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Dark optical solitons with reverse-sign amplitude

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We prove the existence of anti-dark-solitons, i.e., dark solitons of the reverse sign of the amplitude, near the zero-group-dispersion wavelength of normally dispersive optical fibers using direct numerical simulations of the nonlinear Schrödinger equation which includes the third-order dispersion. These solitons were predicted earlier in the small-amplitude limit [Yu. S. Kivshar, Phys. Rev. A **43**, 1677 (1991)] and they may exist only for a certain propagation direction, so that the interaction of dark and anti-dark-solitons propagating in opposite directions is possible; this is probably the only possibility to observe direct collisions of dark and bright solitons. We investigate the collision numerically and demonstrate that it looks elastic at least for small-amplitude solitons.

The possibility of using bright soliton pulses as information carriers in optical communication systems has attracted considerable attention since it was shown theoretically and experimentally that the solitons can propagate in single-mode optical fibers without dispersion broadening (see Ref. [1] and references therein). Recent experimental achievments [2] demonstrate a possibility of real applications to construct soliton-based communication networks. In communication systems it is desirable to work near the zero-group-dispersion (ZGD) point [3], where the second-order dispersion is zero, because there the power required for creating bright solitons is significantly lower. Although exact analytical solutions describing the soliton propagation near the point are not available, numerical [4] and perturbative [3–5] methods have explained the main features of pulse propagation near and at the ZGD point in the anomalous-dispersion regime. In particular, it was shown that bright solitons may exist near the ZGD point but not at it.

Meanwhile, dark solitons have also drawn the attention of several research groups. They are stable localized excitations of a cw background in the normally dispersive, nonlinear medium, and these solitons also have been observed experimentally in optical fibers [6–8] (temporal dark solitons) and laser beams [9] (spatial dark solitons). Additionally, the results of Ref. [10] indicate that dark-soliton propagation may be possible in nonlinear waveguides and such a propagation has probably been observed experimentally [11].

In this paper we demonstrate that dark solitons may exist near the ZGD point and we prove that, as was predicted by Kivshar [12], there is a region of the group-velocity dispersion where *a different type* of optical solitons, the so-called *anti-dark-solitons* (i.e., bright solitons on a pedestal), may exist. Analytical results are based on the small-amplitude approach developed in Kivshar's papers [13,14] but numerical simulations deal with the full nonlinear Schrödinger (NLS) equation including the third-order dispersion. According to analytical and numerical results, anti-dark-solitons exist for a certain propagation direction so that they may interact with dark pulses moving in the opposite direction. We analyze the interaction numerically and demonstrate that it looks elastic at least in the small-amplitude limit.

Using the slowly varying envelope approximation (see, e.g., Refs. [1] and [3]) we may find that the pulse envelope amplitude $\Phi(x,t)$ of the electric field in the neighborhood of the ZGD point satisfies the dimensionless generalized NLS equation

$$i\frac{\partial u}{\partial \xi} - \alpha\frac{\partial^2 u}{\partial s^2} + 2|u|^2 u = i\beta\frac{\partial^3 u}{\partial s^3}. \qquad (1)$$

Here we have used the notation

$$s = (t - k'x)/T, \quad \xi = |k'|x/T, \qquad (2)$$

$$u = \frac{1}{2}\left(\frac{T\omega_0 n_2}{|k'|c}\right)^{1/2}\Phi, \qquad (3)$$

$$\alpha = \frac{k''}{2T|k'|}, \quad \beta = \frac{k'''}{6T^2|k'|}, \qquad (4)$$

where k is the propagation wave number, $k^{(n)} = \partial k^n/\partial\omega^n$, $n = 1,2,3$, n_2 is the Kerr coefficient, ω_0 is the carrier frequency, and T is the pulse duration.

In the case $\beta = 0$ and $\alpha > 0$ Eq.(1) is exactly integrable and it has stable soliton solutions in the form of localized dark pulses propagating on the modulationally stable cw background $|u| = u_0 =$ const. The one-soliton dark pulse is

$$u(\xi,s) = u_0\frac{(\lambda - i\nu)^2 + \exp Z}{1 + \exp Z}\exp(2iu_0^2\xi), \qquad (5)$$

where

$$Z = 2\nu u_0 \alpha^{-1/2}(s - s_0 - 2\lambda\alpha^{1/2}u_0\xi), \quad \lambda^2 = 1 - \nu^2, \quad (6)$$

ν is the amplitude parameter ($\nu^2 \leq 1$), and s_0 is a constant initial phase. At $\lambda = 0$ the solution (5) and (6) describes the so-called fundamental dark soliton, $u(\xi, s) = u_0 \tanh(u_0\alpha^{-1/2}s)\exp(2iu_0^2\xi)$, and for $\nu^2 \ll 1$ it corresponds to the so-called "gray" (small-amplitude) solitons [14].

