Doubly differential cross sections for ionization of helium by electron impact

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The Glauber approximation is used to calculate doubly differential cross sections (DDCS's) for electron-impact ionization of helium at incident energies of 100, 300, and 500 eV. Angular dependences of the cross sections are presented for the primary (scattered) electrons. The present calculation is done for the case where the energy of the primary electron is large compared with that of the secondary (ejected) electron. A comparison is made of the present DDCS with the results of other calculations and experiment.

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In a previous paper [1], we applied the Glauber approximation [2] (GA) to calculate doubly differential cross sections (DDCS's) for electron-impact ionization of helium in the incident energy range 224.58–2824.58 eV. In the case of a symmetric geometry, i.e., the energies of the two outgoing electrons are equal, and for a scattering angle of 45°, the DDCS's obtained in the GA are in reasonably good agreement with experiment. Very recently, we applied the GA to calculate triply differential cross sections (TDCS's) for the $He(e,2e)He^+$ process in the case of coplanar asymmetric geometry [3], i.e., the energy of one of the two emitted particles is very small compared with that of the other, and the angle of scattering is also small. At incident energies of 256 and 600 eV, the TDCS obtained in the GA showed a definite improvement over the first Born result. In view of the success of the GA, we apply it to evaluate the DDCS for the primary electron in the case of asymmetric geometry at incident energies of 100, 300, and 500 eV and compare our results with the distorted-wave Born (DWB) calculation of McCarthy and Zhang [4] and measured data [5,6].

In the GA, the amplitude for ionization of He by electron impact is given by

$$F(\mathbf{q},\mathbf{K}_2)=\frac{iK}{2\pi}\int d\mathbf{b}\,d\mathbf{r}_1 d\mathbf{r}_2 \phi_f^*(\mathbf{r}_1,\mathbf{r}_2)\Gamma(\mathbf{b};\mathbf{r}_1,\mathbf{r}_2)\times\phi_i(\mathbf{r}_1,\mathbf{r}_2)\exp(i\mathbf{q}\cdot\mathbf{b})\ , \qquad (1)$$

where

$$\Gamma(\mathbf{b};\mathbf{r}_1,\mathbf{r}_2)=1-\left[\frac{|\mathbf{b}-\mathbf{s}_1|}{b}\right]^{2i\eta}\left[\frac{|\mathbf{b}-\mathbf{s}_2|}{b}\right]^{2i\eta},$$

$\mathbf{q}=\mathbf{K}-\mathbf{K}_1$ and $\eta=1/K$. Atomic units are used throughout unless otherwise indicated. Here $\mathbf{K}$, $\mathbf{K}_1$, and $\mathbf{K}_2$ are the momenta of the incident, scattered (primary), and ejected (secondary) electrons, respectively; $\mathbf{b}$, $\mathbf{s}_1$, and $\mathbf{s}_2$ are the respective projections of the position vectors of the incident particle and the two bound electrons onto the plane perpendicular to the direction of the Glauber path integration.

