Convective instability with time-varying rotation

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We study experimentally the convective instability in a layer of helium I subject to a time-dependent rotation about a vertical axis $\Omega(t)=\Omega_s+\Omega_0\cos(\tilde{\omega}t)$. Relative stability of the conducting state is primarily determined by $\tilde{\omega}$, with the magnitude of shifts in the convective threshold at fixed $\tilde{\omega}$ increasing with the modulation amplitude Ω_0/Ω_s. Our observations are compared with recent theoretical work.

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In recent years there has been considerable interest in hydrodynamic systems that undergo pattern-forming instabilities in response to a variation of some external parameter, for example, the flow between concentric rotating cylinders (Taylor-Couette flow) and thermal convection in thin horizontal layers heated from below (Rayleigh-Bénard convection). Systems having two or more external control parameters have also generated interest, partly due to the occurrence of novel behavior very near to the threshold of instability where disturbance amplitudes are conveniently small. In the simplest case, stability of the base state is altered in the sense that the transition to a state of lowered symmetry is shifted to higher or lower values of a relevant control parameter [1]. For instance, temporal modulation of the driving buoyancy force in Rayleigh-Bénard convection, which can be accomplished either by means of vibrations in the vertical direction (variations of the effective gravitational acceleration) or the application of time-dependent boundary temperatures (variations in density), has been shown to enhance the stability of the conducting state [2], and for some parameter values causes the bifurcation to become backward (inverted) [3].

Another multiparameter system of interest is thermal convection subject to rotation about a vertical axis. Some fascinating phenomena can result from simple solid-body rotation; for example, the rotation rate can be tuned so that the initial bifurcating flow is time dependent, but nonperiodic (Küppers-Lortz instability) [4].

The present experiments, in one sense, are an extension of the work cited above and were motivated by the recent work of de Nigris, Nicolis, and Frisch [5]. These authors address the question of pattern formation in the presence of slight stirring motions, or fluid mixing, which they model by small amplitude *stochastic* modulation of the rate of rotation of a heated layer of fluid about a vertical axis. The convective transition is shown to be robust, with the amplitude of the *most probable* convective state becoming nonzero for higher values of the static control parameter, relative to the unmodulated system, and occurring via a bifurcation which remains perfect, or sharp. In this regard, we note the receipt of preliminary results [6] on a truncated model which similarly treats a periodic modulation of the rotation rate.

Experimentally, many of the same complications occur with modulated rotation as with oscillating boundary temperatures. For instance, imperfections in the system can force convection and result in a "rounding" of the convective transition. This can effectively mask shifts in the threshold. In our system, as it turns out, there is only a narrow band of modulation frequencies for which positive threshold shifts can be adequately resolved. The major difference, of course, between the two types of modulated systems is the weak mixing of the fluid due to "spin-up and -down" circulations in boundary layers near the surfaces. These circulations result from *changes* in the angular velocity of the container [7].

The experimental cell, which has been described in detail elsewhere [2], consists of a cylindrical layer of helium I of height $d=0.0917$ cm and diameter $D=1.617$ cm, confined between top and bottom copper plates and thin (0.016 cm) stainless-steel sidewalls. The entire cell assembly is situated on an extremely stable 1-m-diam steel turntable rotated by means of a programmable stepper motor. Measurements of the heat transport across the cell are used to determine convective stability, in terms of the Nusselt number Nu, defined as the ratio of the effective thermal conductivity of the fluid, for a given applied heat flux, to its value in the absence of thermal convection.

There are several relevant parameters in this problem. The Rayleigh number $R=g\alpha\Delta Td^3/\nu\kappa$ completely determines stability in the absence of rotation, where g is the gravitational acceleration, α is the isobaric thermal expansion coefficient, ΔT is the temperature difference across the layer, and ν and κ are, respectively, the kinematic viscosity and thermal diffusivity of the fluid. Additionally, we define the Prandtl number $\sigma=\nu/\kappa=0.49$ at the operating temperature of 2.63 K, and a parameter $\Omega=\Omega_D\tau_v$, where Ω_D is the dimensioned rotation rate of the cell, and $\tau_v=d^2/\nu$ is a characteristic vertical viscous diffusion time across the fluid layer.

