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Theory of the stopping power of fast multicharged ions

G. L. Yudin
Institute of Physics and Power Engineering, 249020 Obninsk, U.S.S.R.
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The processes of Coulomb excitation and ionization of atoms by a fast charged particle moving along a classical trajectory are studied. The target electrons are described by the Dirac equation, while the field of the incident particle is described by the Lienard-Wiechert potential. The theory is formulated in the form most convenient for investigation of various characteristics of semiclassical atomic collisions. The theory of sudden perturbations, which is valid at high enough velocities for a high projectile charge, is employed to obtain probabilities and cross sections of the Coulomb excitation and ionization of atomic hydrogen by fast multiply charged ions. Based on the semiclassical sudden Born approximation, the ionization cross section and the average electronic energy loss of a fast ion in a single collision with an atom are investigated over a wide specific energy range from 500 keV/amu to 50 MeV/amu.

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I. INTRODUCTION

The interest in processes of Coulomb excitation of atoms originally arose in the study of energy loss of fast electrons, protons, and α particles in a medium. The appropriate quantum theory has been developed by Bethe [1].

The problem of dependence of individual transition probability and energy loss on the impact parameter in the theory of Coulomb excitation by fast protons has become urgent because of the development of new experimental techniques and a deeper analysis of the channeling effect. However, in the theory dealing with multiply charged ions the problem of the probability dependence on the impact parameter is naturally introduced, in contrast to the case of protons, because the correct results are obtained for different trajectories of relative motion by using different theoretical approaches [2–5].

The two major parameters in the theory of Coulomb excitation are the ratio of the electron velocity in the target atom to the velocity of the incident particle. $\xi=\hbar/mav$ and the value of $\eta=Ze^2/\hbar v$, which for small impact parameter determines the rate of the electron interaction with a moving force center of charge Z.

For fast-proton and multiply-charged-ion impact, when $v \gg e^2/\hbar$ and $\eta \lesssim 1$ the whole range of impact parameter of interest $b \gtrsim a_0 (a_0=\hbar^2/me^2)$ can be considered as follows. Electrons in inner shells of the target atom ($\xi \gtrsim 1$) are affected by a weak external influence because the η value typical for these electrons in the dipole area $b \gtrsim a_0 \gg a$ is about $\eta_{\text{eff}} \approx \eta a/b \ll 1$. Therefore the probabilities of the Coulomb excitation involving these electrons are obtained using the ordinary perturbation theory to the lowest order in η. A general approach to the solution of this problem for outer-shell electrons ($a \approx a_0$, $\xi \ll 1$) is given in Refs. [4] and [5]. For small impact parameter ($b \lesssim a_0$) the Coulomb excitation of discrete spectrum states should be described using the theory of sudden perturbation [6], and in the dipole area ($b \gg a_0$) it should be described by the ordinary theory. The joining of the results obtained in these two cases [which can be made more accurate the better the condition $\eta\xi \ll \gamma$ is satisfied, where $\gamma=(1-v^2/c^2)^{-1/2}$ is the Lorentz factor] makes it possible to correctly calculate the excitation cross section also.

Besides the separate studies of the dipole and nondipole areas in the ionization processes one should also distinguish the channels involving detachment of fast ($ka_0 \gg 1$) and slow ($ka_0 \lesssim 1$) weakly bound ($\xi \ll 1$) electrons ($\hbar\mathbf{k}$ is the momentum of the ejected electron). The theory of sudden perturbations can be applied in the area $b \lesssim a_0$, up to the values of k corresponding to the condition $\hbar k \ll mv$. However, if $\hbar k \gtrsim mv$ the situation simplifies otherwise because one can then use a semisudden method (for details see Ref. [5]).

II. THE ORDINARY PERTURBATION THEORY

The most appropriate formulation of the semiclassical theory of Coulomb excitation of an atom in a relativistic collision is as follows [7]. Atomic electrons are described by the Dirac equation and the external field produced by the charged projectile is described by the Lienard-Wiechert potential (Φ, $\mathbf{A}$). In the first order in η the amplitude of the inelastic transition between the stationary states $|i\rangle$ and $|f\rangle$ of the unperturbed Hamiltonian is given by

$$\mathbf{M}_{fi}^{B}=\frac{ie}{\hbar}\int_{-\infty}^{\infty} dt\, e^{i\Omega t}\langle f|\hat{V}(t)|i\rangle ,$$