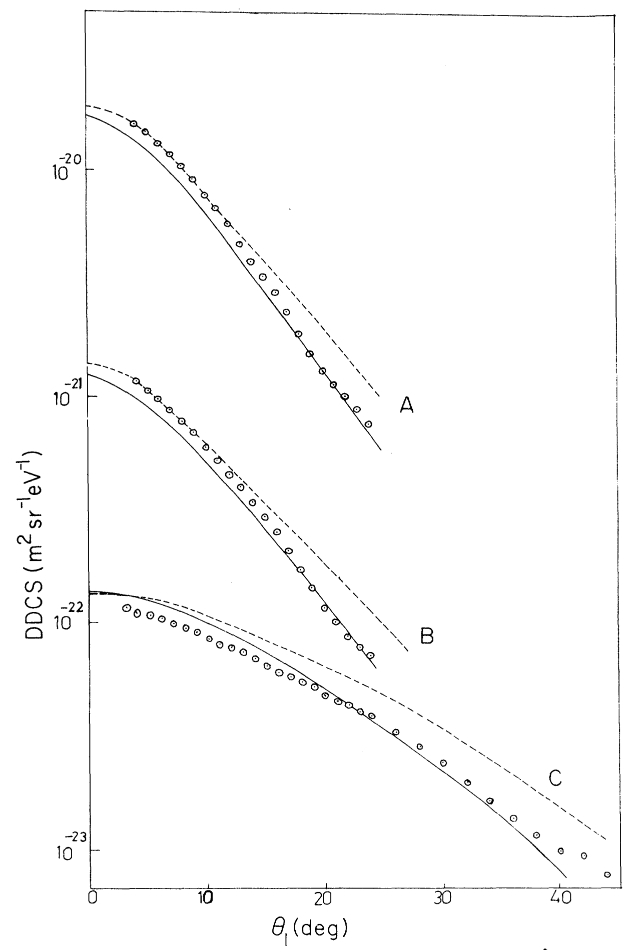

FIG. 1. Doubly differential cross sections $d^2\sigma/d\hat{k}_1 dE_2$ for primary electrons for ionization of He at the incident energy of 100 eV. Experimental data of Müller-Fiedler, Jung, and Ehrhardt [5] are denoted by open circles. The solid and dashed curves represent, respectively, the GA and the DWB cross sections. Cases illustrated are as follows: (a) $E_1=73.4$ eV (cross sections are multiplied by 10) (curve A): (b) $E_1=71.4$ eV (curve B); and (c) $E_1=55.4$ eV (curve C).

TABLE I. Comparison of present DDCS, $d^2\sigma/d\hat{k}_1 dE_2$, in units of $m^2\,sr^{-1}\,eV^{-1}$ obtained in the GA with the DWB calculations of McCarthy and Zhang and measured data of Müller-Fiedler, Jung, and Ehrhardt for electron-impact ionization of He. Square brackets denote powers of 10.

E (eV)	E_1 (eV)	θ_1 (deg)	GA[a]	DWB[b]	Expt.[c]	θ_1 (deg)	GA[a]	DWB[b]	Expt.[c]
100	73.4	0	1.75[−21]	1.88[−21]		40	6.45[−24]	1.44[−23]	
		5	1.26[−21]	1.48[−21]	1.49[−21]	50	3.00[−24]	5.58[−24]	
		10	6.34[−21]	7.95[−22]	7.85[−22]	60	2.09[−24]	3.56[−24]	
		15	2.93[−22]	4.10[−22]	3.38[−22]	90	9.64[−25]	2.59[−24]	
		20	1.32[−23]	2.07[−22]	1.30[−22]	120	5.32[−25]	2.24[−24]	
		25	5.85[−23]	1.05[−22]		150	3.74[−25]	2.10[−24]	
		30	2.60[−23]	5.28[−23]		180	3.33[−25]		
	71.4	0	1.24[−21]	1.40[−21]		40	6.61[−24]	1.44[−23]	
		5	9.43[−22]	1.06[−21]	1.08[−21]	50	2.76[−24]	5.34[−24]	
		10	5.16[−22]	6.18[−22]	5.95[−22]	60	1.84[−24]	3.23[−24]	
		15	2.93[−22]	4.10[−22]	3.38[−22]	90	9.64[−25]	2.59[−24]	
		20	1.25[−22]	1.84[−22]	1.14[−22]	120	4.85[−25]	2.04[−24]	
		25	5.85[−23]	9.74[−23]		150	3.42[−25]	1.92[−24]	
		30	2.71[−23]	5.11[−23]		180	3.05[−25]		
	55.4	0	1.34[−22]	1.32[−22]		40	8.44[−24]	1.58[−23]	1.04[−23]
		5	1.23[−22]	1.28[−22]	1.07[−22]	50	2.87[−24]	6.53[−24]	
		10	9.92[−23]	1.07[−22]	8.55[−23]	60	1.19[−24]	3.20[−24]	
		15	7.36[−23]	8.53[−23]	6.54[−23]	90		2.11[−24]	
		20	5.22[−23]	6.54[−23]	4.82[−23]	120		1.86[−24]	
		25	3.55[−23]	4.88[−23]		150		1.69[−24]	
		30	2.30[−23]	3.49[−23]	2.47[−23]	180			
300	271.4	0	5.06[−21]	4.88[−21]		40	1.36[−24]	8.16[−25]	
		5	1.16[−21]	1.30[−21]	1.41[−21]	50	8.43[−25]	4.73[−25]	
		10	2.62[−22]	3.34[−22]	3.77[−22]	60	5.23[−25]	3.23[−25]	
		15	6.40[−23]	9.75[−23]	1.03[−22]	90	1.64[−25]	1.30[−25]	
		20	1.51[−23]	2.87[−23]	2.51[−23]	120	7.88[−26]	7.29[−26]	
		25	4.53[−24]	8.56[−24]		150	5.28[−26]	5.31[−26]	
		30	2.36[−24]	2.99[−24]		180	4.65[−26]		
	235.4	0	8.72[−23]	7.55[−23]		40	4.54[−25]	7.42[−25]	
		5	6.13[−23]	5.68[−23]	5.01[−23]	50	1.76[−25]	2.07[−25]	
		10	3.57[−23]	3.54[−23]	2.89[−23]	60	1.14[−25]	1.20[−25]	
		15	2.30[−23]	2.49[−23]	1.70[−23]	90	4.13[−26]	5.21[−26]	
		20	1.44[−23]	1.72[−23]	9.95[−24]	120	2.02[−26]	2.97[−26]	
		25	7.45[−24]	9.91[−24]		150	1.32[−26]	2.17[−26]	
		30	3.04[−24]	4.55[−24]		180	1.15[−26]		
500	471.4	0	8.89[−21]	7.39[−21]		40	7.51[−25]	2.49[−25]	
		5	8.07[−22]	9.28[−22]	9.24[−22]	50	4.00[−25]	1.39[−25]	
		10	1.24[−22]	1.74[−22]	1.70[−22]	60	2.28[−25]	8.92[−26]	
		15	1.87[−23]	3.22[−23]	3.06[−23]	90	6.48[−26]	2.81[−26]	
		20	3.87[−24]	7.44[−24]	5.87[−24]	120	3.02[−26]	1.56[−26]	
		25	1.97[−24]	1.72[−24]		150	2.00[−26]	1.22[−26]	
		30	1.41[−24]	6.51[−25]		180	1.75[−26]		

