Particlelike structures and their interactions in spatiotemporal patterns generated by one-dimensional deterministic cellular-automaton rules

N. Boccara
Département de Physique Générale, Service de Physique du Solide et de Résonance Magnétique, Commissariat à l'Energie Atomique, Centre d'Etudes Nucléaires de Saclay, 91191 Gif-sur-Yvette CEDEX, France and Department of Physics, University of Illinois, Chicago, Illinois 60680

J. Nasser and M. Roger
Département de Physique Générale, Service de Physique du Solide et de Résonance Magnétique, Commissariat à l'Energie Atomique, Centre d'Etudes Nucléaires de Saclay, 91191 Gif-sur-Yvette CEDEX, France
(Received 22 October 1990)

Configurations generated by the evolution of some one-dimensional cellular automata may be viewed, after many time steps, as particlelike structures evolving in a regular background. A classification of the most frequently observed "particles" is proposed according to their specific behavior. The simplest—a straightforward generalization of the "kinks" in range-1 Rule 18 earlier studied by Grassberger [Phys. Rev. A **28**, 3666 (1983)]—exhibit a diffusive motion and annihilate according to simple processes. Others have, in contrast, constant (positive, negative, or zero) velocities. The "collision" of particles with different velocities leads to some "reactions" in which some particles are annihilated and others are created. A detailed description of such "reactions" sheds new light on the large-time behavior of range-1 rule 54 with a very slow decrease of the particle number, as $t^{-\gamma}$ ($\gamma \simeq 0.15$). More "exotic" behaviors are sometimes observed. Some particlelike structures radiate other "particles." Some "particles" combine to generate a perturbation whose space extension increases with time and can be annihilated through the interactions with other "particles." These different behaviors could lead to a more precise classification of cellular-automaton rules.

I. INTRODUCTION

Cellular automata are simple systems that often exhibit complex self-organizing behavior [1–4]. They consist of a lattice with a discrete variable at each site. Each site variable evolves in discrete time steps according to a definite rule involving the values of neighboring site variables at previous time steps. The site variables are updated simultaneously.

Cellular automata may be considered as discrete dynamical systems. Let s: $\mathbf{Z} \times \mathbf{N} \mapsto \{0,1\}$ be a function that satisfies the equation

$$s(i,t+1) = f(s(i-r,t), s(i-r+1,t), \ldots, s(i+r,t)), \quad \forall i \in \mathbf{Z},\ \forall t \in \mathbf{N}$$

and such that

$$s(i,0) = s_0(i), \quad \forall i \in \mathbf{Z}$$

where $\mathbf{N}$ is the set of nonnegative integers, $\mathbf{Z}$ the set of all integers, and s_0: $\mathbf{Z} \rightarrow \{0,1\}$ a given function which specifies the initial condition. Such a discrete dynamical system is a one-dimensional cellular automaton (CA). The mapping f determines the dynamics. It is referred to as the local rule of the CA. The positive integer r is the range of the rule. The function S_t: $i \rightarrow s(i,t)$ is the state of the CA at time t. $\mathcal{S} = \{0,1\}^{\mathbf{Z}}$ is the state space. An element of the state space is also called a configuration. Since the state at time $t+1$ is entirely determined by the state at time t and the rule f, f induces a mapping $\mathbf{f}$: $\mathcal{S} \rightarrow \mathcal{S}$, called the global rule, such that

$$S_{t+1} = \mathbf{f}(S_t).$$

Given a rule f, its limit set is defined by

$$\Lambda_f = \lim_{t\to\infty} \mathbf{f}^t(\mathcal{S}) = \bigcap_{t\geq 0} \mathbf{f}^t(\mathcal{S}).$$

Λ_f is clearly invariant, that is, $\mathbf{f}(\Lambda_f) = \Lambda_f$. Since any $\mathbf{f}$-invariant subset belongs to Λ_f, the limit set is the maximal $\mathbf{f}$-invariant subset of $\mathcal{S}$.

Configurations generated by the evolution of some one-dimensional cellular automata may be viewed, after many time steps, as particlelike structures evolving in a regular background. As the result of their interactions, the number of these particles decreases in time.

Consider, for example, range-1 Rule 18—rules are numbered following Wolfram [5, 6]—defined by

$$f_{18}(x_1,x_2,x_3) = \begin{cases} 1 & \text{if}(x_1,x_2,x_3) = (0,0,1) \text{ or } (1,0,0) \\ 0 & \text{otherwise.} \end{cases}$$

For this rule, configurations belonging to the attractor, which is contained in the limit set, consist of sequences of 0's of odd lengths separated by isolated 1's. With respect to this background a sequence of two 1's or a sequence of 0's of even length is a "defect" called a kink by Grassberger [7]. Since two sequences of 0's of odd lengths separated by two neighboring 1's generate, at the next time step, a sequence of 0's of even length, configurations generated by Rule 18 may contain kinks of one type only. During the evolution these kinks move according to a simple diffusion process, and when they meet they annihilate pairwise. This process has been studied by Grassberger [7], who found that, starting from a random initial configuration, the density of kinks decreases as $t^{-1/2}$. These kinks may be viewed as particles. For Rule 18 all particles are of the same type. A particle is its own antiparticle. This is a particularly simple case. In the following sections we describe various particlelike structures and their interactions. We also study the time dependence of their asymptotic density. It is important to note that many particles have a very short lifetime. We shall focus on particles that exist after transients of a few hundred of time steps.

II. DIFFUSIVE PARTICLES

Let us first describe the simplest behavior, which is a straightforward generalization of the "kinks" observed for the first time by Grassberger [7]. As a simple example consider the range-2 totalistic Rule 30 defined by