For periodic modulation we were able to impose on the fluid layer a time-dependent rotation about the vertical axis $\Omega(t)\equiv\Omega_s+\Omega_0\cos(\tilde{\omega}t)$, where $\tilde{\omega}=\omega\tau_v$ with ω the dimensional modulation frequency. In the first set of experiments, $\tilde{\omega}$ was varied over a wide range of values with a fixed mean rotation $\Omega_s=37.6$, and two fixed values of the amplitude ratio $\Delta=\Omega_0/\Omega_s=0.35$ and 0.70. To determine the relative stability of the modulated system we measure the Rayleigh number R relative to the unmodulated threshold $R_c(\Omega_s)$, which we determine separately with $\Omega_0=0$. A reduced Rayleigh number is then denoted by $\epsilon=R/R_c(\Omega_s)-1$. We note that $R_c(\Omega_s)$ increases monotonically with Ω_s [8].

The frequency dependence of the convective threshold ϵ_c is shown in Fig. 1(a) for the amplitude ratio $\Delta=0.70$. At very low modulation frequencies convection is present considerably below the threshold of the unmodulated system. This is not unexpected, and is presumably due to the increasing susceptibility to external noise as the modulation tends toward the quasistatic limit. This apparent destabilization is reduced as the frequency of modulation is increased, and for rapid enough modulation it is possible to resolve positive shifts in the convective threshold. This occurs for modulation periods comparable to τ_v, for which the viscous boundary layers formed by the periodic acceleration and deceleration of the cell attain a thickness $(2\nu/\omega)^{1/2}\cong(d^2/\pi)^{1/2}$ roughly comparable to the cell half height. In the limit of very large ω, we expect modulation effects to be confined to increasingly small boundary layers, and, therefore, the threshold shifts to vanish, resulting in a maximum stabilization at some finite, nonzero frequency. The latter point is evident in the data of Fig. 1(a), but, rather than vanishing at large ω, the threshold shifts again become substantially negative. The additional convective forcing may be associated with crossover to a more nearly "impulsive" spin up of the fluid at these larger accelerations [9], where the time scale of the modulation becomes of the same order as the impulsive Ekman spin-up time $t_{\mathrm{Ek}}=d/(\nu\Omega_s)^{1/2}$.

We repeated the above experiments with $\Delta=0.35$ in order to ascertain the dependence of the convective threshold on the amplitude of modulation. As is ap-

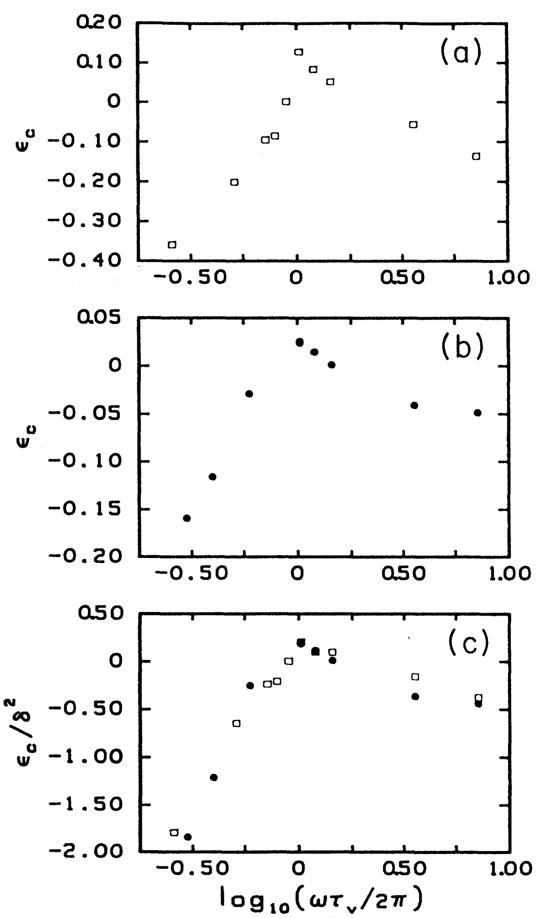

FIG. 1. Convective thresholds for (a) $\Delta=0.70$ (open squares) and (b) $\Delta=0.35$ (filled circles). Dependence on the modulation amplitude of the results in (a) and (b) is suggested in (c), where $\delta=(1+\epsilon_c)\Delta$.