To discuss analytically the dark-soliton dynamics in the neighborhood of the ZGD point, i.e. at $\beta \neq 0$ in Eq.(1) , we look for a solution of Eq.(1) in the form of a small-amplitude excitation of the cw background (see Refs. [12–14])

$$u(\xi, s) = [u_0 + a(\xi, s)]\exp[2iu_0^2\xi + i\phi(\xi, s)]. \quad (7)$$

Substituting Eq.(7) into Eq.(1), we may obtain two equations ($a^2 \ll u_0^2$):

$$\left(\frac{\partial a}{\partial \xi} - \alpha u_0 \frac{\partial^2 \phi}{\partial s^2}\right) - \alpha\left(2\frac{\partial a}{\partial s}\frac{\partial \phi}{\partial s} + a\frac{\partial^2 \phi}{\partial s^2}\right) = \beta\left(\frac{\partial^3 a}{\partial s^3} - 3u_0\frac{\partial \phi}{\partial s}\frac{\partial^2 \phi}{\partial s^2}\right), \quad (8)$$

$$u_0\left(\frac{\partial \phi}{\partial \xi} - 4u_0 a\right) + a\frac{\partial \phi}{\partial \xi} + \alpha\frac{\partial^2 a}{\partial s^2} - \alpha u_0\left(\frac{\partial \phi}{\partial s}\right)^2 - 6u_0 a^2 = \beta\left(3\frac{\partial^2 a}{\partial s^2}\frac{\partial \phi}{\partial s} + 3\frac{\partial a}{\partial s}\frac{\partial^2 \phi}{\partial s^2} + u_0\frac{\partial^3 \phi}{\partial s^3}\right). \quad (9)$$

The method to analyze the system (8) and (9) is described in Ref. [14]. The main idea of the approach is to use new variables

$$\tau = \epsilon(s - C\xi), \quad z = \epsilon^3\xi, \quad (10)$$

ϵ being an arbitrary small parameter connected with the soliton amplitude, and to present the wave fields $a(\tau, z)$ and $\phi(\tau, z)$ in the form of the asymptotic series in the same small parameter ϵ ,

$$a = \epsilon^2 a_0 + \epsilon^4 a_1 + \cdots, \quad \phi = \epsilon\phi_0 + \epsilon^3\phi_0 + \cdots. \quad (11)$$

The parameter C is the limit velocity (in the s space) of linear waves propagating on the cw background:

$$C^2 = 4u_0^2\alpha. \quad (12)$$

Substituting Eqs.(11) into Eqs.(8) and (9) and using the variables (10), we can obtain the Korteweg–de Vries (KdV) equation for the soliton amplitude $a_0(\tau, z)$ (cf. Ref. [12])

$$2C\frac{\partial a_0}{\partial z} + 24\alpha u_0\left(1 + \frac{\beta C}{2\alpha^2}\right)a_0\frac{\partial a_0}{\partial \tau} - (\alpha^2 + 2\beta C)\frac{\partial^3 a_0}{\partial \tau^3} = 0. \quad (13)$$

The sign of the velocity C, $C = \pm 2u_0\alpha^{1/2}$, selects the propagation direction. The soliton solution of the KdV equation (13) has the form

$$a_0(\tau, z) = -\frac{\kappa^2(\alpha^2 + 2\beta C)}{u_0(2\alpha^2 + \beta C)\cosh^2[\kappa\alpha^{1/2}(\tau + 2\kappa^2\frac{(\alpha^2+2\beta C)}{\alpha C}z)]}, \quad (14)$$

κ being an arbitrary parameter related to the KdV soliton amplitude.

As may be seen from Eqs. (13) and (14), the soliton solution depends on the sign of the velocity C. In particular, it means that dark solitons propagating in opposite directions are different, i.e., they have different parameters (different energies) at the same value of the velocity. Moreover, to be a dark soliton, the solution (14) has to correspond to a negative amplitude [cf. Eqs.(7) and (14)]. This is not valid for $C < 0$ in the region

$$1 < \frac{\alpha^{3/2}}{\beta u_0} < 4, \quad (15)$$

when the dark soliton (14) changes the sign of its amplitude and is transformed into an anti-dark-soliton [12].