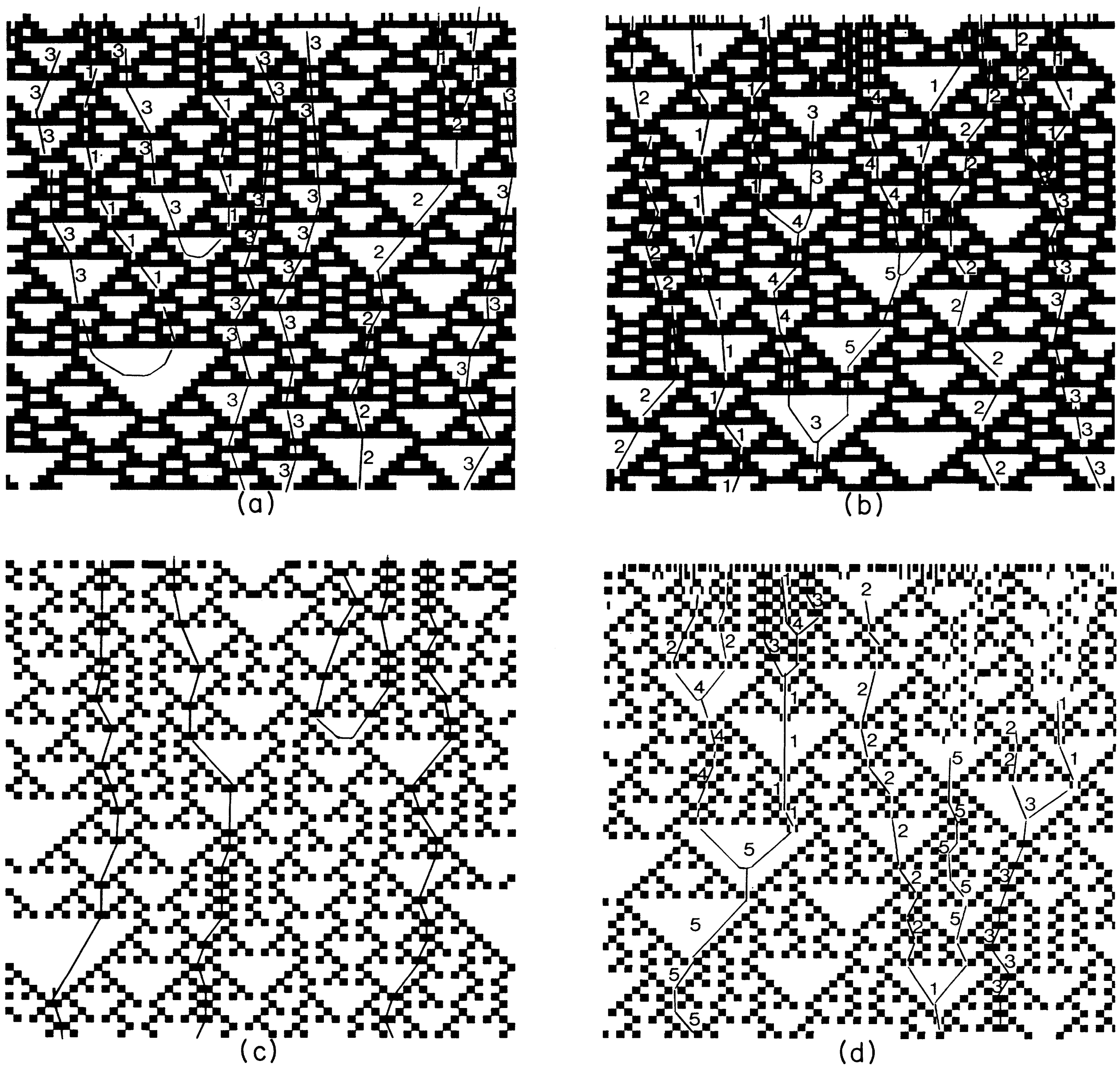

FIG. 1. Spatiotemporal patterns generated by the evolution according to (a) range-2 totalistic Rule 30, (b) range-3 totalistic Rule 126, (c) range-1 Rule 18, (d) block transform of Rule 18 for $b = 3$. Initial configurations are randomly generated. Numbers k refer to types of particles d_k. They combine according to the law $d_i + d_j \rightarrow d_k$ with $k = (i + j)\,\mathrm{mod}\,2b$; b is (a) 2, (b) 3, (c) 1, and (d) 3.

$$f_{30}(x_1,x_2,x_3,x_4,x_5) = \begin{cases} 1 & \text{if } 0 < \sum_{i=1}^{5} x_i < 5 \\ 0 & \text{otherwise.} \end{cases}$$

A configuration belonging to the attractor of this rule consists of alternating sequences of 0's and 1's whose lengths are multiples of 4. The distributions of the sequences of 0's and 1's are identical, and the average number of a sequence of, say 0's, of length $4n$ per site is equal to $1/2^{n+4}$ (Ref. [8]). Any sequence of 0's or 1's whose length is not a multiple of 4 may be viewed as containing a particle. There exist, therefore, three different particles. More precisely, if the length of a sequence of 0's or 1's is equal to $i \bmod 4$, the corresponding particle will be denoted by d_i $(i = 1, 2, 3)$. Figure 1(a) illustrates the reaction

$$d_i + d_j \to d_k$$

with

$$k = (i + j) \bmod 4.$$

d_1 is the antiparticle of d_3, and d_2 is its own antiparticle.

More generally, a configuration belonging to the attractor of the range-r totalistic Rule $2^{2r+1} - 2$, defined by

$$f_{2^{2r+1}-2}(x_1,x_2,\ldots,x_{2r+1}) = \begin{cases} 1 & \text{if } 0 < \sum_{i=1}^{2r+1} x_i < 2r+1 \\ 0 & \text{otherwise,} \end{cases}$$

consists of alternating sequences of 0's and 1's whose lengths are multiples of $2r$ (Ref. [8]). Any sequence of 0's or 1's whose length is not a multiple of $2r$ contains a particle. We may, therefore, define $2r - 1$ particles d_i $(i = 1, 2, \ldots, 2r - 1)$ and we have

$$d_i + d_j \to d_k$$

where

$$k = (i + j) \bmod 2r.$$

Figure 1(b) illustrates the case $r = 3$.

There are many other cases in which particles of this type can be found. Let b be a positive odd integer, and consider the rule f defined by [9]

$$f(x_1, x_2, \ldots, x_{3b}) = 1$$

if, and only if,

$$x_1 + \cdots + x_b < \frac{b}{2}, \quad x_{b+1} + x_{2b} < \frac{b}{2},$$

$$x_{2b+1} + \cdots + x_{3b} > \frac{b}{2},$$

or

$$x_1 + \cdots + x_b > \frac{b}{2}, \quad x_{b+1} + x_{2b} < \frac{b}{2},$$

$$x_{2b+1} + \cdots + x_{3b} < \frac{b}{2}.$$

This rule is a block transform of Rule 18. Its limit set is closely related to the limit set of Rule 18 [Fig. 1 (c)]. Any configuration belonging to the attractor consists of sequences of 0's of lengths equal to $(2n + 1)b$, where n is a non-negative integer, separated by blocks of b 1's. With respect to this background we can define, as above, $2b - 1$ particles d_i $(i = 1, 2, \ldots, 2b - 1)$, and we have

$$d_i + d_j \to d_k$$

where

$$k = (i + j) \bmod 2b.$$

Figure 1(d) illustrates the case $b = 3$.

All the particles described so far are diffusive. That is, they perform a random walk and, as a result of their interactions, their density decreases as $t^{-1/2}$ as t tends to ∞.

