the superradiant pulse continues to grow as it propagates within the electron pulse, its width being narrow in proportion to the inverse of the fourth root of the peak intensity.

In this Rapid Communication we describe the results of modeling a different set of initial pulse conditions; we investigate the evolution of a single (initially low intensity) radiation pulse of finite extent within an electron pulse of effective infinite extent. It will be shown that in such a case we obtain strong superradiant emission in both the trailing *and* leading slippage regions of the radiation pulse.

II. RESULTS

We consider the usual 1D coupled equations [2] for the electron phase $\theta=(k_w+k)z-ckt$ and the scaled complex field amplitude $A=E/(4\pi\rho\bar{n}_e mc^2\gamma_r)^{1/2}$:

$$\frac{\partial^2\theta_j(z_1,z_2)}{\partial z_2^2}=-\{A(z_1,z_2)\exp[i\theta_j(z_1,z_2)]+\text{c.c.}\}\,,$$
$$\frac{\partial A(z_1,z_2)}{\partial z_1}=\chi(z_1)\langle\exp[-i\theta(z_1,z_2)]\rangle\,, \tag{1}$$

where $j=1,\ldots,N_e$. In (1), $z_1=-c(t-z/v_\parallel)/L_c$ and $z_2=c(t-z/c)/L_c$, which are proportional to the retarded times at the speed of the electrons $v_\parallel=c\beta_\parallel$ and the speed of light c, respectively; $L_c=\lambda/4\pi\rho$ is the *cooperation length,* and $z_1+z_2=\bar{z}=z/L_g$ is the scaled coordinate along the wiggler axis, where $L_g=\lambda_w/4\pi\rho$ is the *gain length*; the cooperation length, equal to the slippage in one gain length $[L_c=(\lambda/\lambda_w)L_g]$, can be interpreted as a measure of the minimum distance between which electrons may interact cooperatively via the radiation field; $\lambda=\lambda_w(1-\beta_\parallel)/\beta_\parallel$ is the radiation wavelength and $\rho=\gamma_r^{-1}\times(a_w\lambda_w\omega_p/8\pi c)^{2/3}$ is the fundamental FEL parameter [1], where $\omega_p=(4\pi e^2\bar{n}_e/m)^{1/2}$ is the plasma frequency and $\bar{n}_e$ is the peak electron density, such that $n_e(z-v_\parallel t)=\bar{n}_e\chi(z_1)$, χ being the macroscopic electron-density function normalized to 1 at $\bar{n}_e$; $E(z,t)$ is the slowly varying complex amplitude of the radiation field of wavelength λ; the detuning parameter $\partial\theta_j(z_1,-z_1)/\partial z_2=\delta\equiv(\langle\gamma\rangle_0-\gamma_r)/\rho\gamma_r$ describes the off-resonance of the incident-beam energy $mc^2\langle\gamma\rangle_0$ relative to the resonance energy, $mc^2\gamma_r$, where $\gamma_r=[\lambda_w(1+a_w^2)/2\lambda]^{1/2}$, a_w is the wiggler parameter, and λ_w is the wiggler period. In (1), the average $\langle X\rangle=N^{-1}(z,t)\sum_{j=1}^{N(z,t)}X_j$ is over $N(z,t)$ electrons around the position z at a fixed time t of the general electron variable X.

We investigate the nonlinear evolution of the single-pass FEL amplifier, in the presence of a rectangular optical pulse of length L_r and amplitude A_0 at input, assuming a rectangular electron pulse longer than L_r+L_s, where $L_s=(\lambda/\lambda_w)L_w$ is the slippage length for a wiggler length

L_w. We use a numerical code [2] integrating Eqs. (1), assuming no initial energy spread or noise in the electron beam. We consider, in Fig. 1(a), a long-pulse case ($L_r=20L_c$) for a detuned ($\delta=2$) electron beam. We also consider a short-pulse case ($L_r=L_c$) in Fig. 1(b). We assume $L_w=20L_g$, so that Eqs. (1) are integrated from $\bar{z}=0$ until $\bar{z}=20$ with an initial small signal pulse of field amplitude $A_0=0.01$. In the Figs. 1(a) and 1(b) we report the time profile of the intensity as a function of z_1, at the end of the wiggler; $z_1=0$ corresponds to the position of the trailing edge of the electron pulse which is aligned with the trailing edge of the optical pulse at $\bar{z}=0$. In these figures, the electron pulse envelope is at rest and the radiation propagates from left to right with increasing $\bar{z}$.

