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Deflection of atoms by a quantum field

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We consider the scattering of atoms off a standing electromagnetic wave in a given quantum state and show that the photon statistics can directly manifest itself in the momentum distribution of scattered atoms. Revival-like structures in the distribution function, resulting from quantization, appear as well.

Scatter atoms off a standing electromagnetic wave and ask for the probability to find a given momentum in the scattered beam: Should we treat the principal ingredients, that is, the internal degrees of freedom of the atom, the translational motion, and the electromagnetic field, classically or quantum mechanically? Various combinations of these approaches have been treated previously [1]–[5]. Motivated by the experimental progress in producing nonclassical fields, in this Rapid Communication we consider the influence of a *quantized* electromagnetic field on the momentum distribution of the scattered atoms [6]. The model we study consists of a monokinetic plane wave of two-level atoms propagating perpendicular to a resonant light field. We neglect spontaneous emission and assume that the displacement of the atom induced by the field is small compared to the wavelength of the light. We show that in this case the photon statistics governs the momentum distribution: When the translational motion of the atom is not affected by quantum-mechanical interference and can be treated classically the photon distribution appears directly in the momentum distribution.

A sinusoidally phase modulating structure scatters a plane wave into a distribution of scattered intensities described by the square of Bessel functions [7]. This result carried over to the problem of Bragg scattering of a two-level quantum particle by a resonant *classical* standing electromagnetic wave [5] provides immediately the momentum distribution

$$W_\wp = J_\wp^2(\mathcal{E}\mu\tau/\hbar). \quad (1)$$

Here we consider an atom of dipole moment μ interacting for a time τ with the field of amplitude $\mathcal{E}$ and $\wp = p/(\hbar k)$ denotes the momentum expressed in terms of the photon momentum.

On the other hand, a field in a quantum state $|\psi\rangle = \sum_m w_m|m\rangle$ creates a momentum distribution of scattered atoms [8]

$$W_\wp[|\psi\rangle] = \sum_{m=0}^{\infty} W_m[|\psi\rangle]\, J_\wp^2(\kappa\sqrt{m}). \quad (2)$$

Here $W_m[|\psi\rangle] = |w_m|^2 = |\langle m|\psi\rangle|^2$ denotes the photon statistics of the field. The interaction parameter reads $\kappa = \mu\mathcal{E}_0\tau/\hbar$, where $\mathcal{E}_0$ is the amplitude of vacuum electric field. Equation (2) shows that each number state of the quantum field contributes to the probability of finding a given momentum independently and the contribution from each individual number state, $|m\rangle$, is identical to that of a classical field, Eq. (1), of strength $\mathcal{E} = \mathcal{E}_0\sqrt{m}$.

In the remainder of this article we investigate the influence of the photon statistics on the momentum distribution. The momentum distribution as a readout of the photon distribution is the key result of this Rapid Communication. We illustrate this statement by numerically evaluating the sum, Eq. (2), for (i) a number state, (ii) a coherent state, and (iii) a squeezed state [9]. A semiclassical approach, that is, an asymptotic expansion of the Bessel functions[8], brings to light the striking features revealed by the numerical work.

A field in a single number state $|m\rangle$ reduces the sum over Bessel functions to

$$W_\wp[|m\rangle] = J_\wp^2(\kappa\sqrt{m}). \quad (3)$$

Figure 1(a) shows this distribution for a number state of nine photons, $m = 9$ for an interaction parameter $\kappa = 10$. As a result of the property of Bessel functions, $J_{-\wp} = (-1)^\wp J_\wp$, the momentum distribution is symmetric with respect to $\wp = 0$. We therefore here and in the later figures depict the region of positive $\wp$ values only. We recognize a dominant maximum at $\wp_{\max} = \kappa\langle m\rangle^{1/2} = 10 \times 3 = 30$. For momenta smaller than $\wp_{\max}$ we find a rapid modulation of the smooth dependence, whereas for momenta larger than $\wp_{\max}$ the momentum distribution displays a steep decay. The rapid modulation becomes even more pronounced when we increase the interaction parameter κ, as shown in Fig. 1(b) for $\kappa = 100$. The enlargement reveals in Fig. 1(c) the detailed structure of this oscillation for small momenta, a pattern reminiscent of the Jaynes-Cummings revivals [10].