$$\hat{V}(t)=\Phi(1-\hat{\alpha}\cdot\mathbf{v}/c) \tag{1}$$

where $\Omega=\omega_f-\omega_i$, $\hat{\alpha}=\hat{\gamma}^0\hat{\gamma}$, the $\hat{\gamma}^\mu$ being the Dirac matrices. In the case of a straight-line trajectory, $\mathbf{R}(t)=\mathbf{b}+\mathbf{v}t$, one obtains for the excitation amplitude and the cross section after expanding $\hat{V}(t)$ in a Fourier integral in the coordinates the expressions [7]

$$\mathbf{M}_{fi}^{B}(\mathbf{b})=\frac{i\eta}{\pi}\int\frac{1}{q^2-(\Omega/c)^2}\delta\left[\frac{\mathbf{q}\cdot\mathbf{v}-\Omega}{v}\right]\times e^{-i\mathbf{q}\cdot\mathbf{b}}\mu_{fi}(\mathbf{q})d^3q\ , \tag{2}$$

$$\begin{aligned}\sigma_{fi}^{B}&=\int d^2b\,|\mathbf{M}_{fi}^{B}(\mathbf{b})|^2\\&=4\eta^2\int\frac{1}{[q^2-(\Omega/c)^2]^2}\delta\left[\frac{\mathbf{q}\cdot\mathbf{v}-\Omega}{v}\right]\times|\mu_{fi}(\mathbf{q})|^2d^3q\ .\end{aligned} \tag{3}$$

In the formulas (2) and (3)

$$\begin{aligned}\mu_{fi}(\mathbf{q})&=\langle f|e^{i\mathbf{q}\cdot\mathbf{r}}(1-\hat{\alpha}\cdot\mathbf{v}/c)|i\rangle\\&=\frac{c}{\Omega}\mathbf{Q}\cdot\langle f|\hat{\alpha}e^{i\mathbf{q}\cdot\mathbf{r}}|i\rangle\ .\end{aligned} \tag{4}$$

The vector $\mathbf{Q}=\mathbf{q}-\Omega\mathbf{v}/c^2$ differs from $\mathbf{q}$ because of the presence of the magnetic interaction term in the operator $\hat{V}(t)$. The transverse components of the vectors $\mathbf{Q}$ and $\mathbf{q}$ (in the plane of the impact parameter) coincide, but on the longitudinal direction (along $\mathbf{v}$) $Q_{\parallel}=q_{\parallel}/\gamma^2$.

The expression for $\mu_{fi}(\mathbf{q})$ is significantly simplified in the most-important particular cases. For nonrelativistic velocities and any αZ_a ($\alpha=e^2/\hbar c$ is the fine-structure constant, $Z_a=a_0/a$) one obtains

$$\mu_{fi}(\mathbf{q})\approx M_{fi}(\mathbf{q})\equiv\langle f|e^{i\mathbf{q}\cdot\mathbf{r}}|i\rangle\ . \tag{5}$$

For $\Delta\equiv\Omega a/v\ll\gamma$ the dominant contribution to the amplitude and the cross section is made by $qa\lesssim 1$, for which [7]

$$\mu_{fi}(\mathbf{q})\approx\begin{cases}ia\mathbf{Q}\mathcal{M}_{fi}\ , & qa\ll 1\\ M_{fi}(\mathbf{q})\ , & qa\approx 1\end{cases}\approx M_{fi}(\mathbf{Q})\equiv\langle f|e^{i\mathbf{Q}\cdot\mathbf{r}}|i\rangle\ , \tag{6}$$

$$\mathcal{M}_{fi}\equiv\langle f|\mathbf{r}/\mathrm{a}|\mathrm{i}\rangle\ . \tag{7}$$

The relativistic effects resulting from the closeness of v to c do not reveal themselves for light targets (i.e., for $\xi\ll 1$) in close ($b\lesssim a$) collisions, while for large impact parameters the magnetic interaction of the electron with the projectile almost completely cancels the increase of the role of the retardation in the Coulomb interaction. The effective strength of the Coulomb interaction increases rapidly with increasing γ, a result of the relativistic contraction of the characteristic field range that results in the broadening of the impact parameter range where the excitation of the atom occurs nonadiabatically. But the decreasing of the total interaction caused by the presence of the magnetic term leads to a situation in which the excitation cross section depends weakly (logarithmically) on the Lorentz factor.