III. NONDIFFUSIVE PARTICLES

Another typical and frequently observed behavior corresponds to the following scheme. After a few hundred time steps, the evolution of the CA is characterized by the presence of particles of several types on a periodic background. Each particle moves with a constant (positive, negative, or null) velocity. When two or more particles with different velocities "collide," they lead to a "reaction" in which some particles are annihilated and others are created. In general, due to these collisions, the total number of particles decreases with time. In the infinite time limit, the attractor contains only particles with the same velocity or no particle at all on a periodic background. As a simple illustration, we first describe range-1 Rules 184 and 62, for which the different particles and interaction laws are simple. The rest of the section is devoted to range-1 Rule 54 (one of the most complex range-1 rules). The description of this rule in terms of interacting particles sheds new light on its large-time dynamics.

A. Rule 184

Range-1 Rule 184 has been studied in great detail by Krug and Spohn [10]; it is defined by

$$f_{184}(x_1,x_2,x_3) = \begin{cases} 1 & \text{if } (x_1,x_2,x_3) = (0,1,1), (1,0,0), (1,0,1), \text{ or } (1,1,1) \\ 0 & \text{otherwise.} \end{cases}$$

Starting from a random initial configuration, a spatiotemporal pattern generated by this rule is shown in Fig. 2. The background is a checkerboard of alternating 0's and 1's. Two types of particles may be distinguished. They consist of sequences of 0's or 1's whose lengths $n+1$ is greater than 1. They all propagate with the same constant velocity (equal to 1) but in opposite direction. If we denote these particles, respectively, by $\overrightarrow{0}_n$ and $\overleftarrow{1}_n$, where the arrows refer to the direction of propagation, we have the reactions (Fig. 2)

$$\overrightarrow{0}_{n_0} + \overleftarrow{1}_{n_1} \rightarrow \begin{cases} \overrightarrow{0}_{n_0-n_1} & \text{if } n_0 > n_1 \\ \overleftarrow{1}_{n_1-n_0} & \text{if } n_0 < n_1 . \end{cases}$$

If $n_0 = n_1$, the two particles annihilate. Configurations belonging to the limit set contain particles of only one type. These particles are clearly nondiffusive.

B. Rule 62

Not all particlelike structures are propagating. Consider, for instance, range-1 Rule 62 defined by

$$f_{62}(x_1,x_2,x_3) = \begin{cases} 0 & \text{if } (x_1,x_2,x_3) = (0,0,0), (1,1,0), \text{ or } (1,1,1) \\ 1 & \text{otherwise.} \end{cases}$$

Starting from a random initial configuration, a spatiotemporal pattern generated by this rule is shown in Fig. 3(a). The background is periodic in space and time, both periods being equal to 3. Three types of particles may be distinguished. Two of them are nonpropagating and periodic in time. Their periods are equal to 3. They may be generated by sequences of 0's whose lengths are greater than 2. They will be denoted, respectively, by g_e and g_o (g stands for "gutter"), according to whether they consist of sequences of 0's of even or odd lengths [Fig. 3(b)]. There is also a propagating particle, denoted by w, which may be generated by a sequence of two 0's. Its trajectory in the the two-dimensional space $\mathbf{Z} \times \mathbf{N}$ is the analog of a domain wall. It separates two equivalent patterns of the background. This particle propagates only to the right with a velocity equal to 1.

As represented in Figs. 3(c)–3(e), we have the reactions

$$w + g_e \rightarrow w, \quad w + g_o \rightarrow g_e, \quad 2w + g_e \rightarrow g_o .$$

Figure 3(f) also exhibits the reaction $2w + g_o \rightarrow w$. This is not, however, a new process; it follows directly from the first two reactions. As a result of all these reactions, depending upon the initial configuration, the number of particles decreases rapidly. In the infinite time limit, all the remaining particles have the same velocity.

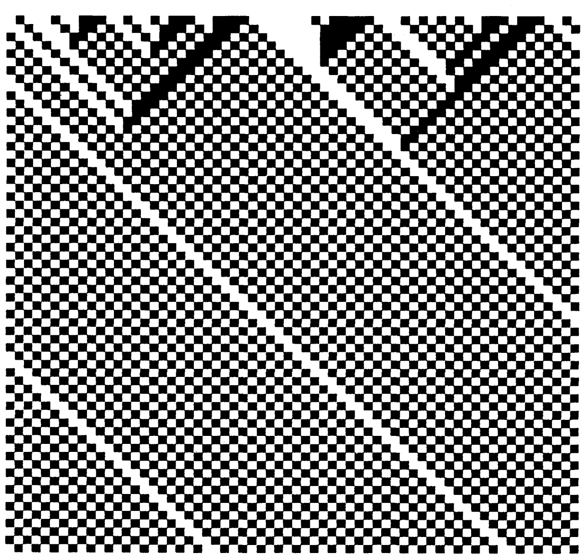

FIG. 2. Spatiotemporal pattern generated by the evolution according to Rule 184. The initial configuration is randomly generated.

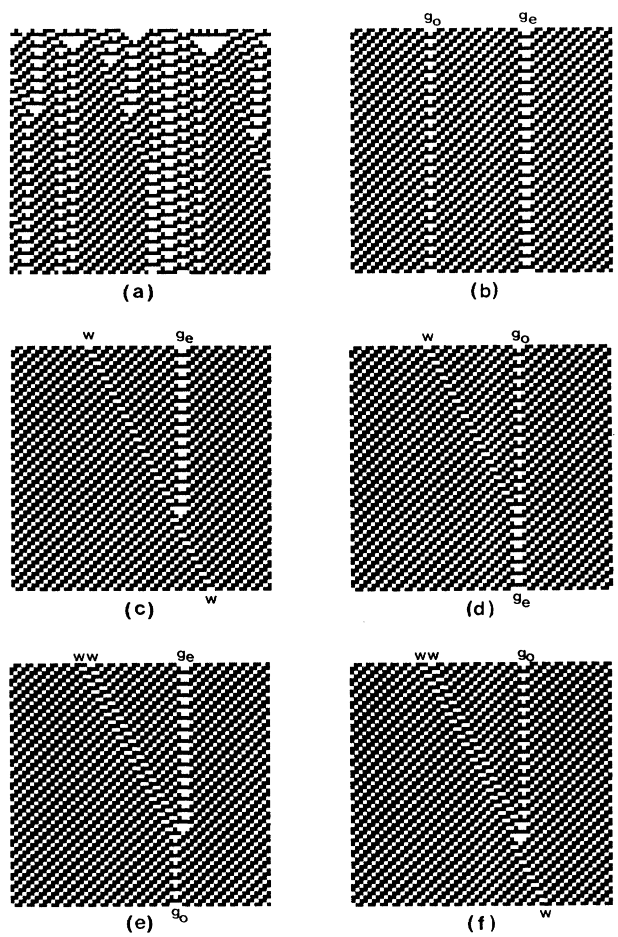

FIG. 3. Rule 62. (a) Evolution from a randomly generated initial configuration. (b) Particles g_e and g_o. (c) Interaction between particles w and g_e. (d) Interaction between particles w and g_o. (e) Interaction between two neighboring particles w and g_e. (f) Interaction between two neighboring particles w and g_o.

