Pump-coupled masers: Transient regime

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We consider a system of two masers that are pumped by the same beam of excited atoms. The atoms pass first through one maser and then through the other. As a result, the first maser modifies the pump going into the second and an effective coupling results. We examine this system in the amplifier configuration, i.e., we do not consider saturation effects. The fields in the two masers are mutually coherent and can, under the proper circumstances, be free of noise in their relative phase. Therefore this system exhibits behavior similar to that of a correlated-emission laser. We also find that the photon number in the second maser grows faster than that in the first. These effects should be observable even if the pump beam has a finite velocity spread.

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I. INTRODUCTION

Recently it has become clear that pump fluctuations have a significant effect on the fluctuations of the light that emerges from a laser. This was first observed and studied in the context of dye lasers, where pump fluctuations lead to a more noisy output than that predicted by standard laser theory [1–8]. A second line of work has been the investigation of quieter, more regular pumps than those considered in the usual theory [9–21]. This has led to the realization that it is possible for lasers to produce light with sub-Poissonian photon statistics.

Standard laser theory takes the pump to be a beam of two-level atoms initially in their excited states. The number of atoms that pass through the laser cavity in a given period of time follows a Poisson distribution. One way of modifying the pump is to consider something other than a Poisson distribution for the atomic arrivals. The usual choice is a distribution in which the fluctuations in the number of atoms are smaller than for a Poisson distribution. This decreases the intensity fluctuations of the output light [9–21]. Another approach is to consider a Poisson arrival distribution but to assume that the states of the atoms entering the laser vary in some way. It is one realization of this scheme that we wish to consider here. The theory we develop is most easily applied to micromasers so that we shall, henceforth, refer to masers rather than lasers.

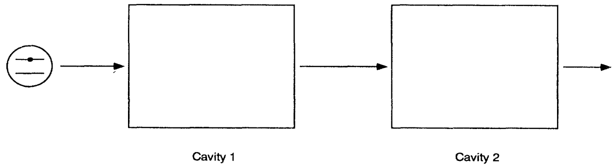

FIG. 1. Schematic arrangement of the pump-coupled micromasers. The pump beam first passes through one maser and then through the other.

The specific system we shall examine consists of two masers pumped by the same atomic beam (Fig. 1). The beam first passes through one maser and then through the other. The atoms are injected into the first maser in their excited states, but by the time they reach the second maser the interaction with the first has changed their states. Therefore, the state of an atom entering the second maser depends upon the state of the field in the first. This means that the first maser modified the pump going into the second, and an effective coupling between the masers is produced.

This coupling produces a coherence between the fields in the two cavities. This would not be the case for independently pumped masers. In fact, the phase of the field in the second cavity is locked to that of the field in the first. One also finds behavior reminiscent of that in correlated emission lasers [22–24]. That is, the noise in the relative phase is smaller than that in either of the individual phases and can vanish under optimal circumstances.

In this paper we shall consider the system in the amplifier configuration, i.e., atomic saturation effects will be ignored. The nonlinear effects and steady-state fields will be examined in a future publication.

II. MASTER EQUATION AND FIELD AMPLITUDE EQUATIONS

We now want to derive a master equation that describes the evolution of the field density matrix for the two cavities. We assume that an average of r two-level atoms per unit time are injected into the first cavity in their excited states. Each atom interacts with the field in the first maser for a time T and then proceeds to the second cavity, where it interacts with the field there, again for a time T. We assume that the time of flight between the cavities is sufficiently short that we can ignore the effect of spontaneous emission for atoms in this region, so that the atom enters the second cavity in the same state as it left the first. We also assume that the interaction time is short compared to the inverses of the Rabi frequencies in the two cavities and to the lifetime of the atoms. Finally, we consider only the case in which there is no detuning between the atoms and the fields. The field mode frequencies are taken to be ω.

The two-level atoms themselves have ground and excited states $|b\rangle$ and $|a\rangle$, respectively. Their interaction with a single field mode is described by the interaction picture Hamiltonian

$$H_I = g(a\sigma^+ + a^\dagger\sigma^-) . \tag{2.1}$$

Here g is the atom-field coupling constant, a and $a^\dagger$ are the mode annihilation and creation operators, and σ^+ and σ^- are the atomic raising and lowering operators. The operator H_I is time independent in the interaction picture.