These phenomena come to light when we put to use the appropriate asymptotic expansion of the Bessel function [8]

$$J_\wp(z) \cong (2/\pi)^{1/2}(z^2 - \wp^2)^{-1/4} \times \cos[(z^2 - \wp^2)^{1/2} - \wp \arccos(\wp/z) - \pi/4], \quad (4)$$

valid for $|\wp| < z$. For $|\wp| > z$ the analytical continuation of this expression leads to an exponential decay. Therefore this region contributes little to the momentum distribution, and in the remainder of this article we put $J_\wp(z) = 0$ for $|\wp| > z$.

This expression for $J_\wp(z)$ is our main tool to gain immediate insight into the behavior of the Bessel function sum, Eq. (2). It brings out two parts in the momentum distribution, Eq. (3), of distinct dependence on $\wp$.

(i) The contribution

$$W_\wp^{(\text{smooth})}[|m\rangle] = \pi^{-1}(\kappa^2 m - \wp^2)^{-1/2} \quad (5)$$

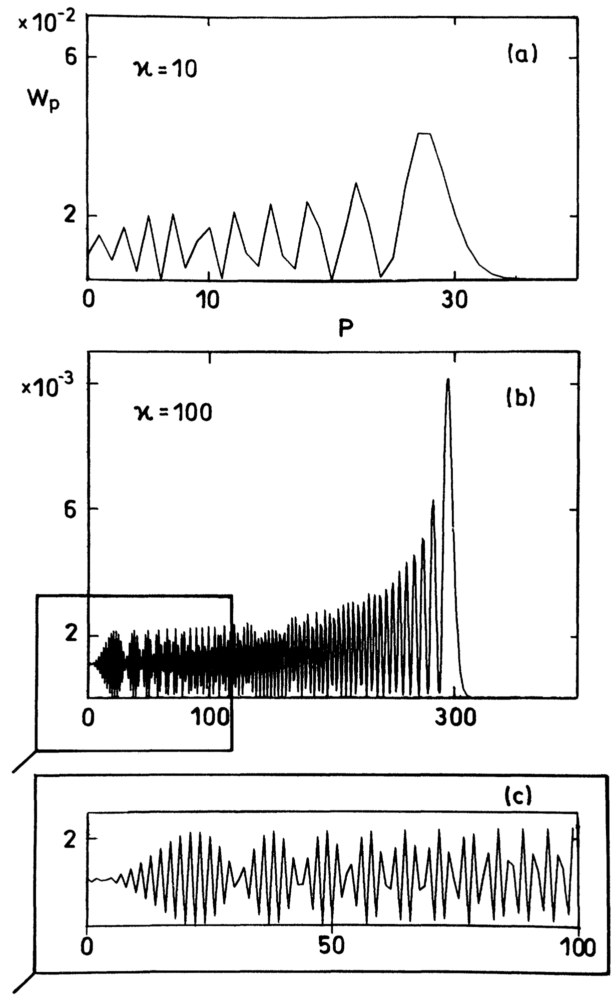

FIG. 1. Momentum distribution $W_\wp[|m\rangle] = J_\wp^2(\kappa\sqrt{m})$ of atoms scattered off an electromagnetic field in a number state of nine photons, $m = 9$ for interaction parameters $\kappa = 10$ (a) and $\kappa = 100$ (b). Both distributions show a dominant peak at $\wp_{\max} = \kappa\sqrt{m}$ and a strong decay for momenta larger than this critical value. For $\wp$ smaller than $\wp_{\max}$ the distribution is oscillatory. These oscillations result from *quantum interference* of translational motion. The envelope follows the classical cross section. A tenfold increase of the interaction parameter rescales the envelope but does not affect its shape. The oscillatory part creates a complex pattern magnified in (c) for small momenta.

is the well-known differential cross section of classical scattering [11]. It is a slowly varying function of $\wp$ and describes the form of the envelope of the momentum distribution.