C. Rule 54

Rule 54 is defined by

$$f_{54}(x_1,x_2,x_3)=\begin{cases}1 & \text{if } (x_1,x_2,x_3)=(0,0,1),(1,0,0),(0,1,0),\text{ or }(1,0,1)\\ 0 & \text{otherwise.}\end{cases}$$

Starting from a random initial configuration, a spatiotemporal pattern generated by this rule is shown in Fig. 4(a). The background is periodic in space and time, both periods being equal to 4. As for Rule 62, three types of particles may be distinguished. Two of them are nonpropagating and periodic in time. Their periods are equal to 4. They may be generated by sequences of 0's whose lengths are greater than 3 [Fig. 4(b)], and are very similar to the particles g_e and g_o described in Sec. III B. We shall denote them by the same symbols according to whether they consist of sequences of 0's of even or odd length. There exists also a propagating particle w, which may be generated by three 0's following three 1's or the converse. This particle may propagate to the right or to the left. Its velocity is equal to 1.

These particles have a rather rich variety of interactions. As represented in Fig. 4(c), we have the reactions

$$\overrightarrow{w}+\overleftarrow{w}\rightarrow g_o,\quad \overrightarrow{w}+g_o\rightarrow \overleftarrow{w},\quad \overleftarrow{w}+g_o\rightarrow \overrightarrow{w},$$

where the arrows over the symbol w indicates the direction of propagation of the corresponding particle. Many other similar reactions can be written down. Figure 4(d) illustrates the interaction of the particles w and g_e. A pair of even gutters g_e may also be annihilated. There are many multiparticle reactions in which a pair of g_e disappear. A few of them are illustrated in Fig. 5. Such reactions are relatively rare because they involve at least four particles (two g_e close together and two or more w). More complex annihilation processes of a pair of g_e involving more particles, in particular the presence of one odd gutter g_o, also occur.

If two g_e are close enough, the impact of one or more neighboring w in precise positions creates a new nonpropagating periodic particle whose period is equal to 32. A few examples of such processes are illustrated in Figs. 6(a)–6(c). This new particle "radiates" continuously particles w: four $\overrightarrow{w}$ and four $\overleftarrow{w}$ during one period. Figure 7(a) shows in which conditions such a particle can be annihilated. Figures 6(d) and 7(b) represent the same processes as Figs. 6(c) and 7(a), but in order to exhibit them on a uniformly black background, we transformed the pattern by the following mapping:

$$\sigma(i,t)=\sum_{k=0}^{3}s(i+k,t)\bmod 2,\quad \forall i,\forall t.$$

Rule 54 is probably the most complex range-1 rule. We have studied its evolution over many generations. We have found that the number of particles decreases very

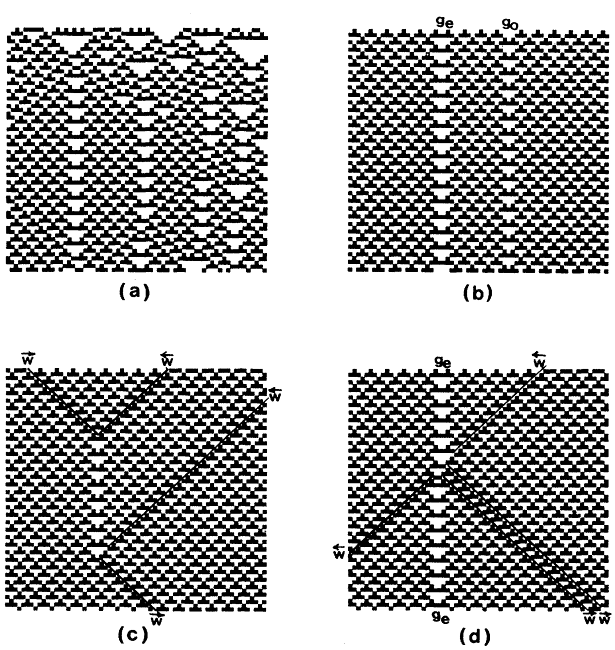

FIG. 4. Rule 54. (a) Evolution from a randomly generated initial configuration. (b) Particles g_e and g_o. (c) Interaction between two particles w with opposite velocities and interaction between one particle w and g_o. (d) Interactions between particles w and g_e.

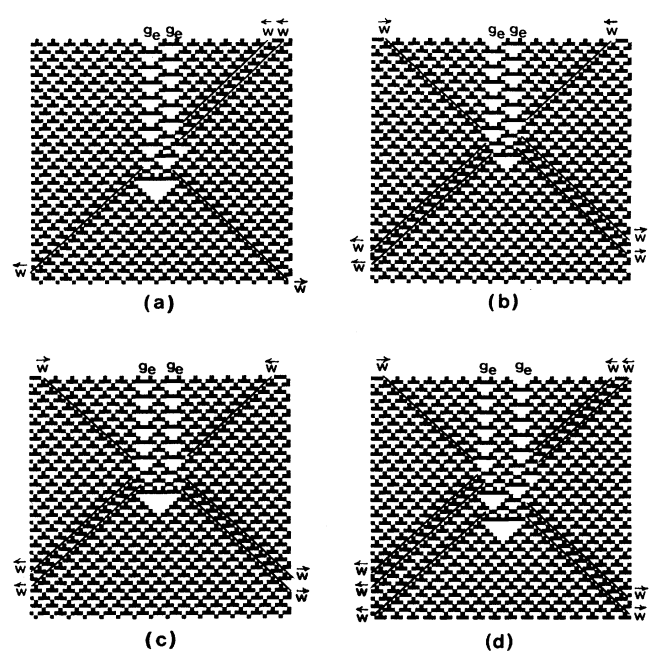

FIG. 5. Rule 54. Pairwise annihilation processes of "even gutters" g_e. More complicated reactions involving for example two even and one odd "gutters" are also possible.

