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Origin of gain in systems without inversion in bare or dressed states

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We analyze the origin of gain in systems that do not exhibit population inversion either in terms of the bare states or dressed states. Such systems involve light amplification by coherence. We show that gain arises from coherence (a) among Fano states in Arkhipkin-Heller-Harris-like systems and (b) between the two dressed states in the Λ system of Imamoğlu, Field, and Harris [Phys. Rev. Lett. **66**, 1154 (1991)].

INTRODUCTION

Many schemes for laser action without population inversion have been proposed.[1-6] While some schemes depend on interference effects, others make use of external fields. The external fields change absorption and emission profiles from those of the bare atoms leading to the possibility of gain even in the absence of population inversion. In several cases[5,6] one finds that the gain can be attributed to the population inversion between two dressed states even though there is no population inversion between the two bare states. For example, consider a two-level system of frequency ω_0 driven strongly by a coherent field of frequency ω_l and amplitude ϵ. One finds that there is gain for a probe field[6] if the probe frequency ω is in the neighborhood of the three-photon Mollow side band, i.e.,

$$\omega = \omega_l - \left\{ (\omega_0 - \omega_l)^2 + 4 \left| \frac{\mathbf{d}\cdot\epsilon}{\hbar} \right|^2 \right\}^{1/2},$$

where $\mathbf{d}$ is the dipole matrix element. One can understand[5] this gain in terms of the inversion in semiclassical dressed states obtained by diagonalization of the Hamiltonian for the two-level system interacting with the external coherent field of frequency ω_l. There are, however, several other schemes in which population inversion does not occur in any basis. We consider two of these schemes.

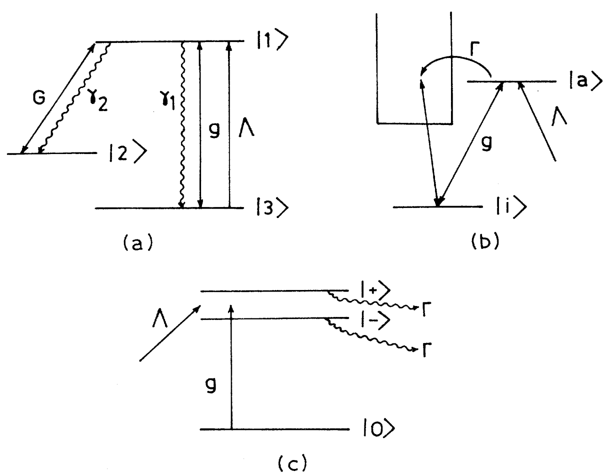

FIG. 1. Schematic representation of different schemes where coherence effects are the key to laser action.

a. Three-level Λ system. Imamoğlu, Field, and Harris[7] have analyzed a three-level Λ system[8] [Fig. 1(a)] and have shown that gain can be obtained under certain conditions. The state $|1\rangle$ is pumped incoherently from both the lower states $|2\rangle$ and $|3\rangle$. In addition, a coherent field couples the transition $|1\rangle \leftrightarrow |2\rangle$. The laser transition, i.e., the transition on which the probe field is amplified, is the transition $|1\rangle \leftrightarrow |3\rangle$. Imamoğlu, Field, and Harris found gain even in the absence of population inversion. In the presence of the strong coherent field on the transition $|1\rangle \leftrightarrow |2\rangle$, the states $|1\rangle$ and $|2\rangle$ go over to the dressed states $|\psi_\pm\rangle$. The bare level $|3\rangle$ is connected to $|\psi_\pm\rangle$ by the weak probe field. The amplification can take place even if the total population of the states $|\psi_+\rangle$ and $|\psi_-\rangle$ is less than that of $|3\rangle$.

b. Autoionizing system. We next consider the scheme[1] involving the pumping of the autoionizing state $|a\rangle$ as shown in Fig. 1(b). The state $|a\rangle$ is pumped at the rate Λ leading to a population distribution in the states $|a\rangle$ and $|i\rangle$ with $\rho_{aa} = \Lambda/\Gamma$. Here, Γ is the rate of autoionization. Even if $\rho_{aa} < \rho_{ii}$, one can have gain for a certain frequency range depending on the parameters Λ, Γ, and the Fano asymmetry parameter q.