In order to find the master equation for the interaction-picture field density matrix ρ_F, we must first find, to second order in g, the change produced in ρ_F by the passage of a single atom through the system [25]. This is calculated from second-order perturbation theory and is given by

$$\delta\rho_F = \sum_{j=1}^{2} [(gT)^2/2](2a_j^\dagger\rho_F a_j - a_j a_j^\dagger \rho_F - \rho_F a_j a_j^\dagger) + (gT)^2(a_2^\dagger\rho_F a_1 + a_1^\dagger\rho_F a_2 - a_1^\dagger a_2\rho_F - \rho_F a_1 a_2^\dagger) . \tag{2.2}$$

Here a_1 and a_2 are the annihilation operators for the fields in the first and second cavities, respectively. The coarse-grained time derivative of the two-mode field density matrix is obtained by multiplying $\delta\rho_F$ by the atomic injection rate r. Finally by adding terms to take into account the cavity losses we obtain the master equation for the interaction-picture field density matrix

$$\begin{aligned}\frac{d\rho_F}{dt} = \sum_{j=1}^{2} &[(A/2)(2a_j^\dagger\rho_F a_j - a_j a_j^\dagger\rho_F - \rho_F a_j a_j^\dagger) \\ &+ (\gamma/2)(2a_j\rho_F a_j^\dagger - a_j^\dagger a_j\rho_F - \rho_F a_j^\dagger a_j)] \\ &+ A(a_2^\dagger\rho_F a_1 + a_1^\dagger\rho_F a_2 - a_1^\dagger a_2\rho_F - \rho_F a_1 a_2^\dagger) .\end{aligned} \tag{2.3}$$

The constant $A = r(gT)^2$ is the gain and γ is the inverse of the cavity lifetime, which is assumed to be the same for both cavities. The terms in the square brackets are just those which describe two independent masers in the linear regime. It is the last four terms that describe the coupling between the fields induced by the common pump.

In the derivation of this equation we have assumed that the change in the field, during the time that a single atom interacts with it, is small. In order to meet this requirement it is necessary that the atomic transit time T be much smaller than the cavity decay time $1/\gamma$. For both masers and micromasers this condition is well satisfied. We are also assuming that the effect of spontaneous emission during the time the atom is between the cavities is negligible. For this to be true the spontaneous lifetimes of the maser levels must be large in comparison to the time it takes for the atom to traverse the distance between the cavities. In micromaser systems, where the masing levels are long-lived Rydberg states, this condition should be satisfied for intercavity distances of the order of centimeters.

We can derive equations of motion for the expectation values of the field amplitudes by multiplying Eq. (2.3) by either a_1 or a_2 and taking the trace. The results are

$$\begin{aligned} d\langle a_1(t)\rangle/dt &= -i\omega\langle a_1(t)\rangle + [(A-\gamma)/2]\langle a_1(t)\rangle , \\ d\langle a_2(t)\rangle/dt &= -i\omega\langle a_2(t)\rangle + [(A-\gamma)/2]\langle a_2(t)\rangle \\ &\quad + A\langle a_1(t)\rangle .\end{aligned} \tag{2.4}$$

Note that, as expected, the first maser is unaffected by the second, but the equation for the field amplitude in the second maser contains a source term that is proportional to the amplitude of the field in the first. These equations are easily solved with the result that

$$\begin{aligned}\langle a_1(t)\rangle &= \langle a_1(0)\rangle e^{[-i\omega+(A-\gamma)/2]t} , \\ \langle a_2(t)\rangle &= [\langle a_2(0)\rangle + (At)\langle a_1(0)\rangle]e^{[-i\omega+(A-\gamma)/2]t} .\end{aligned} \tag{2.5}$$

For $A > \gamma$ the amplitudes of the fields in both masers grow [unless $\langle a_1(0)\rangle = 0$]; but for sufficiently large times we see that the field in the second cavity grows faster than that in the first. In fact, for times t such that

$$(At)|\langle a_1(0)\rangle| \gg |\langle a_2(0)\rangle| ,$$

the amplitudes of the fields are proportional, which implies that they have the same phase. Thus, the phase of the second cavity locks to that of the first. A second way to see this is to express $\langle a_1(t)\rangle$ and $\langle a_2(t)\rangle$ as

$$\begin{aligned}\langle a_1(t)\rangle &= r_1(t)e^{i\phi_1(t)} , \\ \langle a_2(t)\rangle &= r_2(t)e^{i\phi_2(t)} ,\end{aligned} \tag{2.6}$$

and then use Eqs. (2.4) to find equations of motion for r_1, r_2, ϕ_1, and ϕ_2. These in turn can be used to find the equation of motion for $\phi = \phi_1 - \phi_2$, the phase difference between the two fields. We find

$$\frac{d\phi}{dt} = -A(r_1/r_2)\sin\phi , \tag{2.7}$$

which is the standard mode-locking equation [25]. This equation has a single, stable steady-state solution at $\phi = 0$. As t increases $\phi(t)$ tends to this solution with the result that the phases of the field amplitudes become the same.