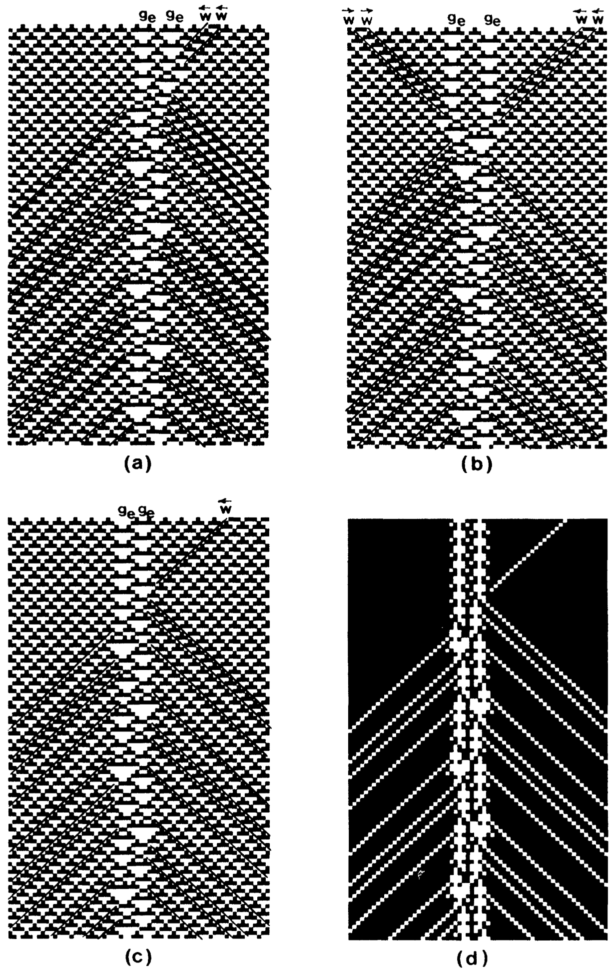

FIG. 6. Rule 54. (a)–(c) A few reactions leading to the creation of a "radiating" particle. (d) The same as (c), the periodic background is eliminated through the mapping $\sigma(i,t) = \sum_{k=0}^{3} s(i+k,t)\,\mathrm{mod}\,2$.

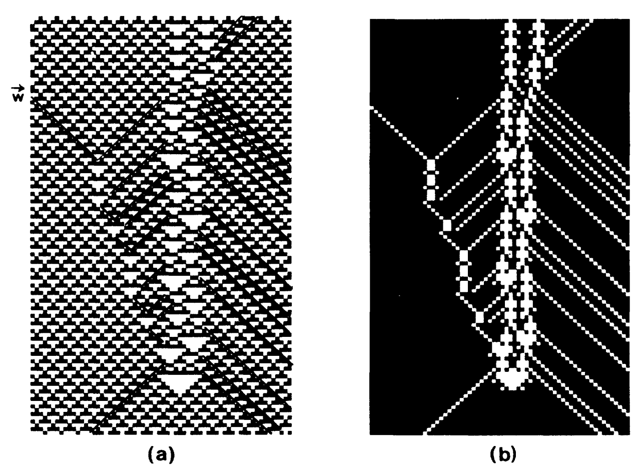

FIG. 7. Rule 54. (a) Annihilation of the radiating particle. (b) The same as (a) with the mapping defined in Fig. 6.

slowly. Simulations done on a lattice of 10^4 sites for a number of time steps of order 10^8 show that the number of even gutters n_{g_e} tends to zero as $t^{-\gamma}$ [Fig. 8(a)]. The exponent γ determined over two decades from 3×10^6 to 3×10^8 time steps is approximately equal to 0.15. Figure 8(b) shows the difference between the concentration of nonzero sites $c(1)$ and its asymptotic limit $c_\infty(1) = 0.5$ as a function of time. It follows the same behavior as the number of even gutters. In order to show that, in this simulation, boundary effects are not important, we also report, on the same figures, results obtained on a 10^5 site chain for a number of time steps of order 10^7.

Figure 9 shows, using a mapping similar to the preceding to obtain a white background, the remaining particles after, approximately, 5×10^2 [Fig. 9(a)] and 3×10^8

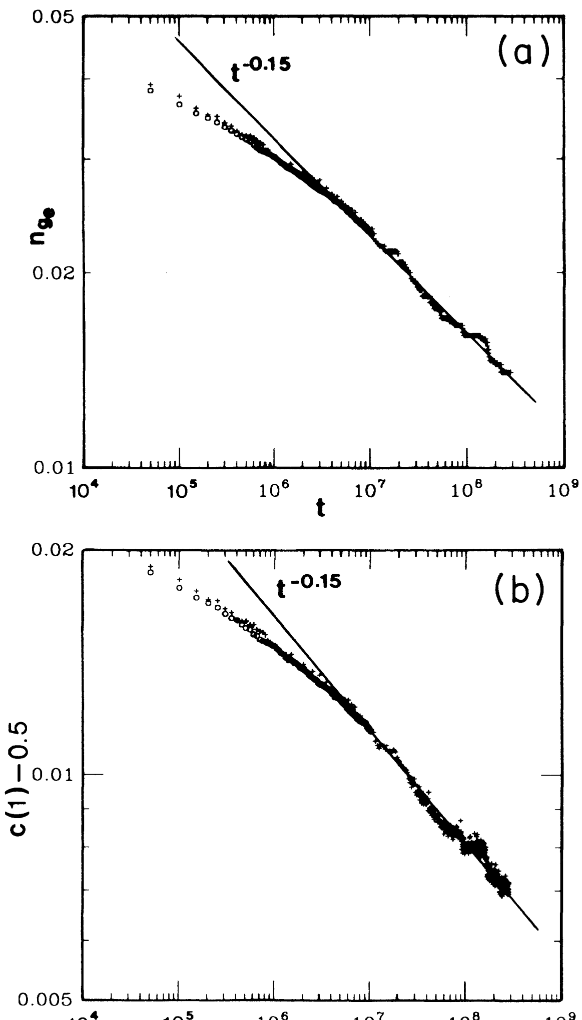

FIG. 8. Rule 54. (a) Average number of even gutters as a function of time. (b) Difference between the density of nonzero site variables $c(1)$ and its asymptotic limit $c_\infty(1) = 0.5$ as a function of time. Time averages are taken over 500 time steps. Circles and crosses are obtained, respectively, from 10^5 and 10^4 chains with periodic boundary conditions. The straight lines represent a $t^{-0.15}$ law.

[Fig. 9(b)] time steps. From these figures, it is clear that the total number of particles is directly related to the number of even gutters and follows the same asymptotic law. The decrease of the number of particles is indeed governed by the pairwise annihilation processes of even gutters.