The question thus arises—what is the source of gain? In this paper, we analyze this question and demonstrate that gain arises from finite "coherence" between the appropriate dressed states $|\psi_+\rangle$ and $|\psi_-\rangle$. The coherence between $|\psi_+\rangle$ and $|\psi_-\rangle$ is induced by the coherent pump on the transition $|1\rangle \leftrightarrow |2\rangle$. Note that $|\psi_+\rangle$ and $|\psi_-\rangle$ are *not* connected by dipole transition. The induced coherence depends on the strength of the coherent field and the rate of incoherent pumping. Systems with such gain can be identified by the term light amplification by coherence. We next give explicit calculations for the two schemes mentioned above.

GAIN IN THE SCHEME OF IMAMOĞLU, FIELD, AND HARRIS

We demonstrate how coherence effects lead to gain in the level scheme of Imamoğlu, Field, and Harris. Our purpose here is not to show the existence of gain, but to establish the origin of such a gain. In order to keep the analysis simple, we consider the case when (i) the coherent field is on resonance with the $|1\rangle \leftrightarrow |2\rangle$ transition and (ii) the level $|1\rangle$ is pumped incoherently only from the level $|3\rangle$. The transition $|1\rangle \leftrightarrow |3\rangle$ is the laser transition. We examine the question of gain of a probe field of frequency ω_l on the transition $|1\rangle \leftrightarrow |3\rangle$. The density-matrix equation in the rotating frame can be written as

$$\frac{\partial \rho}{\partial t} = -\gamma_1(A_{11}\rho - 2\rho_{11}A_{33} + \rho A_{11}) - \gamma_2(A_{11}\rho - 2\rho_{11}A_{22} + \rho A_{11}) - \Lambda(A_{33}\rho - 2\rho_{33}A_{11} + \rho A_{33})$$
$$+ iG[|1\rangle\langle 2| + |2\rangle\langle 1|, \rho] + ig[|1\rangle\langle 3| + |3\rangle\langle 1|, \rho], \quad g = \frac{\mathbf{d}_{31}\cdot\epsilon}{\hbar}, \quad G = \frac{\mathbf{d}_{21}\cdot\epsilon}{\hbar}, \tag{1}$$

where we have assumed that the probe field is also on resonance with the $|1\rangle \leftrightarrow |3\rangle$ transition and $A_{\alpha\beta}$ are the transition operators

$$A_{\alpha\beta} = |\alpha\rangle\langle\beta| . \tag{2}$$

In Eq. (1), $2G(2g)$ is the Rabi frequency of the field on the transition $|1\rangle \leftrightarrow |2\rangle$ (probe field). Assume for simplicity that g and G are real and positive. The induced polarization at the probe frequency ω_l is given by $P(\omega_l) = d_{31}\rho_{13}$. For the purpose of calculation of gain, it is sufficient to know the density-matrix element ρ_{13} to first order in g. Such a calculation is straightforward and one finds the gain for a certain range of parameters even in the absence of inversion.

In order to understand the origin of the gain, we transform to a new basis (dressed states) defined by

$$\psi_\pm = \frac{1}{\sqrt{2}}(|1\rangle \pm |2\rangle), \quad \psi_0 = |3\rangle, \quad |1\rangle = \frac{1}{\sqrt{2}}(\psi_+ + \psi_-) . \tag{3}$$

The induced polarization in the dressed-state basis is

$$P(\omega_l) = \frac{d_{31}}{\sqrt{2}}(\rho_{+0} + \rho_{-0}) . \tag{4}$$

This leads to gain at ω_l if $\mathrm{Im}(\rho_{+0} + \rho_{-0}) < 0$. In the dressed-state basis the equations of motion for $\rho_{\pm 0}$ are

$$\dot\rho_{\pm 0} = \frac{+ig}{\sqrt{2}}(\rho_{00} - \rho_{\pm\pm}) - \frac{ig}{\sqrt{2}}\rho_{\pm\mp} - \Gamma\rho_{\pm 0} - \gamma_0\rho_{\mp 0} \pm iG\rho_{\pm 0} ,$$
$$\Gamma = \Lambda + \gamma_0, \quad \gamma_0 = \frac{\gamma_1 + \gamma_2}{2} . \tag{5}$$