III. INTENSITY EQUATION

. One can also derive from the master equation equations of motion for the intensities of the fields in the cavities. First let us define

$$\begin{aligned} n_1(t) &= \langle a_1^\dagger(t)a_1(t)\rangle, \quad n_2(t) = \langle a_2^\dagger(t)a_2(t)\rangle , \\ n_{12}(t) &= \langle a_1(t)a_2^\dagger(t) + a_1^\dagger(t)a_2(t)\rangle .\end{aligned} \tag{3.1}$$

The quantities $n_1(t)$ and $n_2(t)$ are just the numbers of photons in cavities one and two, respectively. The quantity $n_{12}(t)$ describes the amount of interference the two fields would exhibit if they were superposed. Calculating these expectation values from the master equation we find

$$\frac{dn_1}{dt}=(A-\gamma)n_1+A\ ,$$
$$\frac{dn_{12}}{dt}=(A-\gamma)n_{12}+2A(n_1+1)\ , \tag{3.2}$$
$$\frac{dn_2}{dt}=(A-\gamma)n_2+An_{12}+A\ .$$

These equations can be solved in a straightforward fashion. Before giving the solution, however, note that as in the case of the amplitudes the first maser is independent, but the second is coupled to the first. The solutions to Eqs. (3.2) are

$$n_1(t)=n_1(0)e^{(A-\gamma)t}+\xi_1(t)\ ,$$
$$n_{12}(t)=[n_{12}(0)+2(At)n_1(0)]e^{(A-\gamma)t}+\xi_{12}(t)\ , \tag{3.3}$$
$$n_2(t)=[n_2(0)+(At)n_{12}(0)+(At)^2n_1(0)]e^{(A-\gamma)t}+\xi_2(t)\ ,$$

where

$$\xi_1(t)=[A/(A-\gamma)](e^{(A-\gamma)t}-1)\ ,$$
$$\xi_{12}(t)=2[At-\gamma/(A-\gamma)]\xi_1(t)+2A^2t/(A-\gamma)\ , \tag{3.4}$$
$$\xi_2(t)=[At-\gamma/(A-\gamma)]^2[\xi_1(t)+A/(A-\gamma)]+[A/(A-\gamma)]^2\xi_1(t)-A\gamma^2/(A-\gamma)^3\ .$$

The latter set of quantities are the values that $n_1(t)$, $n_{12}(t)$, and $n_2(t)$ would assume if the initial state is in the vacuum. We shall assume in the following discussion that $A>\gamma$.

Let us note two things about these solutions. The first is that if $n_1(0)=0$ and $n_2(0)=0$, then for times $t>0$ we have that $n_{12}(t)\neq 0$. This means that if the fields were brought together in an interference experiment, then time-stationary interference fringes would be present. This would not be the case if the two masers were pumped independently. Therefore, even if the fields in the two masers are initially in the vacuum state they will be coherent at later times because of the common pump. The second point is that the photon number in the second cavity grows faster than that in the first. In particular, for times such that $(At)\gg 1$, the intensity in the first cavity grows as $e^{(A-\gamma)t}$, while that in the second grows as $(At)^2e^{(A-\gamma)t}$.

IV. INTERPRETATION OF RESULTS

Here we want to discuss some of the main features of the two-maser system and to show in a simple way how they come about. In particular we wish to explain the time dependence of the fields in the two cavities and the growth of coherence between them.

Let us consider first the time dependence of the field amplitudes. As we saw in Sec. II the field in the first cavity obeys the same equation of motion as that in a single maser, but the field amplitude in the second cavity deviates from this behavior. This can be understood in terms of the atomic coherence produced in the pump beam by the field in the first cavity.