[a]Present Glauber approximation.
[b]Distorted-wave Born calculation of McCarthy and Zhang (Ref. [4]).
[c]Reference [5].

TABLE II. Comparison of present DDCS, $d^2\sigma/d\hat{k}_1 dE_2$, in units of $m^2\,sr^{-1}\,eV^{-1}$ obtained in the GA with the DWB calculations of McCarthy and Zhang, the measured data of Müller-Fiedler, Jung, and Ehrhardt, and those of Avaldi *et al.* for electron-impact ionization of He. Square brackets denote powers of 10.

E (eV)	E_1 (eV)	θ_1 (deg)	GA[a]	DWB[b]	Data of Müller-Fiedler, Jung, and Ehrhardt[c]	Data of Avaldi *et al.*[d]
500	435.4	0	1.52[−22]	1.28[−22]		
		5	6.63[−23]	6.10[−23]	5.06[−23]	
		10	3.07[−23]	3.13[−23]	1.85[−23]	3.90[−23]
		15	1.70[−23]	1.94[−23]	9.00[−24]	2.40[−23]
		20	6.89[−24]	9.02[−24]	3.55[−24]	1.04[−23]
		25	1.87[−24]	2.89[−24]		3.69[−24]
		30	4.89[−25]	8.15[−25]		9.81[−25]
		40	1.32[−25]	1.19[−25]		2.50[−25]
		50	7.72[−26]	5.21[−26]		1.29[−25]
		60	4.74[−26]	3.20[−26]		7.17[−26]
		90	1.41[−26]	1.08[−26]		2.97[−26]
		120	6.34[−27]	5.37[−27]		1.60[−26]
		150	4.12[−27]	5.85[−27]		
		180	3.60[−27]			

[a]Present Glauber approximation.
[b]Distorted-wave Born calculation of McCarthy and Zhang (Ref. [4]).
[c]Reference [5].
[d]Reference [6].