From the previous results of a linear theory [6], it is possible to identify three distinct regions of the optical pulse in the long-pulse case: a trailing-edge slippage region, $0<z_1<\bar{z}$, a steady-state region $\bar{z}<z_1<L_r'=L_r/L_c$, and a leading-edge slippage region $L_r'<z_1<L_r'+\bar{z}$, for $\bar{z}<L_r'$ (or $L_s<L_r$). In Fig. 1(a), $\bar{z}=L_r'$, so that the steady-state region has disappeared and only the two slippage regions are visible.

The basic difference between a resonant and a detuned case arises as the high-gain steady-state emission occurs only for $\delta<3/2^{2/3}\sim1.89$ [1]. In the resonant case, not shown here, the electrons in the trailing slippage region $0<z_1<L_r'=20$ have interacted first with radiation which has a steady-state evolution, and after with the superradiant radiation propagating from the back of the pulse; the steady-state radiation extracts energy from the electrons so perturbing their initial state and introducing an energy spread in the beam. On the contrary, in the detuned case shown in Fig. 1(a), no steady-state interaction occurs, and the electrons are effectively unperturbed from their initial state on entering the trailing-edge slippage region. The appearance of the spike in the leading-edge slippage region $L_r'<z_1<2L_r'$ of Fig. 1(a) can be described in terms analogous to that of the trailing-edge slippage region in the detuned case: electrons entering the leading-edge slippage region are unperturbed from their initial state, and then interact only with the superradiant radiation generated at the leading edge of the optical pulse. As shown in Fig. 1(a), the trailing-edge and leading-edge slippage regions exhibit two almost identical superradiant spikes, as both the superradiant pulses interact with unperturbed electrons.

We now consider the short-pulse case in Fig. 1(b). The initial conditions at $\bar{z}=0$ of the optical pulse are intensity $|A_0|^2$ with trailing edge at $z_1=0$ and leading edge at $z_1=1$. In this case the linear analysis [6] predicts the steady-state region to be negligible, as the slippage $L_s=(L_w/L_g)L_c=20L_c$ is much greater than the initial optical pulse length $L_r=L_c$. In Fig. 1(b) the final optical pulse extends from $z_1=0$ to $z_1=(L_s+L_r)/L_c=21$. We see a high spike in the leading edge of the pulse. The same characteristics of the spikes observed in the long-pulse case of Fig. 1(a) are evident.

In order to test for superradiance as defined in Ref. [2], we consider the radiation intensity $|A|^2$ as a function of the position $\bar{z}$ through the wiggler, at a fixed position z_1 within the electron pulse. As an example, in Fig. 2 we show the evolution of $|A|^2$ as a function of $\bar{z}$, in the long-pulse ($L_r=20L_c$) resonant case, at the position $z_1=30$ within the electron pulse. These electrons do not interact with any radiation until $\bar{z}=10$, the value of $\bar{z}$ required to allow the leading edge of the radiation to propagate to $z_1=30$. After $\bar{z}=10$ the electrons interact with the spike in the leading slippage region and transfer energy to the radiation. In Fig. 3 we plot the peak value $|A|_p$ of the radiation (for all $0<\bar{z}<20$) at position z_1 within the electron beam, as a function of z_1, in the long-pulse simulation. For example, in Fig. 2, for $z_1=30$, the peak value is 3.2 at position $\bar{z}=13.8$. The solid line in Fig. 3 is for a detuned case, $\delta=2$, and the dashed line is for a resonant case, $\delta=0$. The horizontal line at $|A|_p=1.2$ is the limit of the maximum amplitude reached by the steady-state field at resonance. Due to the scaling of Eqs. (1), any superradiant emission will exhibit a linear dependence on z_1 when