This very slow decrease of the number of even gutters can be, at least qualitatively, understood through the following arguments. From Fig. 4(d), it is clear that the space translation of an even gutter is related to the impact of a (simple or multiple) wall w. Having observed that the number of walls w is proportional to the number of even gutters n_{g_e}, the mean velocity for the displacement of even gutters is proportional to n_{g_e}. Moreover, when two even gutters come close together, their pairwise annihilation requires the occurrence of at least two simultaneous events, i.e., the impact of at least two walls w in very precise positions as illustrated in Figs. 5(b) and 5(c) (otherwise, for example, through the impact of a single wall w, they move away). Since the number of walls is proportional to n_{g_e}, let us assume that the probability of such a double event is proportional to $n_{g_e}^2$. From these conjectures, the mean time required for two neighboring even gutters to annihilate is roughly proportional to $n_{g_e}^{-2} n_{g_e}^{-1} n_{g_e}^{-2}$ (the first factor represents a simple diffusion process with constant velocity, the second term takes into account the decrease of the velocity as n_{g_e} and the third contribution includes the probability of the double event previously described) and the number of even gutters n_{g_e} is expected to decrease roughly as $t^{-1/5}$. Taking into account the crudeness of these arguments compared to the complexity of the problem involving a large number of intricate annihilation processes, the behavior so estimated in $t^{-0.20}$ reasonably agrees with the result of the simulation ($t^{-0.15}$).

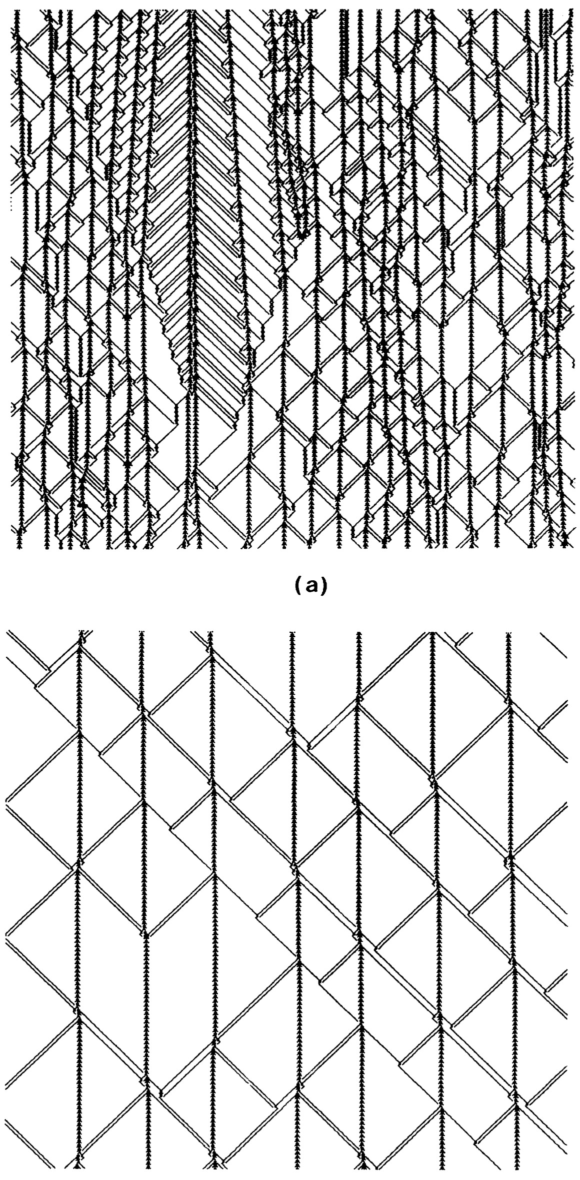

FIG. 9. Rule 54. Remaining particles after approximately (a) 500 time steps, (b) 3×10^8 time steps. The figures show the evolution of 512 lattice sites from a 10^4 site chain during 512 time steps. A similar mapping as defined in the preceding figures is used to suppress the periodic background. (a) shows, in particular, the pairwise annihilation of two even gutters and the presence of a radiating particle that transforms after some reaction into two walls and a pair of even gutters. In (b), the density of particles has decreased by a factor of about 3.

IV. PARTICLES GENERATING EXTENDED PERTURBATIONS

In the study of some block transformations [11], we frequently find a peculiarly interesting behavior. Some localized particles as described in Sec. II are present, but they generate during their interactions other objects which cannot be called "particles" because their space extension increases as a function of time. Nevertheless, such extended perturbations do behave during their interactions just as the particles described in the preceding sections, in particular they can be destroyed though the interaction with particles or other extended perturbations, according to simple rules.

Let H_2 be a homomorphism defined on the state space $\mathcal{S}$ by

$$H_2(\ldots, x_i, x_{i+1}, x_{i+2}, \ldots)$$
$$= \ldots, x_i, x_i, x_{i+1}, x_{i+1}, x_{i+2}, x_{i+2}, \ldots,$$

where $\ldots, x_i, x_{i+1}, x_{i+2}, \ldots$ is a configuration in $\mathcal{S}$; and consider, for example, range-1 Rule 18 defined above. Is it possible to find a range-2 rule, denoted $T_2 f_{18}$, such that its limit set coincides with $H_2(\Lambda_{f_{18}})$? In this case, configurations belonging to the limit set of rule $T_2 f_{18}$ would consist of sequences of $4n+2$ 0's, where n is a nonnegative integer, separated by pairs of 1's. The function

$$(x_1, x_2, x_3, x_4, x_5) \mapsto T_2 f_{18}(x_1, x_2, x_3, x_4, x_5)$$

should, therefore, be such that, for all $(x_1, x_2, x_3) \in \{0, 1\}^3$,

$$T_2 f_{18}(x_1, x_1, x_2, x_2, x_3) = T_2 f_{18}(x_1, x_2, x_2, x_3, x_3)$$
$$= f_{18}(x_1, x_2, x_3).$$

These relations do not determine a unique function $T_2 f_{18}$. Only the images of 14 quintuplets $(x_1, x_2, x_3, x_4, x_5)$ are determined. The images of the remaining 18 quintuplets are arbitrary. There exist, therefore, 2^{18} rules $T_2 f_{18}$ that fulfill the condition $T_2 \mathbf{f}_{18} \circ H_2 = H_2 \circ \mathbf{f}_{18}$. 2^{12} of them are legal. Starting from a random initial configuration, some of these rules generate spatiotemporal patterns similar to the pattern represented in Fig. 10(a). After a space contraction of a factor 2, in a background identical to the pattern generated by the evolution of Rule 18 [Fig. 1 (c)], two types of objects may be distinguished.

(i) The defects d_1 and d_2 are diffusive particles, as described in Sec. II. They are generated by sequences of $(2+i)\,\mathrm{mod}4$ $(i = 1, 2)$ 0's.