In Eq. (1), $\mathbf{q}$, $\mathbf{b}$, $\mathbf{s}_1$, and $\mathbf{s}_2$ are all coplanar. $\phi_i(\mathbf{r}_1,\mathbf{r}_2)$ and $\phi_f(\mathbf{r}_1,\mathbf{r}_2)$ represent the wave functions of the initial and final states of the target, respectively. For the initial state of He, we have chosen the analytical fit to the Hartree-Fock wave function given by Byron and Joachain [7]:

$$\phi_i(\mathbf{r}_1,\mathbf{r}_2)=U(\mathbf{r}_1)U(\mathbf{r}_2)\ , \tag{2}$$

where

$$U(r)=(4\pi)^{-1/2}(Ae^{-\alpha r}+Be^{-\beta r})\ ,$$

$$A=2.065\,05,\quad B=2.081\,44,\quad \alpha=1.41,\quad \beta=2.61\ .$$

For the final-state wave function we have used a symmetrized product of the He^+ ground-state wave function for the bound electron times a Coulomb wave $\phi_{\mathbf{K}_2}$ orthogonalized to the ground-state orbital

$$\phi_f(\mathbf{r}_1,\mathbf{r}_2)=2^{-1/2}[\phi_{\mathbf{K}_2}(\mathbf{r}_1)\nu(\mathbf{r}_2)+\nu(\mathbf{r}_1)\phi_{\mathbf{K}_2}(\mathbf{r}_2)]\ , \tag{3}$$

where

$$\nu(\mathbf{r})=(\lambda')^{3/2}\pi^{-1/2}e^{-\lambda' r}\ ,$$

$$\phi_{\mathbf{K}_2}(\mathbf{r})=\chi^-_{\mathbf{K}_2}(\mathbf{r})-\langle U(\mathbf{r}')|\chi^-_{\mathbf{K}_2}(\mathbf{r}')\rangle U(\mathbf{r})\ ,$$

$$\chi^-_{\mathbf{K}_2}(\mathbf{r})=(2\pi)^{-3/2}\exp(\tfrac{1}{2}\gamma\pi)\Gamma(1+i\gamma)\exp(i\mathbf{K}_2\cdot\mathbf{r})\times{}_1F_1(-i\gamma,1,-i\mathbf{K}_2\cdot\mathbf{r})\ ,$$

$$\gamma=1/K_2,\quad \lambda'=2\ .$$

The triply differential cross section is given by

$$\frac{d^3\sigma}{d\hat{k}_1 d\hat{k}_2 dE_2}=\frac{k_1k_2}{k}|F(\mathbf{q},\mathbf{k}_2)|^2\ . \tag{4}$$

The doubly differential cross section for the primary (scattered) electron is obtained by integrating the TDCS

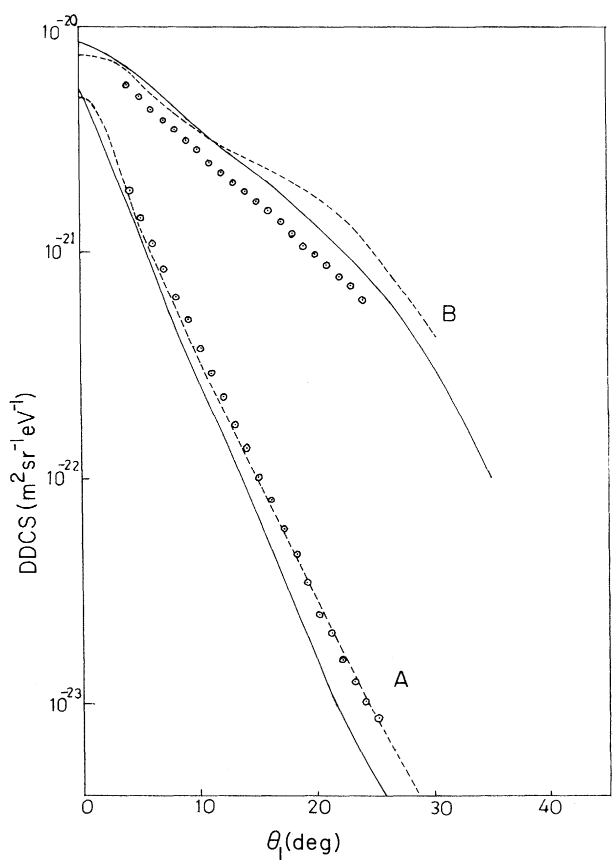

FIG. 2. Same as in Fig. 1 except for incident energy $E=300$ eV. Cases illustrated are as follows: (a) $E_1=271.4$ eV (curve *A*); and (b) $E_1=235.4$ eV (cross sections are multiplied by 100) (curve *B*).