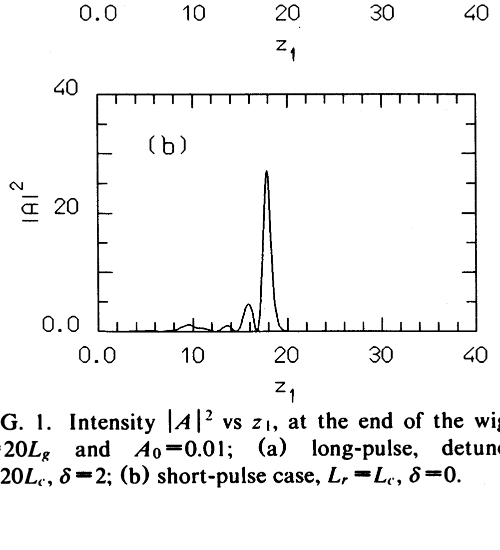

FIG. 1. Intensity $|A|^2$ vs z_1, at the end of the wiggler, for $L_w=20L_g$ and $A_0=0.01$; (a) long-pulse, detuned case, $L_r=20L_c$, $\delta=2$; (b) short-pulse case, $L_r=L_c$, $\delta=0$.

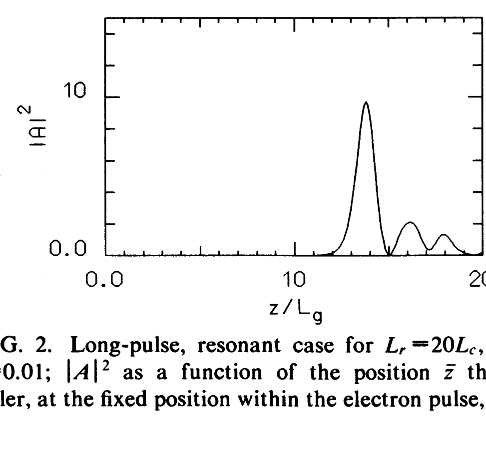

FIG. 2. Long-pulse, resonant case for $L_r=20L_c$, $\delta=0$ and $A_0=0.01$; $|A|^2$ as a function of the position $\bar{z}$ through the wiggler, at the fixed position within the electron pulse, $z_1=30$.

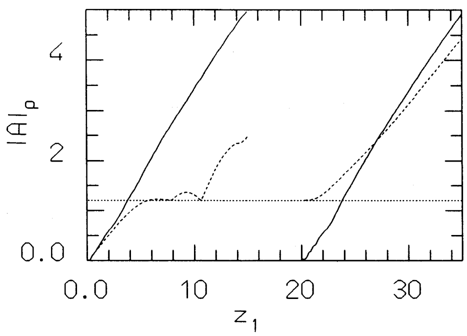

FIG. 3. Peak amplitude $|A|_p$ as a function of z_1 for $\delta=2$ (solid line) and $\delta=0$ (dashed line); $L_r=20L_c$ and $A_0=0.01$.

the electron density $\bar{n}_e$ (and hence the cooperation length) is changed [2]. In fact, as $|E|^2 \propto \rho \bar{n}_e |A|^2$ and $z_1 \propto \rho \propto \bar{n}_e^{1/3}$, then, if $|A|_p$ is proportional to z_1, $|E|^2$ is proportional to the square of the electron density $\bar{n}_e$ (superradiance). The curves on the left-hand side of Fig. 3, for $0<z_1<15$, correspond to the spikes in the trailing edge: for $\delta=0$, $|A|_p$ is linear for small z_1 (weak superradiance), and flatten out for larger values of z_1 (steady-state emission); however, for larger values of z_1, the trailing-edge spike reaches a peak value greater than the steady-state saturation value and shows an almost linear dependence (strong superradiance); for $\delta=2$, there is a continuous transition from the weak superradiance to the strong superradiance. The linear dependence on z_1 of $|A|_p$ is in this case more evident. The curves on the right-hand side of Fig. 3, for $20<z_1<35$, correspond to the spikes in the leading edge: for $\delta=0$, $|A|_p$ grows linearly from the saturated steady-state value 1.2, whereas for $\delta=2$ $|A|_p$ grows from the initial value $|A_0|$. Hence, we can conclude that the spikes observed are superradiant. A linear analysis of superradiance [3,4] leads to the following principal results: (i) The optical intensity in the slippage regions grows as $|A|^2 \propto \exp(3\sqrt{3}y^{2/3})$, for all values of the detuning parameter δ, with $y=(z_1)^{1/2}z_2$, instead of the usual exponential growth as $|A|^2 \propto \exp(\sqrt{3}\bar{z})$ at resonance ($\delta=0$) in the steady-state region. (ii) There exists a particular solution of the propagation equations (1) with the following self-similar form:

$$A(z_1,z_2)=z_1\exp(-i\delta z_2)A_1(y)\,, \qquad (2a)$$

$$\theta_j(z_1,z_2)=\theta_{1j}(y)+\delta z_2\,, \qquad (2b)$$

where A_1 and θ_{1j} are functions on y. By substituting (2) in the propagation equations (1), we obtain a set of ordinary differential equations for the variables A_1 and θ_1:

$$\frac{d^2\theta_{1j}}{dy^2}=-[A_1\exp(i\theta_{1j})+\text{c.c.}]\,, \qquad (3a)$$

$$\frac{y}{2}\frac{dA_1}{dy}+A_1=\langle\exp(-i\theta_1)\rangle\,. \qquad (3b)$$

The particular solution (2) describes a solitary wave, with the peak intensity proportional to z_1^2 and the width proportional to $z_1^{-1/2}$, i.e., inversely proportional to the fourth root of the peak intensity.

It has been shown [7] that the solution of Eqs. (3) is described approximately by the expression:

$$|A_1|^2=(3\sqrt{3}/4)(2/y)^{4/3} \times \text{sech}^2\{(3\sqrt{3}/2)(y/2)^{2/3} - \tfrac{1}{2}\ln[36\pi\sqrt{3}(y/2)^{2/3}/|b_0|^2]\}\,, \qquad (4)$$

where b_0 is the initial value $A_1(0)=\langle\exp[-i\theta_1(0)]\rangle$. The results of Fig. 3 are in agreement with the similarity law of Eq. (2a), $|A|=z_1|A_1|$. Furthermore, it is possible to show numerically [4] that all the spikes observed in the superradiant regime, such as those in Figs. 1 and 2, are very well described by the particular solution (2a), where A_1 is obtained by numerical integration of Eqs. (3) or, for the main pulse, by Eq. (4).

III. DISCUSSION

In this paper we report analytic and numerical evidence of superradiant spiking phenomena in a radiation pulse which propagates within a (longer) electron pulse in a high-gain FEL. This completes the previous analysis [2–4], which refers to an initial excitation distributed over all of the electron pulse, and in which only a trailing-edge spike exists. On the contrary, if the electron pulse is longer than the initial excitation, spiking is observed in both the trailing *and* leading slippage regions of the pulse, provided that the initial radiation pulse is much longer than the cooperation length L_c. If the radiation pulse length is of the order of, or shorter than, the cooperation length, only one spike is observed.

Central to the description of the mechanism in both trailing- and leading-edge superradiance for long radiation pulses is the relative slippage of the radiation and electron beams. In the trailing superradiant region, no radiation propagates into the slippage region at the rear of the electron pulse. The electrons in this region do not interact with a steadly growing radiation field, and no steady-state saturation mechanism occurs. This leads to the establishment of a weak superradiant pulse of radiation, which may be amplified further on propagating through the electron pulse, giving rise to spiking behavior [2]. This spiking exists for a wide range of values of the detuning parameter [8], and in particular for values greater than the threshold value above which no steady-state type exponential growth of the radiation field exists. Better amplification of the superradiant pulse occurs when it interacts with unperturbed electrons, i.e., those that have not been bunched nor have an energy spread, as in the detuned case. A slippage region also occurs one slippage distance forward of the initial leading edge of a radiation pulse propagating within a longer electron pulse. This region leads to the strong superradiance emission of radiation. What is happening in this region can be described in terms analogous to that of the trailing-edge slippage region in the detuned case: unperturbed electrons propagate into the slippage region at the front of the radiation pulse where no steady-state mechanism is possible. This slippage again gives rise to typical superradiant behavior.

The superradiant pulses narrow as the peak of the intensity increases, with a width inversely proportional to the fourth root of the peak intensity. Moreover, the pulse does not change in shape, but conserves a well-defined profile that can be obtained from the solution of the self-similar equations (3).