(ii) The other objects are extended perturbations whose space extensions increase as a function of time: d_3, which results from the interaction of d_1 and d_2, has a space extension increasing in time with a velocity equal to 2. d_4 is the result of the interaction of d_1 and d_3. Its space extension depends upon the space extension of d_3 at the time of the interaction. It may propagate to the right or to the left and its space extension exhibits a slow variation.

The use of similar notations for particles (i) and extended perturbations (ii) is justified by their mutual particlelike interactions. As shown in Figs. 10(b)–10(i), we have the following reactions:

$$d_1 + d_1 \rightarrow d_2,$$
$$d_2 + d_2 \rightarrow d_0,$$
$$d_1 + d_2 \rightarrow d_3,$$
$$d_1 + d_3 \rightarrow \overrightarrow{d_4},$$

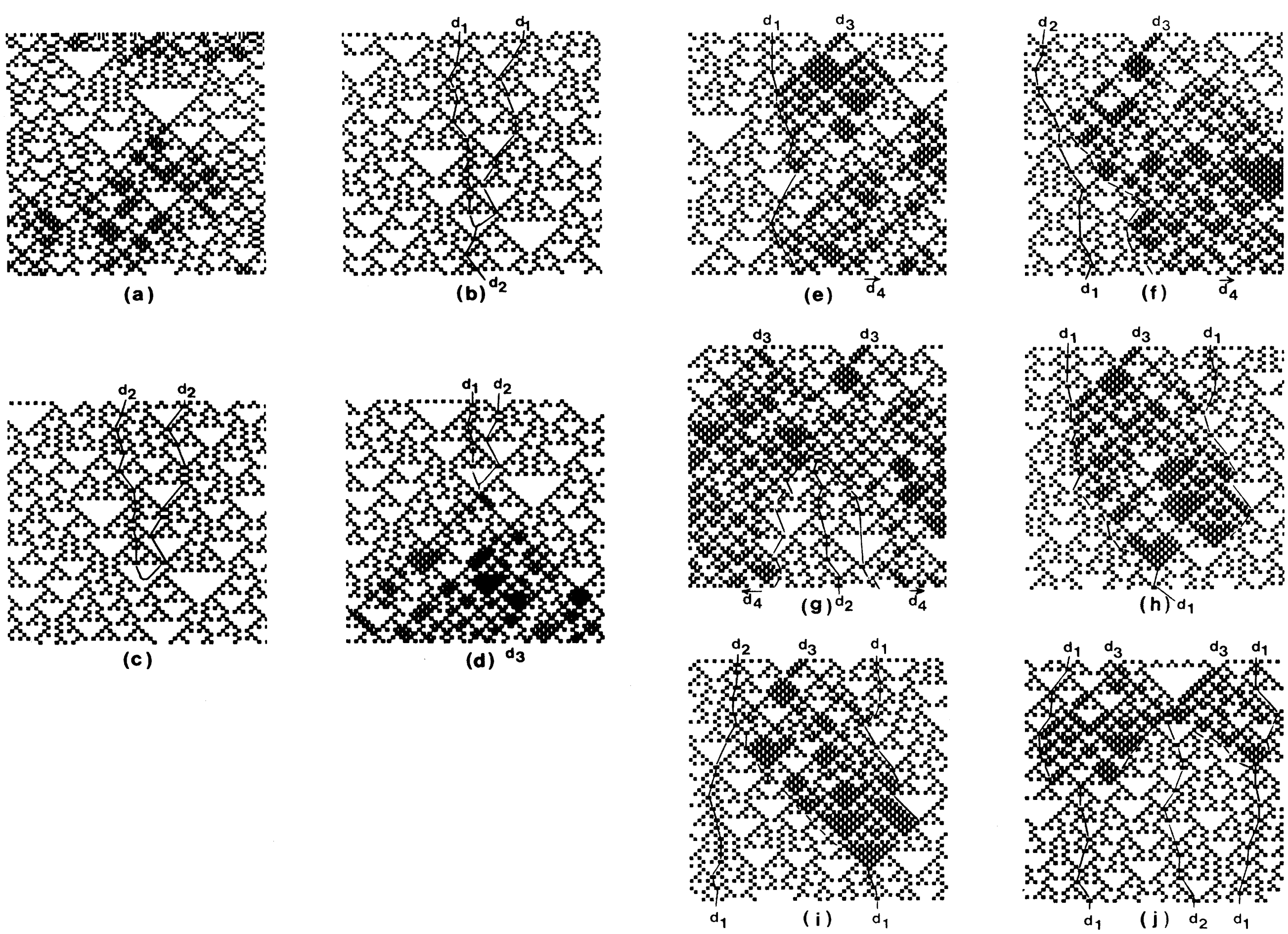

FIG. 10. (a) Pattern generated by the evolution of range-2 Rule 769861902. The initial configuration is randomly generated. (b) Interaction between two d_1 particles. (c) Annihilation of two d_2 particles. (d) Interaction between particles d_1 and d_2 leading to the extended perturbation d_3. (e) Interaction between particle d_1 and the extended perturbation d_3 leading to another extended object $\overrightarrow{d}_4$. (f) Interaction between d_2 and d_3. (g) Interaction between two d_3. (h) Interaction between $\overrightarrow{d}_4$ and d_1. (i) Interaction between d_2 and $\overleftarrow{d}_4$. (j) Interaction between $\overrightarrow{d}_4$ and $\overleftarrow{d}_4$. Note that, for all of these reactions, the sum of the subscripts on the left-hand side is equal to the sum of the subscripts on the right-hand side mod4.

$$d_2 + d_3 \rightarrow d_1 + \overrightarrow{d_4},$$
$$d_3 + d_3 \rightarrow \overleftarrow{d_4} + d_2 + \overrightarrow{d_4},$$
$$\overrightarrow{d_4} + d_1 \rightarrow d_1,$$
$$d_2 + \overleftarrow{d_4} \rightarrow d_1 + d_1.$$

d_0 represents the vacuum. The arrows on the symbol d_4 indicates the direction of propagation of the corresponding object. The order in which the particles and extended perturbations appear in the reactions listed above is important and corresponds to the relative spatial positions of these objects from the left to the right in the corresponding figures. As a consequence of the above reactions, we could write down more reactions. For example, we have [Fig. 10(j)]

$$\overrightarrow{d_4} + \overleftarrow{d_4} \rightarrow d_1 + d_2 + d_1.$$

Note that, for all these reactions, the sum of the subscripts on the left-hand side is equal to the sum of the subscripts on the right-hand side mod4.