In order to see this directly let us consider a maser with injected atomic coherence. The master equation in the interaction picture for such a maser in the linear regime is [26]

$$\frac{d\rho_F}{dt}=(A/2)(2a^\dagger\rho_Fa-aa^\dagger\rho_F-\rho_Faa^\dagger)+(\gamma/2)(2a\rho_Fa^\dagger-a^\dagger a\rho_F-\rho_Fa^\dagger a)-i[(sa^\dagger+s^*a),\rho_F]\ . \tag{4.1}$$

Here atoms are injected into the maser in a state described by an atomic density matrix $\rho^{(\mathrm{at})}$. The parameter s is given by $s(t)=grT\rho_{ab}^{(\mathrm{at})}(t)$, where T is again the time the atom interacts with the field and $\rho_{ab}^{(\mathrm{at})}$ is given by $\rho_{ab}^{(\mathrm{at})}=\mathrm{Tr}[\rho^{(\mathrm{at})}(t)\sigma^-(t)]$ (the operators in this expression are in the interaction picture). We discuss the derivation of Eq. (4.1) in the Appendix. The equation of motion for the field amplitude that results from this master equation is

$$\frac{d\langle a(t)\rangle}{dt}=-i\omega\langle a(t)\rangle+[(A-\gamma)/2]\langle a(t)\rangle-is\ . \tag{4.2}$$

Let us now go back to the two-cavity problem and consider the amount of atomic coherence that an atom acquires by passing through the first cavity. An atom injected into the first cavity at time $t-T-\tau$, where τ is the time of flight between the two cavities, will arrive at the second cavity at time t. Using first-order perturbation theory we find that the total density matrix of the system at time t is

$$\rho(t)=|a\rangle\langle a|\otimes\rho_F(t-T-\tau)-igT[\sigma^-(t-T-\tau)a_1^\dagger(t-T-\tau)\rho_F(t-T-\tau)-\sigma^+(t-T-\tau)\rho_F(t-T-\tau)a_1(t-T-\tau)]\ . \tag{4.3}$$

For the off-diagonal element of the atomic density matrix $\rho_{ab}^{(\mathrm{at})}$ we find

$$\begin{aligned}\rho_{ab}^{(\mathrm{at})}(t)&=\mathrm{Tr}[\rho(t)\sigma^-(t)]\\&=igT\,\mathrm{Tr}_A[\sigma^+(t-T-\tau)\sigma^-(t)]\\&\quad\times\mathrm{Tr}_F[a_1(t-T-\tau)\rho_F(t-T-\tau)]\\&=igTe^{-i\omega(T+\tau)}\langle a_1(t-T-\tau)\rangle\ .\end{aligned} \tag{4.4}$$

Here the subscripts on the traces indicate whether they are over atomic or field states, and in evaluating the trace over the atomic variables we have used the fact in the interaction picture

$$\sigma^+(t)=e^{i\omega t}\sigma^+(0),\quad \sigma^-(t)=e^{-i\omega t}\sigma^-(0)\ . \tag{4.5}$$

We can now use the relation

$$\langle a_1(t)\rangle=e^{-i\omega(T+\tau)}\langle a_1(t-T-\tau)\rangle+O(g) \tag{4.6}$$

to give

$$\rho_{ab}^{\rm at}(t)=igT\langle a_1(t)\rangle+O(g^2)\ . \tag{4.7}$$

This result can now be substituted into the field amplitude equation (4.2). We first note that

$$s=ir(gT)^2\langle a_1(t)\rangle+O(g^3)=iA\langle a_1(t)\rangle+O(g^3)\ , \tag{4.8}$$

which, dropping terms of greater than second order in g, gives us for Eq. (4.2)

$$d\langle a_2(t)\rangle dt=-i\omega\langle a_2(t)\rangle-[(A-\gamma)/2]\langle a_2(t)\rangle + A\langle a_1(t)\rangle\ . \tag{4.9}$$

This is just the second of Eqs. (2.4). We see, then, that the origin of the extra term in the equation of motion for the field amplitude in the second cavity is a result of the atomic coherence, which is induced in the pump beam by the field in the first cavity.

This fact is the explanation for the unusual time dependence of the field amplitude in the second cavity. As the field in the first cavity grows the amount of atomic coherence injected into the second cavity increases. This leads to the rapid $(At)e^{(A-\gamma)t/2}$ increase of the field in the second cavity.