The results obtained in the framework of the perturbation theory in the soliton amplitude have to be proved by direct numerical simulations of the full NLS equation (1). To do this, let us present our approximate solution defined by Eqs. (7), (10)-(12), and (14) in the terms of the variables s and ξ for the wave field $u(\xi, s)$. To simplify the resulting formulas, we have introduced the parameters

$$\mu = \frac{\epsilon\kappa\alpha^{1/2}}{u_0}, \quad \gamma = \frac{\alpha^{3/2}}{\beta u_0}, \quad (16)$$

and also used the notation

$$\sigma = \operatorname{sgn} C. \quad (17)$$

As was demonstrated in Ref. [12], in the lowest approximation of the asymptotic expansion the soliton phase ϕ is determined by its amplitude,

$$\frac{\partial \phi_0}{\partial \tau} = -\frac{Ca_0}{u_0}, \quad (18)$$

i.e., the phase $\phi_0(\tau)$ may be found as the following:

$$\phi_0(\tau) = -2\sigma\alpha^{1/2}\int a_0\, d\tau. \quad (19)$$

Therefore, using Eqs. (7), (10)–(12), (14), and (19), and introducing the notation defined by Eqs. (16) and (17) , we may demonstrate that near the ZGD point of a normally dispersive optical fiber the small-amplitude solitons are described by the following expression:

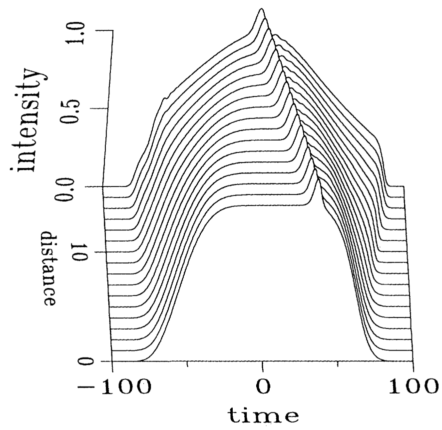

FIG. 1. Propagation of the anti-dark-soliton at $\mu = 0.3$. The dispersion parameters are chosen to be the following: $\alpha = 1$ and $\beta = 0.5$.

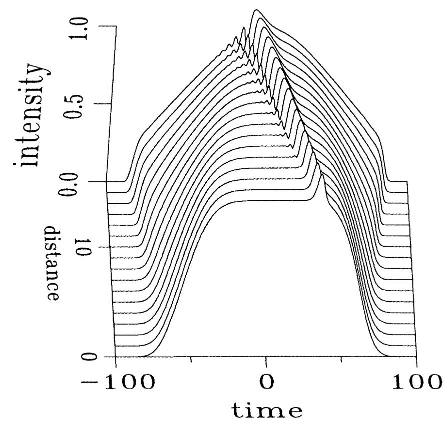

FIG. 2. The same as in Fig.1 but at $\beta = 0$. The input pulse decays because anti-dark-solitons cannot exist for this parameter region.

$$u_s(\xi, s) = u_0 \left(1 - \frac{\mu^2(\gamma + 4\sigma)}{2\alpha(\gamma + \sigma)\cosh^2 Z}\right) \exp[2iu_0^2\xi + i\phi(Z)], \tag{20}$$

$$\phi(Z) = -\frac{\sigma\mu(\gamma + 4\sigma)}{\alpha^{1/2}(\gamma + \sigma)} \tanh Z + \phi(0), \tag{21}$$

where

$$Z \approx \mu u_0(s - 2u_0\sigma\alpha^{1/2}\xi) \tag{22}$$

and $\phi(0)$ is an arbitrary constant.

We have used the pulse (20)–(22) for numerical simulations of the NLS equation (1) at $\alpha = 1$ and $u_0 = 1$. According to analytical predictions, the anti-dark-solitons may exist in the region $1 < \gamma < 4$, so that we put $\beta = 0.5$ (i.e., $\gamma = 2$). Figure 1 shows the propagation of the anti-dark-soliton at $\mu = 0.3$. The input pulse was taken in the form of the soliton (20) and (21) on a finite-extent (Gaussian-like) pulse

$$u(0, s) = u_s(0, s) \exp[-(s/T_*)^8], \tag{23}$$

T_* being sufficiently large. For comparison, we present also the same picture for the case when the third-order dispersion is absent, i.e., at $\beta = 0$ (Fig. 2). In the latter case the initial anti-dark-pulse decays very fast as a dispersive wave packet.