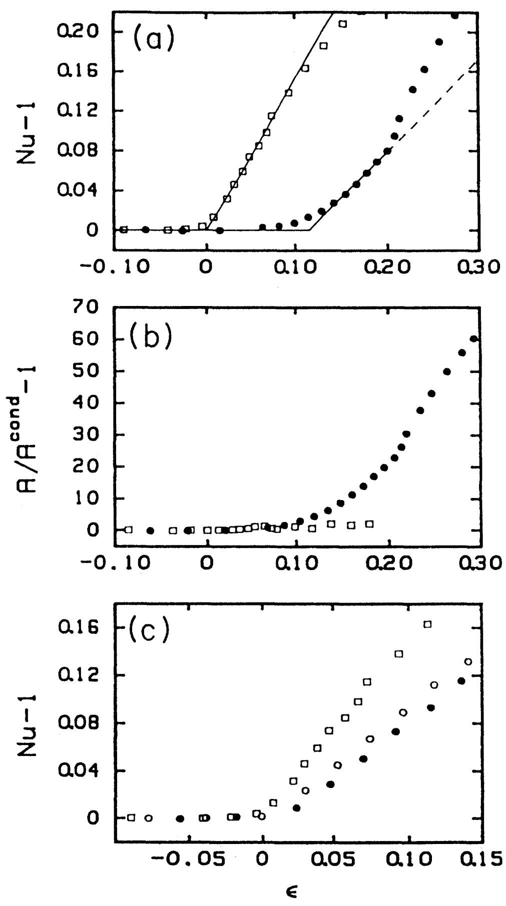

FIG. 2. (a) Nusselt number vs reduced Rayleigh number for sinusoidal modulation about $\Omega_s=37.6$, with $\tilde{\omega}=6.45$. Open squares, $\Delta=0$; filled circles, $\Delta=0.70$. (b) rms amplitude A of temperature fluctuations in the bottom plate for the same runs as in (a). The normalizing value of A in the conducting state is $A^{\mathrm{cond}}=1.4\times10^{-6}$ K. The very slight rise in A for the $\Delta=0$ run at onset is due to weakly turbulent flow resulting from the Küppers-Lortz instability. (c) Small amplitude stochastic modulation about $\Omega_s=37.6$ with a cutoff frequency $\tilde{\omega}_{\max}\approx34$. rms values of Δ are 0.10, filled circles; 0.05, open circles; 0, open squares.

parent in Fig. 1(b), the results are qualitatively very similar, including a maximum positive threshold shift near $\tilde{\omega}\approx 2\pi$. Quantitatively, however, the lower amplitude modulation has a significantly smaller effect on both the positive and negative onset shifts, as is clearly seen by noting the difference in ordinate scales. In both cases the apparent destabilization is in disagreement with the model behavior, for which the onset is shifted in the positive direction an amount which should vary quadratically with amplitude [3,6]. With regard to the latter point, we observe that if we introduce a modified amplitude ratio $\delta=(1+\epsilon_c)\Delta$ ($\delta\cong\Delta$ for the positive onset shifts), then the threshold data in Figs. 1(a) and 1(b), including the negative shifts, effectively scale in an analogous manner, as shown in Fig. 1(c). For the purpose of comparison, preliminary data from Ref. [6] for $\tilde{\omega}=2\pi$, where perhaps the convective forcing has its least effect on the position of the onset, and for similar parameters $\Omega_s=35.4$ and $\sigma=\frac{2}{3}$, gives $\epsilon_c/\Delta^2\cong 1.6\times 10^{-1}$ which compares encouragingly well with the corresponding data in Fig. 1(c).

Heat transfer data corresponding to $\Delta=0.70$, $\Omega_s=37.6$, and $\tilde{\omega}=6.45$ are plotted in Fig. 2(a), along with data obtained in the absence of modulation ($\Delta=0$), as an example of both the observed stabilization due to modulation, and the imperfection in the bifurcation due to convective forcing. The latter manifests itself in the form of a rounding, or smearing out, of the transition to convection. The convecting fluid evidently becomes unstable to a second flow configuration very near threshold [about $\epsilon\cong 0.2$ in Fig. 2(a)]. Figure 2(b) illustrates a significant feature of the flow—the strength of the convective circulation depends strongly on the *instantaneous* rate of rotation, giving rise to large fluctuations in the heat transport commensurate with the modulation. We have plotted the rms amplitude of temperature fluctuations in the bottom plate, relative to their background value below the onset, as a function of ϵ for the same runs shown in Fig. 2(a). Note that even for $\Delta=0$ there is a measurable increase in the amplitude of temperature fluctuations immediately above the convective threshold, although clearly very slight on the scale of the perturbed modulated flow. This low amplitude noise in the absence of modulation probably results from the Küppers-Lortz instability [4], which, at this rotation rate, renders the convective rolls unstable directly at onset. Its "noisy" appearance results from the short integration period coupled with the nonperiodicity of the flow.