The steady-state polarization can be obtained from Eq. (5) and from the values of ρ_{00}, $\rho_{\pm\pm}$, and ρ_{+-} in the absence of the probe field. Before we examine gain of the probe field, we examine the character of the steady state in the absence of the probe field. The populations of the dressed states in the limit $G \gg \gamma_1, \gamma_2, \Lambda$ satisfy

$$\dot\rho_{\pm\pm} = \Lambda\rho_{00} \mp \frac{\gamma_2}{2}(\rho_{++} - \rho_{--}) - \gamma_1\rho_{\pm\pm} , \tag{6}$$

which leads to

$$\rho_{00} = \frac{\gamma_1}{\gamma_1 + 2\Lambda} + O\left(\frac{\gamma}{G}\right)^2, \quad \rho_{++} = \rho_{--} = \tfrac{1}{2}(1 - \rho_{00}) , \tag{7}$$

and hence

$$\rho_{++} + \rho_{--} - \rho_{00} = \frac{2\Lambda - \gamma_1}{2\Lambda + \gamma_1} . \tag{8}$$

Thus, the population in the dressed states $|\psi_+\rangle$ and $|\psi_-\rangle$ will be less than that in the ground state if

$$\gamma_1 > 2\Lambda . \tag{9}$$

In the steady-state, Eq. (5) shows that $\rho_{\pm 0} = 0$ if $g = 0$. The equation of motion for ρ_{+-} leads to

$$\rho_{+-} = (\gamma_1 + 3\gamma_2/2 - 2iG)^{-1}[\rho_{00}(\Lambda + \gamma_2 + \gamma_1/2) - (\gamma_2 + \gamma_1/2)] \sim 0(\gamma/G) . \tag{10}$$

We thus find that the steady state is such that (i) there is no *population inversion* if $\gamma_1 > 2\Lambda$; (ii) there are *no optical coherences,* i.e., $\rho_{\pm,0} = 0$; (iii) the *coherence* ρ_{+-} *between two dressed states is nonzero.* In the following, we show that the *gain arises from nonzero coherence* ρ_{+-}. We write the approximate solution of Eq. (5) as

$$\mathrm{Im}\rho_{\pm 0} = (+g/\sqrt{2}G^2)[\Gamma(\rho_{00} - \rho_{\pm\pm}) - \gamma_0(\rho_{00} - \rho_{\mp\mp}) + \mathrm{Re}(\mp iG\rho_{\pm\mp})] , \tag{11}$$

and hence

$$\mathrm{Im}(\rho_{+0} + \rho_{-0}) = (+g/\sqrt{2}G^2)[(3\rho_{00} - 1)\Lambda - \mathrm{Re}(iG\rho_{+-} - iG\rho_{-+})] . \tag{12}$$

Note that if we had *ignored* the contribution from the dressed-state coherence, then

$$\mathrm{Im}(\rho_{+0} + \rho_{-0}) = (+g\Lambda/\sqrt{2}G^2)(3\rho_{00} - 1) \equiv +2g\Lambda(\gamma_1 - \Lambda)/\sqrt{2}G^2(\gamma_1 + 2\Lambda) . \tag{13}$$

One has no possibility of gain as γ_1 by definition is bigger than Λ. The coherence contribution in Eq. (12) using (10) becomes

$$-iG\rho_{+-} + \text{c.c.} = \rho_{00}(\Lambda + \gamma_2 + \gamma_1/2) - (\gamma_2 + \gamma_1/2) = -2\gamma_2\Lambda/(\gamma_1 + 2\Lambda) . \tag{14}$$

It is important to note that the coherence contribution in Eq. (12) has a sign opposite to that of the population term and thus if the *contribution of the coherence term exceeds that of the population term,* then one can have gain. Upon substituting Eqs. (13) and (14) in (12) we obtain

$$\mathrm{Im}(\rho_{+0} + \rho_{-0}) = (+2g/\sqrt{2}G^2)\left[\frac{\Lambda(\gamma_1 - \Lambda) - \gamma_2\Lambda}{\gamma_1 + 2\Lambda}\right] . \tag{15}$$

Hence, the model will exhibit gain if

$$\gamma_2 > (\gamma_1 - \Lambda) . \tag{16}$$

Condition (16) implies that the rate of spontaneous emission for the transition $|1\rangle \leftrightarrow |2\rangle$ is bigger than the rate for the transition $|1\rangle \leftrightarrow |3\rangle$. This condition is identical to that given in Ref. 7. Finally, note that this system in the dressed-state basis ψ_+, ψ_-, and ψ_0 is reminescent of the Hanle system where it is known how the Hanle coherence

(ρ_{+-} in the present case) affects the radiative properties of the system.