As we can see directly from Eqs. (20) and (21), the anti-dark-soliton exists only for $\sigma = -1$, i.e., for a certain propagation direction. It means that there is a possibility of observing interactions between *different types* of optical solitons, in fact, between dark and anti-dark-solitons, the former solitons are really dark when, at the same values of dispersion parameters, they propagate in the opposite direction [for such a soliton $\sigma = +1$ in Eqs. (20)–(22)]. Figures 3 and 4 show the collision between dark and anti-dark-solitons of equal intensities; in this case the interaction between solitons is more strong. In the case when anti-dark-solitons exist [see Fig. 3(a)], such a

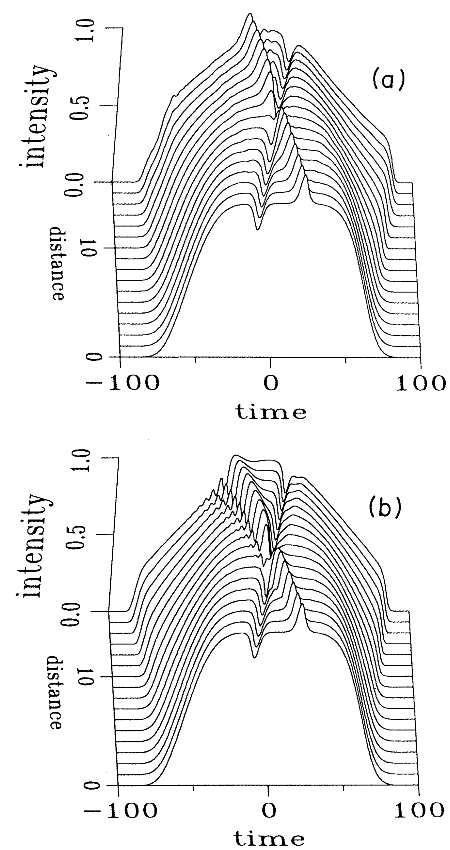

FIG. 3. Collision of dark and anti-dark-solitons at $\mu = 0.3$, when (a) $\beta = 0.5$ and (b) $\beta = 0$. In the former case the anti-dark-pulse is not destroyed due to the collision because anti-dark-solitons are stable in such a region of parameters. The collision is inelastic and radiation is observed.

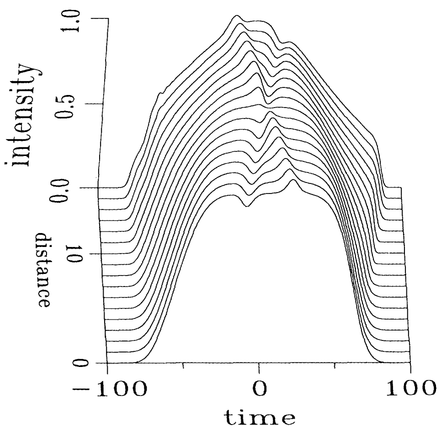

FIG. 4. The same as in Fig. 3(a) but at $\mu = 0.2$. The collision in this small-amplitude case looks quite elastic.

collision generates only a small emission which may be explained by nonintegrability of the NLS equation including the third-order dispersion (see, e.g., Ref. [5], where the nonintegrability was discussed for the anomalous dispersion region). However, in the limit of small-amplitude pulses (e.g., at $\mu \leq 0.2$), the collision looks almost elastic (see Fig. 4), and the pulses are much more stable. This is in agreement with the analytical predictions that dark and anti-dark-solitons in the small-amplitude limit are described by the exactly integrable KdV equations (13).

In conclusion, we have proved analytically and by direct numerical simulations of the nonlinear Schrödinger equation including the third-order dispersion that near the ZGD point the so-called anti-dark-solitons may propagate. These solitons are similar to dark ones, but they have the reverse sign of the amplitude, so that they may be considered as bright solitons on a pedestal. However, these solitons are not usual bright optical solitons because they are one-parameter ones, and in the small-amplitude limit the solitons are described by the KdV equation. Since anti-dark-solitons exist only for a certain propagation direction, they may interact with usual dark pulses propagating in the opposite direction; probably, this is the only possibility to observe direct interactions of dark and bright (in fact, anti-dark) solitons. As may be seen from numerical simulations, the collision of small-amplitude dark and anti-dark-solitons looks elastic; however, for larger soliton amplitudes the soliton interaction demonstrates emission of small radiation, which may be explained by nonintegrability of the nonlinear Schrödinger equation including the third-order dispersion.

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