Although nearly all frequencies of modulation resulted in apparent destabilization under periodic modulation of the rotation rate, we found that the net effect of a small amplitude *broadband* modulation (up to a cutoff frequency $\tilde{\omega}_{\max}\approx 34$) was to enhance the conductive stability. This modulation was achieved by driving the programmable stepper motor with a pseudorandom wave form digitized from a HP 3722A noise generator. Heat transfer data for runs with rms values of $\Delta=0.10$ and 0.05 are shown in Fig. 2(c).

Finally, we also investigated the effect of periodic modulation having *zero* mean, i.e., $\Omega_s=0$. In this case the angular velocity of the cell was varied not only in magnitude, but in *direction* as well during a modulation cycle. Threshold data are shown in Fig. 3(a) for two different modulation frequencies, $\tilde{\omega}=7.53$ and 3.76, where we have plotted the measured threshold relative to the critical Rayleigh number in the absence of rotation, $r_c=R_c/R_c(\Omega_0=0,\Omega_s=0)$, as a function of Ω_0. We also show in Fig. 3(a) data corresponding to steady rotation at Ω_s for comparison. As illustrated in the figure, the threshold data with modulation increase with Ω_0 in a manner similar to the increase in threshold with Ω_s for steady rotation. In fact, we find that the convective thresholds with modulation are consistent with those due to an *effective* steady rotation rate $\alpha\Omega_0$ equal in magnitude to the rms value of the modulation amplitude. This is illustrated in Fig. 3(b), where we have chosen $\alpha=0.707$.

In conclusion, we have investigated the effect of both periodic and stochastic modulation of the rate of rotation on the convective onset of a thin layer of helium *I*. Un-

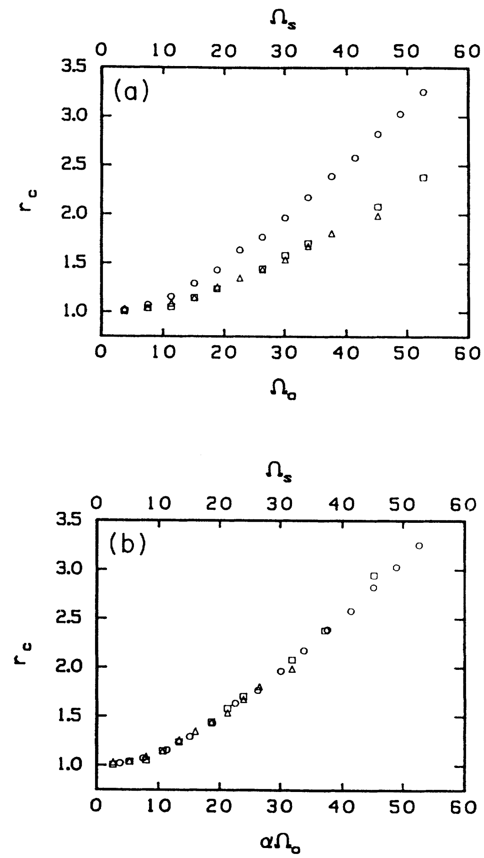

FIG. 3. (a) Open circles represent convective threshold due to steady rotation and are plotted as a function of $\Omega_s(\Omega_0=0)$. The open squares and open triangles represent thresholds due to modulation about $\Omega_s=0$, corresponding to $\tilde{\omega}=7.53$ and 3.76, respectively, and are plotted as a function of Ω_0. (b) Scaling of the threshold data with modulation in (a) by plotting against $\alpha\Omega_0$, where $\alpha=0.707$. That is, the convective thresholds with modulation appear to be those corresponding to an *effective* steady rotation rate $\alpha\Omega_0$. In both (a) and (b) the relative Rayleigh number r_c has as its normalization the critical Rayleigh number *in the absence of rotation.*

der periodic modulation the effect is similar to that observed in temporal modulation of the plate temperatures, with the addition of strong forcing at high frequencies which may be associated with an approach to "impulsive" spin up. Positive threshold shifts are observed only over a very narrow range of modulation frequencies and their magnitude increases in proportion to the square of the modulation amplitude as expected. Modulation about zero rotation with amplitude Ω_0 produces convective flow at the same critical temperature gradient as for steady rotation at a rate $\alpha\Omega_0$, with $\alpha=0.707$.

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