GAIN IN ARKHIPKIN-HELLER-HARRIS-LIKE SCHEME

Having seen how the excited-state coherence can lead to gain in the scheme of Fig. 1(a), we examine the origin of gain in the scheme which uses autoionizing states. As discussed elsewhere,[1] the absorption and emission profiles for the system [Fig. 1(b)] are such that gain is possible if

$$\frac{\Lambda(1+q^2)}{1+\delta^2} > \frac{\Gamma(\delta+q)^2}{1+\delta^2}, \quad \delta=\frac{2}{\Gamma}(E_a-E_i-\omega_0). \tag{17}$$

The pumping of the state $|a\rangle$ produces a population $p_a=\Lambda/\Gamma$ in this state. For studying the dynamics, it is convenient to diagonalize the configuration interaction. This was already done by Fano.[9] One introduces states $|E\rangle$ (structured continuum) related to the unperturbed continuum via

$$|E\rangle=\frac{\sin\Delta}{\pi V_{E_a}}|a\rangle+\int [V_{E'_a}\sin\Delta/\pi V_{E_a}(E-E') -\cos\Delta\delta(E-E')]|E'\rangle dE', \tag{18}$$

$$\tan\Delta=-\pi|V_{E_a}|^2/(E-E_a), \quad \langle a|E\rangle=b(E,a).$$

Note that a steady-state population in the state $|a\rangle$ leads to

$$\rho_{E_1E_2}=\langle E_1|\rho|E_2\rangle=(\sin\Delta_{E_1}\sin\Delta_{E_2}/\pi^2V_{E_1a}V_{E_2a})p_a. \tag{19}$$

Equation (19) shows that the pumping of the autoionizing state $|a\rangle$ leads to *coherence in the structured continuum.* However, there is *no optical coherence,* i.e., $\rho_{Ei}=0$. It is the coherence $\rho_{EE'}$ which leads to gain even if the total population p_a in the excited states is less than the population in the ground state $|i\rangle$. The Laplace transform of the density matrix for this system can be written in the form

$$\tilde{\rho}(z)=p_i(z-L)^{-1}|i\rangle\langle i|+p_a\frac{\Gamma}{z}(z-L)^{-1} \times\int dE_1\int dE_2|E_1\rangle\langle E_2|b(E_1,a)b^*(E_2,a), \tag{20}$$

where L is the Liouville operator for the system in the absence of pumping. The contribution from the coherence terms can be evaluated by standard techniques leading to the known expression for gain.

CONCLUSIONS

We conclude this paper by pointing out how coherence effects change the entire situation in the framework of a simple model. Consider the model shown schematically in Fig. 1(c). We assume that appropriate diagonalization, etc., has been done so that the states $|+\rangle$, $|-\rangle$, etc., *correspond to dressed states.* The gain $\mathcal{G}$ in this model can be shown to be

$$\mathcal{G}=|d_{+0}|^2(\rho_{++}-\rho_{00})\frac{\Gamma/2}{(\Gamma/2)^2+\Delta_{+0}^2}+\mathrm{Re}\left[\frac{d_{0+}d_{-0}\rho_{+-}}{\Gamma/2+i\Delta_{+0}}\right] +(+\leftrightarrow-), \quad \Delta_{+0}=E_+-E_0-\omega_l \tag{21}$$

where $(+\leftrightarrow-)$ means the preceding terms with subscript + replaced by −. Note that the populations, coherences ρ_{+-}, dipole matrix elements, Δ_{+0}, etc., depend on the strength of the external fields used to create dressed states. Equation (21) shows the role of the coherence term in producing gain ($\mathcal{G}>0$) even if there is no population inversion.

In conclusion, we have shown how gain in systems that have no inversion in any atomic basis can be understood in terms of the coherence in appropriate basis of states.

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