The theoretical investigations of fast-ion stopping are basically devoted to the processes involving bare nuclei as projectiles. The empirical methods of introducing the effective ion charge depending on the collision velocity are also used. The comprehensive accounting of electron screening is usually made only for the lightest atoms, though multielectron systems are of the greatest interest. A successful model of projectile bare-nucleus screening has been used [8] for classical calculations of the stopping power in a polarized medium. Following [8], I consider a projectile nucleus of charge Z, with the inherent N electrons distributed around the nucleus with density

$$\rho(r)=\frac{N}{4\pi\lambda^3}\frac{\lambda}{r}\exp(-r/\lambda)\ , \tag{8}$$

where λ is the screening parameter, equal to

$$\lambda\approx 0.48\frac{(N/Z)^{2/3}}{1-N/7Z}\frac{a_0}{Z^{1/3}}\ . \tag{9}$$

For a straight-line trajectory the excitation amplitude is calculated by direct summation of the amplitudes of the excitation by point charge particles (5) with distribution (8) and (9):

$$\mathbf{M}_{fi}^{B}(\mathbf{b})=\frac{i}{\pi}\left[\frac{e^2}{\hbar v}\right]\int\frac{d^3q}{q^2-(\Omega/c)^2}Z_q\delta\left[\frac{\mathbf{q}\cdot\mathbf{v}-\Omega}{v}\right]\times e^{-i\mathbf{q}\cdot\mathbf{b}}\mu_{fi}(\mathbf{q})\ . \tag{10}$$

All the symbols here have the same meaning, as above, but effective charge Z_q has to depend on momentum transfer:

$$Z_q=Z-\frac{N}{1+q^2\lambda^2}\ . \tag{11}$$

The corresponding excitation cross section is

$$\sigma_{fi}^{B}=4\left[\frac{e^2}{\hbar v}\right]^2\int\frac{d^3q}{[q^2-(\Omega/c)^2]^2}Z_q^2\delta\left[\frac{\mathbf{q}\cdot\mathbf{v}-\Omega}{v}\right]\times|\mu_{fi}(\mathbf{q})|^2\ . \tag{12}$$

In the dipole area $b\gg a+\lambda$ for small momentum transfer the projectile ion is completely screened by inherent electrons, $Z_q\approx Z-N$. The more complex case takes place for large momentum transfer. The features of screening effects in the cross sections of bound-bound and bound-continuum transitions are discussed in detail in Ref. [9].

In the dipole area for a small momentum transfer the total excitation-plus-ionization probability

$$W_{\mathrm{inel}}^{B}(\mathbf{b})\approx\frac{4(Z-N)^2}{v^2\gamma^4}\left[\frac{e^2}{\hbar v}\right]^2\times\sum_{\substack{f,i\\f\neq i}}\Omega^2|\langle f|r|i\rangle|^2\times\left[\gamma^2\mathcal{K}_1^2\left[\frac{\Delta b}{\gamma a}\right]+\mathcal{K}_0^2\left[\frac{\Delta b}{\gamma a}\right]\right] \tag{13}$$

one can substitute by approximate expression using the average frequency method

$$\begin{aligned}&\hbar\Omega\to\hbar\overline{\Omega}\equiv\epsilon(\hbar^2/ma^2)\ ,\\&W_{\mathrm{inel}}^{B}(\mathbf{b})\to\overline{W}_{\mathrm{inel}}^{B}(\mathbf{b})\ ,\end{aligned} \tag{14}$$

where $\epsilon\approx 1$ is the parameter to be determined. For the atomic targets with zero dipole momentum in the ground state

$$\overline{W}^B_{\rm inel}(\mathbf{b})=\tfrac{4}{3}(Z-N)^2\left[\frac{\epsilon\xi e^2}{\gamma^2\hbar v}\right]^2\langle i|r^2/a^2|i\rangle \times[\gamma^2\mathcal{K}_1^2(\beta\epsilon)+\mathcal{K}_0^2(\beta\epsilon)] \quad (15)$$

where $\beta=b\xi/a\gamma$. Since the Bessel function $\mathcal{K}_n(z)$ for $z\gg1$ decreases exponentially, the transition amplitudes have nonvanishing values up to $\omega_f^{\max}\approx v\gamma/b$. Due to the condition $b\gg a$ for $\hbar v\gamma/b\ll mc^2$, the nonrelativistic approximation for the final state wave function is correct for practically every Ω. The comparison of the two calculation methods shows that for fixed Lorentz factor γ the value ϵ in (14) is a monotonic function of the parameter β only (see Ref. [9] for details), which mainly takes values from 0.4 to 0.5. In Ref. [10] the average frequency method has been extended over the whole impact parameter range. The average value $\bar{\epsilon}$ was found by comparison of the total nonrelativistic cross section of inelastic channels