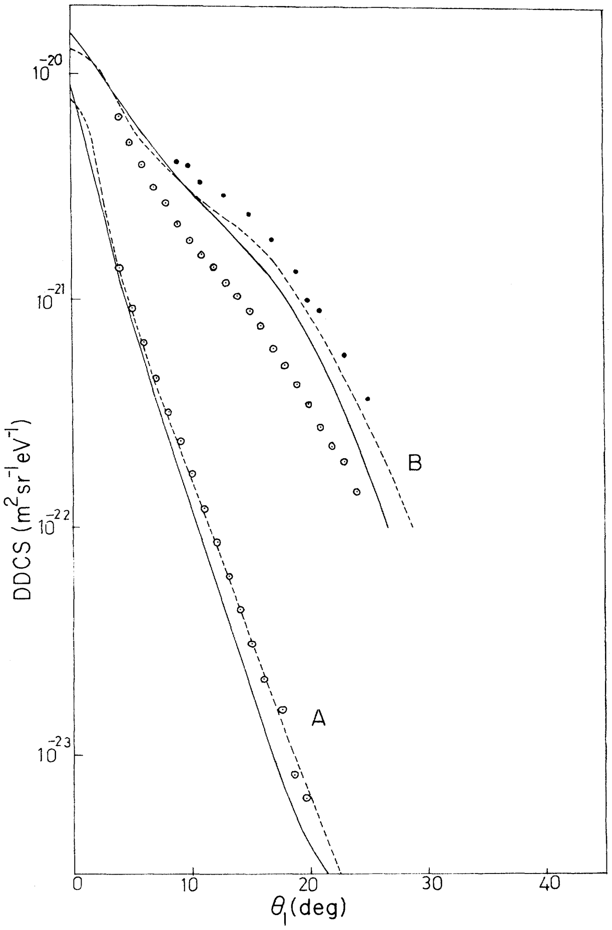

FIG. 3. Same as Fig. 1 except for incident energy $E=500$ eV. Cases illustrated are as follows: (a) $E_1=471.4$ eV (curve A); and (b) $E_1=435.4$ eV (cross sections are multiplied by 100) (curve B). The solid circles represent the experimental data of Avaldi *et al.* [6].

over the solid angle for the secondary (ejected) electron:

$$\frac{d^2\sigma}{d\hat{k}_1 dE_2}=\frac{k_1k_2}{k}\int d\hat{k}_2|F(\mathbf{q},\mathbf{k}_2)|^2 \; . \tag{5}$$

The present calculation is performed using the technique of Roy, Das, and Sil [8] that reduces the eight-dimensional Glauber amplitude for the He$(e,2e)$He$^+$ process to a three-dimensional integral. Tables I and II present our GA results together with the DWB cross sections of McCarthy and Zhang and the available experimental data for the ionization of He by electron impact at the incident energy E of 100, 300, and 500 eV. At $E=100$ eV, we have considered the energy E_1 of the primary electrons to be 73.4, 71.4, and 35.4 eV. At $E=300$ eV, E_1 is taken to be 271.4 and 235.4 eV while at $E=500$ eV, the primary energies considered are 471.4 and 435.4 eV. It may be noted that the experimental data of Müller-Fiedler, Jung, and Ehrhardt [5] are available in all the cases mentioned above while the data of Avaldi *et al.* [6] exist only when $E=500$ eV and $E_1=435.4$ eV.

Figures 1–3 exhibit a graphical comparison of the GA and the DWB results with the experimental data of Müller-Fiedler, Jung, and Ehrhardt and of Avaldi *et al.* We see from Tables I and II as well as Figs. 1–3 that the GA cross sections are in reasonably good agreement with experiment. The experimental data of Avaldi *et al.* agree much more closely with the GA and the DWB calculations than the data of Müller-Fiedler, Jung, and Ehrhardt. The DWB cross sections are found to be in better accord with the data of Avaldi *et al.* than the GA results especially at larger scattering angles. However, a comparison of the GA and the DWB cross sections with the data of Müller-Fiedler, Jung, and Ehrhardt shows that the GA cross sections are in better agreement with experiment than the DWB at lower energies of primary electrons whereas the DWB approximation is slightly better than the GA at higher values of E_1, especially at smaller angles of the primary electrons.

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