(ii) The rapidly oscillating term

$$W_\wp^{(\text{rapid})}[|m\rangle] = \pi^{-1}(\kappa^2 m - \wp^2)^{-1/2} \times \cos\left(2\{(\kappa^2 m - \wp^2)^{1/2} - \wp \arccos[\wp/(\kappa\sqrt{m})] - \pi/4\}\right) \quad (6)$$

results from semiclassical considerations of Bragg scattering. For small momenta, $|\wp| \ll \kappa\sqrt{m}$, an expansion of the argument of the cosine function in Eq. (6),

$$W_\wp^{(\text{rapid})}[|m\rangle] \cong (\pi\kappa\sqrt{m})^{-1}(-1)^\wp \times \sin\left[2\kappa\sqrt{m} + \frac{\kappa}{\sqrt{m}}\left(\frac{\wp}{\kappa}\right)^2\right] \quad (7)$$

reveals the revival-like behavior [10, 12] of the distribution close to the origin.

Motivated by these results we now return to the case of arbitrary statistics as given by the Bessel function sum, Eq. (2). We therefore substitute Eqs. (5) and (6) into Eq. (2) and find [8]

$$W_\wp[|\psi\rangle] \cong W_\wp^{(\text{smooth})} + W_\wp^{(\text{rapid})}.$$

The smooth part

$$W_\wp^{(\text{smooth})} = 2(\pi\kappa)^{-1}\int_0^\infty dy\, W_{m=y^2+(\wp/\kappa)^2} \quad (8)$$

follows from replacing the discrete summation over m by integration over m and introducing the new integration variable $y = [m - (\wp/\kappa)^2]^{1/2}$. The part $W_\wp^{(\text{rapid})}$ originates from Eq. (6) and is more complicated [8].

We now discuss the general features of $W_\wp^{(\text{smooth})}$. This contribution possesses properties of a probability distribution, that is, it is normalized to unity and accounts for the lower moments of the momentum distribution. For instance, the separation of the dominant maximum from the origin, as expressed by the second moment $\langle\wp^2\rangle$, reads $\langle\wp^2\rangle = \frac{1}{2}\kappa^2\langle m\rangle$. This is in full accord with the exact expression following [8] from Eq. (2). Moreover, the normalized width of the photon distribution, $\sigma^2 \equiv \langle m^2\rangle/\langle m\rangle - \langle m\rangle$ rules the value of the smooth part at $\wp = 0$, that is,

$$W_{\wp=0}^{(\text{smooth})} = (\pi\kappa)^{-1}\langle m\rangle^{-1/2}\left(1 + \frac{3}{8}\frac{\sigma^2}{\langle m\rangle} + \cdots\right).$$

The scaling law

$$W_\wp^{(\text{smooth})}(\lambda\kappa) = \lambda^{-1} W_{\wp/\lambda}^{(\text{smooth})}(\kappa)$$

is an immediate consequence of Eq. (8) and is apparent in Figs. 2 and 3. It follows from classical mechanics: The particle acquires a momentum identical to the product of the force and the interaction time, that is, a quantity proportional to the interaction parameter κ.