Now let us examine the coherence between the fields in the two cavities. One might think that the coherence would be produced, as in the preceding discussion, by the atomic coherence that the field in the first maser induces in the pump beam. This is, however, in general not true. One can see this by considering the case in which $\langle a_1(0)\rangle=0$. The first of Eqs. (2.4) implies that $\langle a_1(t)\rangle=0$, and Eq. (4.7) then gives us that $\rho_{ab}^{(\rm at)}(t)=0$. This means that there is no atomic coherence in the beam. On the other hand, from Eq. (3.3) we see that $n_{12}(t)\neq 0$ so that there is coherence between the two fields. Therefore, a different explanation of the field coherence is necessary.

It is possible from a simple example to see how the passage of an atom through the cavities creates coherence between the fields. Let us suppose that both cavities are in number states with the first cavity containing n_1 photons and the second n_2 photons. This implies that initially the fields are incoherent and that $\langle a_1a_2^\dagger\rangle=0$. At $t=0$ we inject an atom in its upper state $|a\rangle$ into the first cavity. The wave function of the system at $t=0$ is

$$\Psi=|a,n_1,n_2\rangle\ . \tag{4.10}$$

After the atom has passed through the first cavity it has either emitted a photon into cavity one, or it has not. Let the amplitude for the first event be c_b and that for the second be c_a. The state of the system when the atom is between the cavities is then

$$\Psi=c_a|a,n_1,n_2\rangle+c_b|b,n_1+1,n_2\rangle\ . \tag{4.11}$$

The atom now passes through cavity two. We can think of each of the terms in Eq. (4.11) as evolving independently. If the atom enters the second cavity in its upper state, then it can either emit a photon into cavity two or not. Let the amplitude for these events be d_{ab} and d_{aa}, respectively. Similarly, if it enters in its lower state, then it can either absorb a photon or not. The amplitudes in this case we shall call d_{ba} and d_{bb}, respectively. Therefore, the state of the system after the atom has passed through both cavities is

$$\Psi=c_a(d_{aa}|a,n_1,n_2\rangle+d_{ab}|b,n_1,n_2+1\rangle) + c_b(d_{ba}|a,n_1+1,n_2-1\rangle+d_{bb}|b,n_1+1,n_2\rangle)\ . \tag{4.12}$$

If we now calculate $\langle a_1a_2^\dagger\rangle$ we find

$$\langle a_1a_2^\dagger\rangle=(c_ad_{aa})^*(c_bd_{ba})\sqrt{n_1+1}\sqrt{n_2} + (c_ad_{ab})^*(c_bd_{bb})\sqrt{n_1+1}\sqrt{n_2+1}\ . \tag{4.13}$$

The passage of a single atom has led to coherence between the fields.

This coherence can be thought of as arising from the interference of two atomic paths. Suppose that we consider only the situation where the atom emerges from the second cavity in its excited state, i.e., we measure the atom at the output of the second cavity and only perform a measurement on the fields if the atom has come out in its upper state. Then only the first term in Eq. (4.13) will be present, but we still have $\langle a_1a_2^\dagger\rangle\neq 0$. In this case the atom starts in the upper state; between the cavities it has a nonzero amplitude to be in either state, and it leaves the second cavity in its upper state. We can think of this as two possible paths for the atom, one in which it is in its upper state between the cavities and the other in which it is in its lower state there. Therefore, the atom can follow two different paths to reach the same final state. The amplitudes for these two paths interfere leading to the nonzero value of $\langle a_1a_2{}^\dagger\rangle$.

Note that if we measured the atom between the cavities we would know whether the atom was in its upper or lower state there, and we would know which path the atom followed. This means that after the measurement we would have either $c_a=0$ or $c_b=0$. A quick glance at Eq. (4.13) shows that this would result in $\langle a_1a_2^\dagger\rangle=0$, i.e., the coherence would be destroyed. Therefore, as expected, a measurement of which path the atom takes eliminates the interference effect.