We remark on two aspects in which the work presented here provides a basis of ongoing research: (i) If a single radiation pulse evolving in an effectively infinite length electron pulse gives rise to superradiant evolution, as demonstrated above, it is possible to envisage conditions under which perturbations to a uniform intensity, infinite length radiation pulse (which would normally evolve with a SS behavior) may give rise to superradiant evolution. (A single radiation pulse, as shown, gives rise to superradiant evolution and can be considered as a perturbation to a zero intensity infinite length pulse.) The conditions under which such superradiant evolution may be observed is currently under investigation. This also suggests a connection between superradiance and the so-called "sideband" instability [9]. (ii) The above results may also be relevant to the understanding of spiking behavior in the FEL oscillator [10,11] in the short electron pulse regime (when $L_e \sim L_s$). Consider the case of the propagation of a short radiation pulse through an electron pulse of length $L_e \sim L_s$ over many round-trips within the cavity of a FEL oscillator: If, at the wiggler entrance, the radiation pulse is on the trailing edge of the electron pulse then it will propagate through to the leading edge at the end of the wiggler. This process is to be repeated for the number of round-trips within the cavity. This evolution may be considered tentatively equivalent to a single-pass FEL with a long wiggler in which a short initial radiation pulse propagates through a long electron pulse starting from its trailing edge. In this way, the single pass, short-pulse superradiant evolution considered above [Fig. 1(b)] may be related to the short-pulse case in the oscillator indicating possible superradiant behavior in a low-gain oscillator system. (Initial results indeed confirm the equivalence of the two systems.) Superradiance may also explain spiking behavior occurring in oscillators with electron pulses of length $L_e \gg L_s$ [12,13]. (This phenomena has usually been related to the sideband instability.) A periodic structure of radiation spikes, with period equal to the slippage length L_s, has been observed [12,13]. The electron and radiation interaction of such a long-pulse structure may be split into consecutive regions each of length of the slippage distance L_s. Each section then evolves independently of the others every cavity round trip. We hypothesize that due to some mechanism [such as outlined in (i), above] spikes may begin to develop within each section of the radiation pulse. Those spikes separated by a slippage length will dominate and the radiation pulse will then begin to evolve as a series of short pulses, each of length L_s and developing a single spike.

Work is progressing in these areas and initial results appear to concur with the hypothesis outlined above. A more detailed discussion and results will appear in a future paper.

ACKNOWLEDGMENTS

This work was supported by the Istituto Nazionale di Fisica Nulceare (INFN), sezione di Milano.

[1] R.Bonifacio, C. Pellegrini, and L. Narducci, Opt. Commun. **50**, 373 (1984).
[2] R. Bonifacio, B. W. J. McNeil, and P. Pierini, Phys. Rev. A **40**, 4467 (1989).
[3] R. Bonifacio, C. Maroli, and N. Piovella, Opt. Commun. **68**, 369 (1988).
[4] R. Bonifacio, L. De Salvo Souza, P. Pierini, and N. Piovella, Nucl. Instrum. Methods Phys. Res. Sect. A **296**, 358 (1990).
[5] G. Moore, Nucl. Instrum. Methods Phys. Res. Sect. A **239**, 19 (1985).
[6] S. Y. Cai, J. Cao, and A. Bhattacharjee, Phys. Rev. A **42**, 4120 (1990).
[7] N. Piovella, Opt. Commun. **83**, 92 (1991).
[8] W. M. Sharp, W. M. Fawley, S. S. Yu, A. M. Sessler, R. Bonifacio, and L. De Salvo, Nucl. Instrum. Methods Phys. Res. Sect. A **285**, 217 (1989).
[9] N. M. Kroll, H. V. Wong, and B. N. Moore, Phys. Fluids B **2**, 1635 (1990).
[10] W. B. Colson and A. Renieri, J. Phys. (Paris) Colloq. **44**, C1-11 (1983).
[11] G. Moore and J. C. Goldstein, Nucl. Instrum. Methods Phys. Res. Sect. A **285**, 176 (1989).
[12] R. W. Warren, J. C. Goldstein, and B. E. Newnam, Nucl. Instrum. Methods Phys. Res. Sect. A **250**, 19 (1986).
[13] R. W. Warren and J. C. Goldstein, Nucl. Instrum. Methods Phys. Res. Sect. A **272**, 155 (1988).