$$\sigma^B_{\rm inel}=4\eta^2\int\frac{1}{q^4}\sum_{\substack{f,i\\ f\neq i}}\delta\left[\frac{\mathbf{q}\cdot\mathbf{v}-\Omega}{v}\right]|\langle f|e^{i\mathbf{q}\cdot\mathbf{r}}|i\rangle|^2d^3q\ , \quad (16)$$

with the corresponding cross section calculated in the mean frequency approximation

$$\bar{\sigma}^B_{\rm inel}=4\eta^2\int\frac{1}{q^4}\delta\left[\frac{\mathbf{q}\cdot\mathbf{v}-\Omega}{v}\right](1-|\langle i|e^{i\mathbf{q}\cdot\mathbf{r}}|i\rangle|^2)d^3q\ . \quad (17)$$

It has been found that $\bar{\epsilon}\approx0.465$.

Note that for the straight-line trajectories and bare-nucleus projectile Fourier transform of the Lienard-Wiechert potential used in amplitude (1) can be rewritten in the following form [10]:

$$\frac{i}{\hbar}\hat{V}_\Omega=\frac{1}{\hbar}\int_{-\infty}^{\infty}dt\,e^{i\Omega t}\hat{V}(t) =-2i\eta(1-\hat{\alpha}\cdot\mathbf{v}/c)\left[\exp\left[\frac{i\Omega\mathbf{v}\cdot\mathbf{r}}{v^2}\right]\mathcal{K}_0\left[\frac{\Omega}{v\gamma}|\mathbf{b}-\mathbf{s}|\right]-\mathcal{K}_0\left[\frac{\Omega b}{v\gamma}\right]\right]. \quad (18)$$

The vector $\mathbf{s}$ is the projection of the electron radius-vector $\mathbf{r}$ on the impact parameter plane.

III. THE THEORY OF SUDDEN PERTURBATIONS

In nondipole area $b\lesssim a$ effective interaction is determined by η, which for large Z is about unity up to a high projectile velocity. Here one can use the theory of sudden perturbations [6], which is correct if $\omega\tau\approx\xi$ is small and $V\tau\lesssim\hbar$. The transition amplitude in zeroth order in $\omega\tau$ is

$$\mathbf{M}_{fi}(\mathbf{b})\approx\mathbf{M}^S_{fi}(\mathbf{b})=\left\langle f\left|\exp\left[-\frac{i}{\hbar}\int_{-\infty}^{\infty}dt\ \hat{V}(t)\right]\right|i\right\rangle\ . \quad (19)$$

It is expressed by the Fourier transform (18) at zero frequency:

$$\mathbf{M}^S_{fi}(\mathbf{b})=\langle f||(\mathbf{b}-\mathbf{s})/b|^{-2i\eta}|i\rangle\ . \quad (20)$$

For example, the Coulomb-ionization differential probability for a hydrogenlike atom

$$\frac{d^2W^S(\mathbf{b})}{d(ka)d\Omega}=\frac{(ka)^2}{(2\pi)^3}|\mathbf{M}(\mathbf{k})|^2 \quad (21)$$

is calculated by use of the amplitude

$$\mathbf{M}(\mathbf{k})=a^{-3/2}\int d^3r\,\Psi_{\mathbf{k}}^{(-)*}(\mathbf{r})|(\mathbf{b}-\mathbf{s})/b|^{-2i\eta}\Psi_{1S}(\mathbf{r})\ , \quad (22)$$

where the final state wave function is normalized to unity volume. It will be shown further that the arbitrary transition amplitude asymptote for $b\ll a$ (which has been obtained in Refs. [4] and [5]) are of practical interest. The shaking probabilities do not exceed unity even for $\eta\approx1$, that results from the unitarity of the time evolution operator used in the theory of sudden perturbations.