V. PHASE DIFFUSION

Using standard techniques, we can derive from the master equation (2.3) a Fokker-Planck equation for the P representation of the two-mode field $P(\alpha_1,\alpha_2,t)$ [25]. We find that

$$\frac{\partial P}{\partial t}=\sum_{j=1}^{2}\left[A\frac{\partial^2 P}{\partial\alpha_j^*\partial\alpha_j}-\tfrac{1}{2}(A-\gamma)\frac{\partial}{\partial\alpha_j}(\alpha_jP) -\tfrac{1}{2}(A-\gamma)\frac{\partial}{\partial\alpha_j^*}(\alpha_j^*P)\right] + A\left[\frac{\partial^2 P}{\partial\alpha_1^*\partial\alpha_2}+\frac{\partial^2 P}{\partial\alpha_1\partial\alpha_2^*}-\alpha_1\frac{\partial P}{\partial\alpha_2}-\alpha_1^*\frac{\partial P}{\partial\alpha_2^*}\right]\ . \tag{5.1}$$

If we set $\alpha_1=\rho_1 e^{i\theta_1}$ and $\alpha_2=\rho_2 e^{i\theta_2}$ we can rewrite Eq. (5.1) in polar form as an equation for the P representation considered as a function of ρ_1, ρ_2, θ_1, and θ_2. The discussion in Sec. II shows that the quantity of primary interest is the difference between the phases of the two fields. Therefore, we define the variables

$$\mu=(\theta_1+\theta_2)/2, \quad \theta=\theta_1-\theta_2 , \tag{5.2}$$

and find the Fokker-Planck equation for P considered as a function of ρ_1, ρ_2, μ, and θ. The coefficient of the $\partial^2 P/\partial^2\theta$ term $D(\theta)$ is the diffusion constant for the relative phase of the fields θ and is given by

$$D(\theta)=(A/4)[(1/\rho_1)^2+(1/\rho_2)^2-(2/\rho_1\rho_2)\cos\theta] . \tag{5.3}$$

If $\rho_1=\rho_2$ and $\theta=0$, we see that $D(\theta)=0$. In fact, under these conditions the angular drift and diffusion terms of the Fokker-Planck equation become

$$\frac{A}{4\rho_2^2}\frac{\partial^2 P}{\partial\mu^2}-A\sin\theta\left[\frac{1}{2}\frac{\partial P}{\partial\mu}-\frac{\partial P}{\partial\theta}\right] . \tag{5.4}$$

Thus, the diffusion matrix for the angular variables is diagonal and the element corresponding to θ, $D(\theta)$, is zero. This means that there is no noise in the relative phase of the fields in the two cavities. This kind of behavior is characteristic of correlated-emission lasers [22–24]. In order to determine how close one can come to achieving these conditions in the oscillator rather than the amplifier configuration, one needs to develop the nonlinear theory for this system. We do know, however, from Eq. (2.7) or Eq. (5.4) that θ will lock to zero. What we cannot determine from the linear theory is the ratio ρ_1/ρ_2 at the operating point.

VI. ATOMIC VELOCITY SPREAD

Finally, let us discuss the effect that a velocity spread in the atomic pump beam has on the performance of the masers. The question which arises is whether the finite spread destroys the coherence between the fields in the two cavities. In order to provide an answer we note that the atomic velocity enters the description of the system via three parameters. We consider each of these in turn.

First, the interaction time T depends on the velocity of the atoms. An examination of the expressions in the previous sections shows that the effect of a velocity spread is to replace T by an average value in the expression for the gain A. In the Appendix we consider this issue from the point of view of injected atomic coherence using the model in Ref. [26]. There we find that if the field in the first cavity is strong, so that the interaction time is of the order of the inverse Rabi frequency, then the velocity spread does lead to a mild suppression of the coherence between the fields. In the two-maser model we have assumed that the Rabi frequencies in both cavities are much smaller than the reciprocal of the interaction time, so that in this regime a velocity spread has little effect.

The second parameter that is affected is the transit time of the atoms between the two cavities. This parameter does not appear in any of the expressions describing the coupled maser system on resonance. This is also the case if the first cavity contains a strong classical field that generates atomic coherence in the beam [27]. This is discussed in the Appendix. The reason that this quantity does not appear is that on resonance the free evolution of the phase of the field and the phase of the atomic coherence are the same. Thus, no matter how long the atom takes to go between the cavities it arrives at the second cavity in phase with the field there.