The diffusive particles d_1 and d_2 dominate the asymptotic behavior of the number of defects at large time. The number of particles and extended perturbations also decreases as $t^{-1/2}$ [Fig. 11(a)]. However, as shown in Fig. 11(b), there are large fluctuations of the concentration of nonzero sites $c(1)$. For a chain with a fixed length, the amplitude of these fluctuations increases with time. This is easily understood, since when the number of particles decreases, the lifetime and space extension of d_3 and d_4 objects increase and these extended perturbations have a concentration of nonzero sites much larger than the background: $c_\infty(1) = 0.5$.

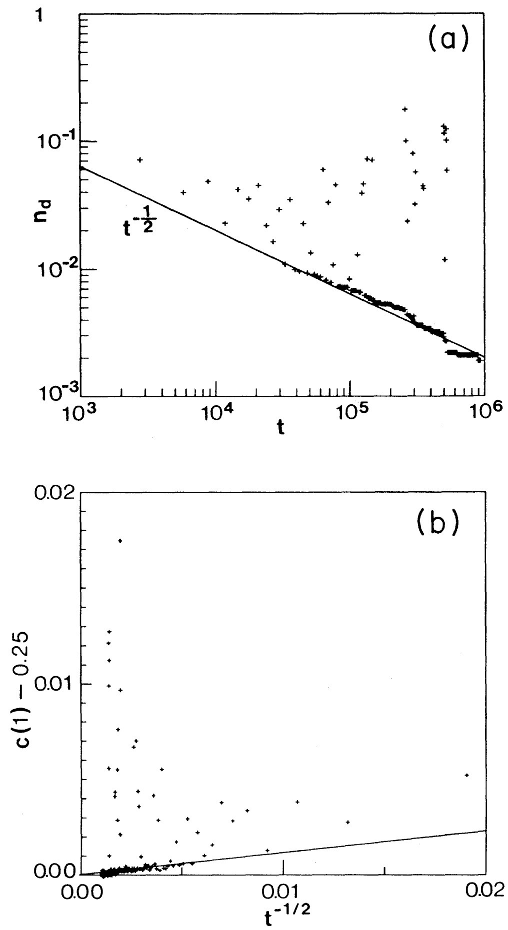

FIG. 11. Range-2 Rule 769861902. (a) Average number of defects n_d defined as the total number of sequences of 0's and 1's, whose length is not equal to 2 mod4, as a function of time t (log-log scale). (b) Difference between the concentration of nonzero sites $c(1)$ and its asymptotic value $c_\infty(1) = 1/4$ as a function of $t^{-1/2}$. The size of the lattice is 10^4; time averages over 500 time steps are taken. The straight lines represent a $t^{-1/2}$ law. Both figures indicate that the number of particles decreases as $t^{-1/2}$. The large fluctuations in the concentration of nonzero sites are due to the fact the space extension and lifetime of extended perturbations d_3 are roughly inversely proportional to the number of particles, and consequently increase with time.

V. CONCLUSION

Configurations generated by the evolution of some one-dimensional cellular automata may be viewed, after many time steps, as particlelike structures evolving in a regular background. We have described the behavior of some typical particles. Sometimes these particles behave in a rather simple way, but more complex behaviors are often found. There exist, for instance, both diffusive and nondiffusive particles. Among the nondiffusive particles we have found nonpropagating particles, complex particles radiating periodically in time other simple particles, etc. Mutual annihilation of particles causes particle number to decrease during the evolution of an automaton. The number of particles does not, however, necessarily go to zero even in the infinite time limit. Rule 54, which is probably the most complex range-1 cellular automaton rule, has been studied in detail. The number of nonpropagating particles has been found to tend, as a function of time t, to zero approximately as $t^{-0.15}$ over two decades from 3×10^6 to 3×10^8 time steps.

Based on investigation of a large sample of CA's, Wolfram [6] has shown that, according to their asymptotic behavior, CA rules appear to fall into four qualitative classes. Class-1 CA's evolve, from almost all initial states, to a unique homogeneous state in which all sites have the same value. Class-2 CA's yield separated simple stable or periodic structures. Class-3 CA's exhibit chaotic patterns. The statistical properties of these patterns are typically the same for almost all initial states. In particular, the density of nonzero site variables tends to a fixed value as time t tends to ∞. The evolution of class-4 CA's leads to complex localized or propagating structures. Range-1 Rules 18, 54, and 62 and range-2 totalistic Rule 30 are considered to be class-3. The evolution of the corresponding CA's analyzed in terms of interacting particles shows that that Rules 18 and 30 are very different from Rules 54 and 62. In the first case, the various particles are diffusive, whereas in the latter they are not. We have the feeling that the nature of the particles identified during the evolution of a CA could lead to a sharper classification of CA rules.

Particles in two-state cellular automata may be used to simulate many-state cellular automata [12]. Each particle would represent a state and the interactions between particles would simulate the rule. Behaviors that are thought to occur only in the case of many-state cellular automata could, therefore, exist in two-state cellular automata. Many types of particles probably remain to be identified, and it is of course not at all obvious that the evolution of any CA can always be interpreted in terms of interacting particles.

ACKNOWLEDGMENTS

One of us (N.B.) is grateful for numerous interesting discussions with Eric Goles, Howard Gutowitz, and James Crutchfield during his stay at the Institute for Scientific Interchange in Turin, Italy.

[1] *Cellular Automata*, Proceedings of an Interdisciplinary Workshop, Los Alamos, 1984, edited by D. Farmer, T. Toffoli, and S. Wolfram (North-Holland, Amsterdam, 1984).

[2] S. Wolfram, *Theory and Applications of Cellular Automata* (World Scientific, Singapore, 1986).

[3] *Cellular Automata and Modeling of Complex Physical Systems,* Proceedings of a Workshop, Les Houches, edited by P. Manneville, N. Boccara, G. Vichniac, and R. Bidaux (Springer-Verlag, Heidelberg, 1989).

[4] *Cellular Automata Theory and Experiment*, Proceedings of a Workshop sponsored by the Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, 1989, edited by H. Gutowitz (North-Holland, Amsterdam, 1990).

[5] S. Wolfram, Rev. Mod. Phys. **55**, 601 (1983).

[6] S.Wolfram, Physica D **10**, 1 (1984).

[7] P. Grassberger, Phys. Rev. A **28**, 3666 (1983).

[8] N. Boccara, J. Nasser, and M. Roger, in *Complexity and Evolution*, Proceedings of a Winter Workshop, Les Houches, 1990, edited by R. Livi *et al.* (Nova Science, Commack, NY, 1991); see also Europhys. Lett. **13**, 489 (1990).

[9] N. Boccara, J. Phys. A **22**, L393 (1989).

[10] J. Krug and H. Spohn, Phys. Rev. A **38**, 4271 (1988).

[11] N. Boccara and M. Roger, J. Phys. A **24**, 1849 (1991).

[12] N. Boccara and E. Goles (unpublished).