In the general case beyond the first order of ordinary perturbation theory, instead of summing up the transition amplitudes (similar to the accounting for the screening effects, see Sec. II) the summation should be carried out in the interaction operator included in the total Hamiltonian of the system. In the lowest order of the theory of sudden perturbations [9],

$$\mathbf{M}^S_{fi}(\mathbf{b})=\langle f|\exp[i\chi_0(N/Z)]|i\rangle\ , \quad (23)$$

where

$$\chi_0(z)=-2\eta(1-\hat{\alpha}\cdot\mathbf{v}/c)\times\left\{(1-z)\ln|(\mathbf{b}-\mathbf{s})/b|+z\left[\mathcal{K}_0\left[\frac{b}{\lambda}\right]-\mathcal{K}_0\left[\frac{|\mathbf{b}-\mathbf{s}|}{\lambda}\right]\right]\right\}. \quad (24)$$

In the important particular case of the target excitation by nonrelativistic neutral-atom impact

$$\chi_0(1)=-\eta\left[\mathcal{K}_0\left[\frac{b}{\lambda}\right]-\mathcal{K}_0\left[\frac{|\mathbf{b}-\mathbf{s}|}{\lambda}\right]\right]. \quad (25)$$

For systematic calculations one should mostly pay attention to the calculation of asymptote

$$w^S_{fi}(\mathbf{b})=|\mathbf{M}^S_{fi}(b\ll a)|^2\ .$$

Due to the slow decrease of the Coulomb potential at large distances, even for $b\lesssim a$ it would be correct to use the Born approximation at the farthest parts of projectile trajectory instead of the sudden perturbation theory. To consider this fact one should return to the general problem of sudden-perturbation-theory series construction, the formulation of which is explained in Ref. [6]. Let us project a differential equation for the time evolution operator in the interaction representation $\hat{S}(t,t')$ on the given initial and final states of the unperturbed system and sum over the complete set of unperturbed Hamiltonian functions $|s\rangle$:

$$\left\langle f \left| i\hbar \frac{\partial \hat{S}(t,t')}{\partial t} \right| i \right\rangle = \sum_s e^{i(E_f-E_s)t/\hbar} \langle f|\hat{V}(t)|s\rangle \times \langle s|\hat{S}(t,t')|i\rangle \ . \tag{26}$$

In the problem of Coulomb excitation of atoms a simplified solution of (26) is physically realizable though not strictly demonstrable. In this solution the difference $E_f - E_s$ is replaced with some mean value $E_f - E_s \to \hbar\bar{\Omega} \approx \hbar\Omega_{fi}$, independently of the intermediate states and determined only by the energies of initial and final states E_i and E_f [11]. The joining of the results for different parts of the trajectory can be successfully fulfilled by exponential operator [11–14]

$$W^e_{fi}(\mathbf{b}) = \left| \left\langle f \left| \exp\left[-\frac{i}{\hbar} \int_{-\infty}^{\infty} dt \begin{bmatrix} \cos\Omega t \\ \sin\Omega t \end{bmatrix} \hat{V}(t) \right] \right| i \right\rangle \right|^2 . \tag{27}$$

One should take $\cos\Omega t$ in Eq. (27) for even $l_i + m_i + l_f + m_f$ and $\sin\Omega t$ otherwise, where l_i and l_f are orbitals and m_i and m_f are magnetic quantum numbers of initial and final eigenstates, respectively.

For the appropriate asymptote

$$w^e_{fi}(\mathbf{b}) = W^e_{fi}(b \ll a)$$

in the case of screened projectile (27) yields

$$w^e_{fi}(\mathbf{b}) = |\langle f|(1+\Delta_0)\exp(i\gamma_0)|i\rangle|^2 \ , \tag{28}$$

$$\gamma_0 = \frac{2e^2}{\hbar v} \begin{bmatrix} \cos(\Omega \mathbf{r}\cdot\mathbf{v}/v^2) \\ \sin(\Omega \mathbf{r}\cdot\mathbf{v}/v^2) \end{bmatrix} \times \left[(Z-N)\mathcal{K}_0 \left[\frac{\Omega s}{v} \right] + N\mathcal{K}_0 \left[\frac{\Omega s \kappa}{v} \right] \right] , \tag{29}$$

$$\Delta_0 = \frac{2ie^2}{\hbar v} \Omega \frac{\mathbf{b}\cdot\mathbf{s}}{vs} \begin{bmatrix} \cos(\Omega \mathbf{r}\cdot\mathbf{v}/v^2) \\ \sin(\Omega \mathbf{r}\cdot\mathbf{v}/v^2) \end{bmatrix} \times \left[(Z-N)\mathcal{K}_1 \left[\frac{\Omega s}{v} \right] + N\mathcal{K}_1 \left[\frac{\Omega s \kappa}{v} \right] \right] , \tag{30}$$

where

$$\kappa = \frac{v}{\Omega\lambda} [1 + (\Omega\lambda/v)^2]^{1/2} \ .$$

The first term in matrix element (28) is responsible for $S \to S, P^{(0)}$ transitions and the second one for $S \to P^{(\pm 1)}, D^{(\pm 1)}$ transitions.