The third place where the velocity enters is in the resonance frequency. Atoms moving with different velocities will have different resonance frequencies due to the Doppler shift. A full consideration of this issue requires an analysis of the situation in which the atom and the field modes are detuned, and inhomogeneous broadening is taken into account. A similar analysis, however, has been carried out in the case of a more conventional correlated-emission laser (CEL), and it has been shown that only the atoms whose resonant frequencies are close to that of the cavity interact significantly with the light [28]. The physical reason is obvious; the gain is strongly reduced for atoms with larger detunings. Thus, only the resonant velocity group is selected, and the CEL effect (vanishing phase diffusion constant) remains. By analogy, we anticipate that the situation with the two-maser system is similar. Only the resonant velocity group contributes significantly to the gain, and, from the discussion in the Appendix, it follows that for this group the atomic coherence is in phase with the field upon entering the second cavity. The full pump beam can be utilized if $T\Delta<1$ (Δ being the inhomogeneous width). Otherwise, only that portion for which the detuning times T is less than 1 will contribute to the gain. A complete analysis of the two-maser problem including nonlinear operation as well as inhomogeneous broadening will be presented in a future publication.

VII. CONCLUSION

We have seen that the fields of two masers pumped by a common beam of excited atoms will be coherent. The first maser induces atomic coherence in the beam passing through it. As a result the atoms carry information about the field in the first cavity. This information is then transferred to the field in the second cavity, thereby producing coherence between the fields. In particular, the relative phase of the two fields locks to zero and can, under the proper conditions, have a diffusion constant that is zero. This makes the pump-coupled maser system an example of a correlated-emission laser. If the effect of atomic decay between the cavities is taken into account this conclusion is modified slightly. If the lifetimes of the masing levels are large in comparison to the time that the atom spends between the cavities, then at equal intensities the diffusion coefficient will not vanish but can be made very small.

We also found that the field in the second cavity grows faster than that in the first. This is caused by the fact that as the field in the first cavity grows, the amount of atomic coherence injected into the second cavity grows as well. This leads to a greater than exponential increase with time of the photon number in the second cavity.

It should be noted that in the case of micromasers, due to the lack of photocounters in the microwave regime and the fact that the fields are weak, it is impossible to directly measure the field in the cavity. The state of the field is inferred from the state of appropriately prepared probe atoms as they exit the cavity, as measured by state-selective field ionization [29–32]. In the case of the two-cavity scheme being considered here, the coherence of the two fields can be deduced from a scheme involving several probe atoms. For example, let $p_1(p_2)$ denote the probability that probe atoms initially prepared in their upper state and sent through the first (second) cavity are detected in their lower state upon exiting that cavity, and p_{12} be the probability that an atom injected into the first cavity in its upper state emerges in its lower state after having passed through both cavities. A short calculation using second-order perturbation theory shows that the quantity $p_{12}-p_1-p_2$ is equal to $(gT)^2 n_{12}$, where all cavity transit times have been taken to have the same value T. Therefore, by using a three-probe-atom scheme one can find n_{12}.

Previous schemes for coupling two lasers have involved taking a part of the output light from one laser and injecting it into the second [33]. This produces coherence between the output of the two lasers. What has been shown here is that coupling via a common pump can produce similar results.

Finally, we note that a finite velocity spread in the pump beam should not present a problem in observing the effects discussed above. As a result, it should be possible to realize this system in the laboratory.

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APPENDIX

Here we would like to discuss two aspects of the master equation for a maser with injected atomic coherence. First, the definition of the parameter s appearing in Eq. (4.1) is slightly different from that given in Ref. [26]. We shall derive the definition we use. The second point concerns the sensitivity of a maser with injected atomic coherence to the velocity spread of the atomic beam. In particular, if the atomic coherence is generated by a strong classical field in the first cavity, then, on resonance and to the level of approximation used in this paper, the dependence of the behavior of the maser on the velocity spread is relatively weak.

The atomic coherence produces the last term in Eq. (4.1). It is derived by considering, to first order in g, the effect of an atom with density matrix $\rho^{(\mathrm{at})}(t)$ entering the maser cavity at time t and interacting with the field for a time T. The other terms that appear in the master equation are standard. Thus, the change in the maser field produced by the atomic coherence is

$$\begin{aligned}\delta\rho_F(t)_{\mathrm{coh}} &= -i\int_t^{t+T} dt'[H_I(t'),\rho(t)] \\ &= -iT\,\mathrm{Tr}_A\{[H_I(t),\rho^{(\mathrm{at})}(t)\otimes\rho_F(t)]\} \\ &= -igT[\{\mathrm{Tr}_A[\rho^{(\mathrm{at})}(t)\sigma^-(t)]a^\dagger(t) \\ &\quad + \mathrm{Tr}_A[\rho^{(\mathrm{at})}(t)\sigma^+(t)]a(t)\},\rho_F(t)] \;. \end{aligned} \tag{A1}$$

Here we have used the fact that H_I is time independent and have chosen to evaluate it at t. The contribution of the atomic coherence to the coarse-grained time derivative of ρ_F is given by multiplying $\delta\rho_F(t)_{\mathrm{coh}}$ by the atomic injection rate r. Doing so immediately gives the last term of Eq. (4.1) with the definition of s quoted after Eq. (4.1).