The photon distribution W_m determines the momentum distribution $W_\wp$, Eq. (2). But is the converse true? Is it possible to recognize W_m in $W_\wp$? The two examples shown in Figs. 2 and 3 of a coherent state and of a highly squeezed state seem to suggest that. The photon distribution explicitly manifests itselfs in the smooth part of the momentum distribution. The strong correlation between W_m and $W_\wp$ stands out most clearly for moderate values of the interaction parameter κ, that is, when κ is large enough to sharpen the smooth part of $J_\wp^2(\kappa\sqrt{m})$ allowing it to trace the variation of W_m, and still small enough as to keep $W_\wp^{(\mathrm{rapid})}$ small. For a rather small or a rather large value of κ, however, this inverse problem, that is, the determination of W_m from $W_\wp$ does not have such an explicit graphical solution.

Now we turn to the rapid part of the distribution function that results from quantum interference and makes its appearance for large values of κ. The mathematical analysis of this contribution is much more difficult compared to the smooth one and a detailed treatment will be presented elsewhere [8]. In this Rapid Communication we confine ourselves to the case of $(\wp/\kappa)^2 \ll 1$. When we substitute Eq. (7) into Eq. (2) we arrive at

$$W_\wp^{(\mathrm{rapid})} \cong (-1)^\wp(\pi\kappa\sqrt{\langle m\rangle})^{-1}\left\{\frac{1}{2i}\left(\sum_{m=0}^{\infty} W_m e^{i2\kappa\sqrt{m}}\right)\exp\left[i\frac{\kappa}{\sqrt{\langle m\rangle}}\left(\frac{\wp}{\kappa}\right)^2\right]+\text{c.c.}\right\},$$

where we have replaced the summation index m by the average number of photons $\langle m\rangle$ in the slowly varying parts of Eq. (7). The sum in large parentheses is the well-known sum giving birth to the quantum revivals in the Jaynes-Cummings model [10, 12]. This sum, represented in the form [13] $\mathcal{A}(\kappa)e^{i\varphi(\kappa)}$, yields

$$W_\wp^{(\mathrm{rapid})} = (-1)^\wp(\pi\kappa\sqrt{\langle m\rangle})^{-1} \times\mathcal{A}(\kappa)\sin\left[\varphi(\kappa)+\frac{\kappa}{\sqrt{\langle m\rangle}}\left(\frac{\wp}{\kappa}\right)^2\right], \quad (9)$$

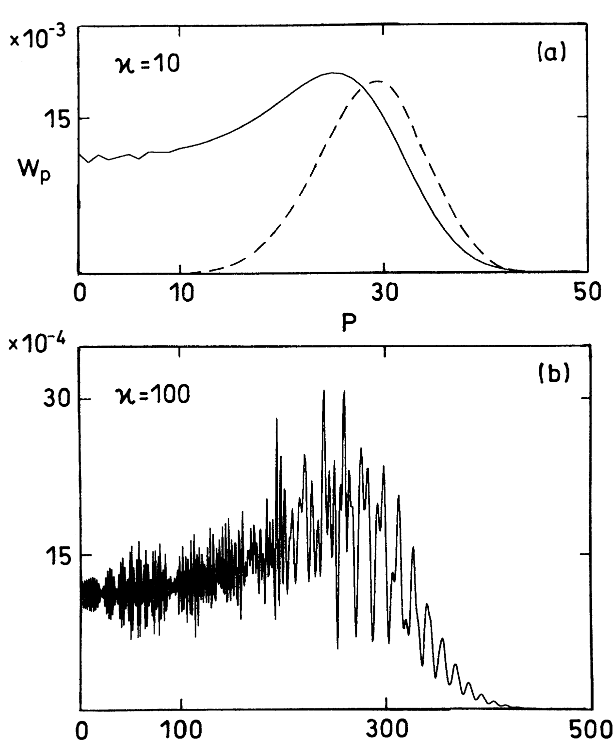

FIG. 2. Influence of the photon distribution W_m of a coherent state of average number of photons $\langle m\rangle = 9$ on the momentum distribution $W_\wp$. The Poissonian photon distribution—dashed curve in (a)—creates for $\kappa = 10$ a smooth momentum distribution $W_\wp$—solid line in (a). The maximum of W_m governs the maximum of $W_\wp$, that is, $\wp_{\max} \cong \kappa\langle m\rangle^{1/2}$. The right edge of W_m controls the right edge of $W_\wp$. The smooth part of $W_\wp$ obeys a scaling law while the rapidly oscillating part which is only minutely present for small p when $\kappa = 10$ (a) is significantly enhanced for $\kappa = 100$ (b).