Let us now consider the case in which the first cavity contains a strong field that can be described classically. The interaction of the atom with this field can be described by the interaction picture Hamiltonian

$$H_{I1}(t)=g'E_0[e^{i\omega t}\sigma^-(t)+e^{-i\omega t}\sigma^+(t)] \;, \tag{A2}$$

where resonance between the atom and the field has been assumed. Here E_0 is the amplitude of the field and g' is one-half the transition dipole matrix element. The Hamiltonian $H_{I1}(t)$ is, in fact, time independent because the time dependences of the operators are cancelled by the time-dependent exponential factors.

Let us inject an atom into the first cavity at $t-\tau-T$. This means that it will arrive at the second cavity at time t. Our object is to find $s=grT\rho_{\mathrm{ab}}^{(\mathrm{at})}(t)$. The atom is injected in its upper state so that at time $t-\tau-T$ we have

$$\rho^{(\mathrm{at})}(t-\tau-T)=|a\rangle\langle a| \;. \tag{A3}$$

Upon leaving the first cavity at time $t-\tau$ the atomic density matrix is

$$\rho^{(\mathrm{at})}(t-\tau)=\exp(-iTH_{I1})\rho^{(\mathrm{at})}(t-\tau-T)\exp(iTH_{I1}), \tag{A4}$$

from which we find

$$\begin{aligned}\rho_{ab}^{(\mathrm{at})}(t-\tau) &= \mathrm{Tr}_A[\rho^{(\mathrm{at})}(t-\tau)\sigma^-(t-\tau)] \\ &= ie^{-i\omega(t-\tau)}\sin(g'E_0T)\cos(g'E_0T) \;. \end{aligned} \tag{A5}$$

The atom now propagates freely reaching the second cavity at time t. Because there is no interaction during this period we have $\rho^{(\mathrm{at})}(t)=\rho^{(\mathrm{at})}(t-\tau)$. On the other hand,

$$\sigma^-(t)=e^{-i\omega\tau}\sigma^-(t-\tau) \;, \tag{A6}$$

so that

$$\begin{aligned}\rho_{ab}^{(\mathrm{at})}(t) &= e^{-i\omega\tau}\rho_{ab}^{(\mathrm{at})}(t-\tau) \\ &= ie^{-i\omega t}\sin(g'E_0T)\cos(g'E_0T) \;. \end{aligned} \tag{A7}$$

For s we then have

$$s(t)=igrTe^{-i\omega t}\sin(g'E_0T)\cos(g'E_0T) \;. \tag{A8}$$

Note that the combinations $s(t)a^\dagger(t)$ and $s(t)^*a(t)$ that appear in the master equation are time independent. The time dependence of s cancels the time dependence of the interaction-picture operator $a(t)$.

The only dependence on the atomic velocity in Eq. (A8) is through the interaction time T. If the atomic beam has a velocity spread, then the expression for $s(t)$ must be averaged over the values of T corresponding to the range of atomic velocities. In particular consider an atomic beam with an average velocity $\overline{v}$ and velocities between $\overline{v}-\Delta v/2$ and $\overline{v}+\Delta v/2$, where we assume that $\Delta v/\overline{v} \ll 1$. If an atom with velocity $\overline{v}$ has an interaction time of $\overline{T}$, then the interaction times lie in the range between $\overline{T}-\Delta T/2$ and $\overline{T}+\Delta T/2$, where $\Delta T=(\Delta v/\overline{v})\overline{T}$. If we average s over the interaction times we find that if $\Omega\Delta T \ll 1$, then s is of order $gr\overline{T}$. If $\Omega\Delta T$ is of order one or greater, then s is of order $gr\overline{T}/\Omega\Delta T$. Thus, we see that while the dependence of the atomic coherence term in the master equation is rather weak, the effects of atomic coherence will be greatest if $\Omega\Delta T=(\Delta v/\overline{v})\Omega\overline{T}$ is of order one or less.

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