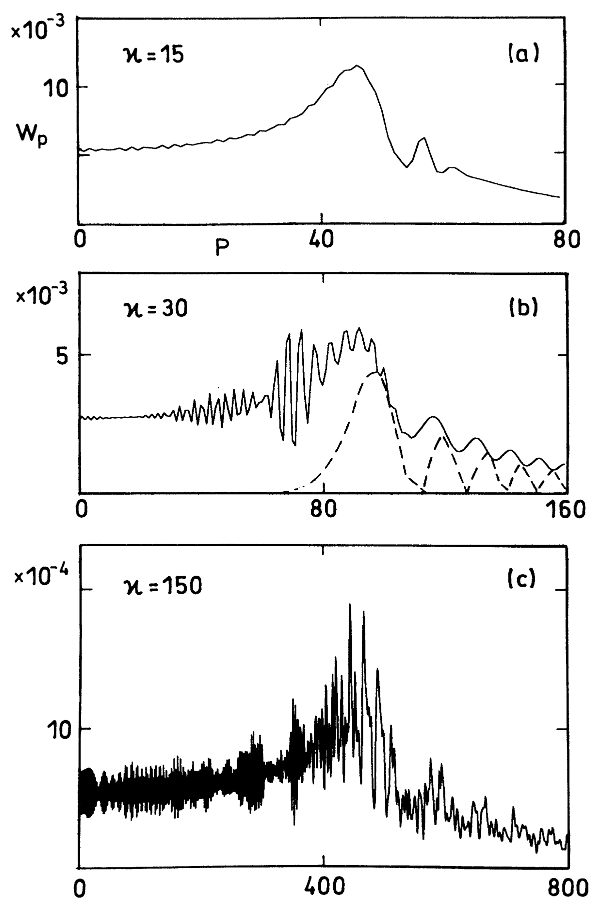

FIG. 3. Influence of the photon distribution W_m of a highly squeezed state of displacement $\alpha^2 = 9$ and squeezing $s = 50$ for three interaction parameters $\kappa = 15$ (a), $\kappa = 30$ (b), and $\kappa = 150$ (c). The oscillatory photon distribution—dashed curve in (b)—provokes modulations in the smooth part of the momentum distribution. Maxima of W_m correspond to maxima in $W_\wp$. This correspondence comes out most clearly for intermediate values of κ such as those shown in (b).

valid for $(\wp/\kappa)^2 \ll 1$. The quantum state of the field enters into Eq. (9) only via the amplitude $\mathcal{A}$ and the phase φ. Indeed, the revival-like patterns along the momentum axis created by the three different field states and presented in the three figures are all alike in the vicinity of origin. They only differ in the amplitude and the shift of these structures.

We conclude this article by summarizing our main results. The interplay between the internal degrees of freedom of the atom, its translational motion and the electromagnetic field is the central point of this problem. Throughout this article we consider two-level atoms, that is, particles with quantized internal degrees of freedom. When the *field* is *classical* or in a *photon number state*, and the translational *motion* of the two-level atom is treated *quantum mechanically* the momentum distribution is the well-known square of the Bessel function, Eqs. (1) and (3). This distribution consists of a smooth part representing classical motion and a rapidly oscillating contribution resulting from quantum interference of trajectories. The latter resembles the familiar quantum revivals of the Jaynes-Cummings model. In the case of *quantum motion* in a *quantum field* we find an explicit manifestation of the photon statistics in the smooth part of the momentum distribution. Interfering atomic trajectories again create revival-like structures in the probability curve for the momentum.

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