IV. NORMALIZED PROBABILITIES AND CROSS SECTIONS

The unitary semiclassical theory of Coulomb excitation of atoms by multiply-charged-ion impact is based on joining the inelastic channel probabilities $W^e_{fi}(\mathbf{b})$ calculated for small impact parameters in the framework of the sudden perturbation method with the Born probabilities $W^B_{fi}(\mathbf{b})$, which are relevant in the dipole area up to high projectile charges. The most convenient method of probability joining (also the natural one) is the Born probability renormalization:

$$W_{fi}(\mathbf{b}) = \frac{W^B_{fi}(\mathbf{b})}{1 + g_{fi}(\mathbf{b}) W^B_{fi}(\mathbf{b})} \ , \tag{31}$$

$$g_{fi}(\mathbf{b}) = \frac{1}{w^e_{fi}(\mathbf{b})} - \frac{1}{w^B_{fi}(\mathbf{b})} \ . \tag{32}$$

The renormalized probabilities $W_{fi}(\mathbf{b})$ are equal to those calculated by sudden perturbation theory for $b \ll a$. This boundary condition leads to the expressions for probabilities, which are correct in the whole range of impact parameter. The appropriate cross sections are

$$\sigma_{fi} = \int d^2b \ W_{fi}(\mathbf{b}) \ . \tag{33}$$

Using the exponential asymptotes (28)–(30) in the renormalization method (31)–(33) is of the most interest for calculation of collision processes involving multicharged ions. In the estimation of the Coulomb-excitation cross sections they allow one to advance to the middle energy range, where the "suddenness" condition gradually ceases to hold and one can scarcely expect success.

The probability-joining procedure can be applied not only to partial probabilities of bound-bound transitions but also to the ionization probability. The simplest method is as follows. By a method like (13) and (14) one can find in the exponential approximation (27)

$$\overline{W}^e_{\text{inel}}(\mathbf{b}) = 1 - \left| \left\langle i \left| \exp\left[-\frac{i}{\hbar} \int_{-\infty}^{\infty} dt \cos(\bar{\Omega} t) \hat{V}(t) \right] \right| i \right\rangle \right|^2 , \tag{34}$$

where $\hbar\bar{\Omega} = \bar{\epsilon}\hbar^2 m a_0^2$. The ionization probability $W^e_{\text{ion}}(\mathbf{b})$ can be obtained from $W^e_{\text{inel}}(\mathbf{b})$ by subtraction of the probabilities of inelastic bound-bound channels. One can estimate the probabilities of the transitions to states with $n = 5, 6, 7, \ldots$ from the asymptotes $W_{1S\to n} \sim n^{-3}$. The ionization probability $w^e_{\text{ion}}(\mathbf{b})$ is used for the joining with $W^B_{\text{ion}}(\mathbf{b})$ (see, for example Fig. 1). The methods of calculation of $W^B_{\text{ion}}(\mathbf{b})$ are discussed in Refs. [15]–[17]. Still the probabilities and cross sections can be obtained from Eqs. (2) and (3). It is interesting that the scaling relation for the probabilities $W^B_{\text{ion}}(\mathbf{b})$ occurs [12]. This relation can be presented as follows:

$$W^B_{\text{ion}}(\mathbf{b}) \approx W^B_{\text{ion}}(0) \frac{\exp[-(v_0/v)G(b/a_0)]}{F(b/a_0)} \ , \quad v_0 = e^2/\hbar \ . \tag{35}$$

The functions $F(z)$ and $G(z)$ are displayed in [12].

The probabilities for shaking allowed transitions, the ionization cross sections σ_{ion} and σ^B_{ion} and also the Glauber cross sections σ^G_{ion} [18] are given in [12]. It should be noticed that the inequality $\sigma_{\text{ion}} > \sigma_{\text{ion}_B}$, obtained in [12] for very small η and very high velocities of multicharged ions, has no physical cause. It stands to reason that in this case the shaking forbidden channels of $S \to P^{(\pm 1)}, D^{(\pm 1)}$ transitions should be taken into consideration during the estimation of ionization probability

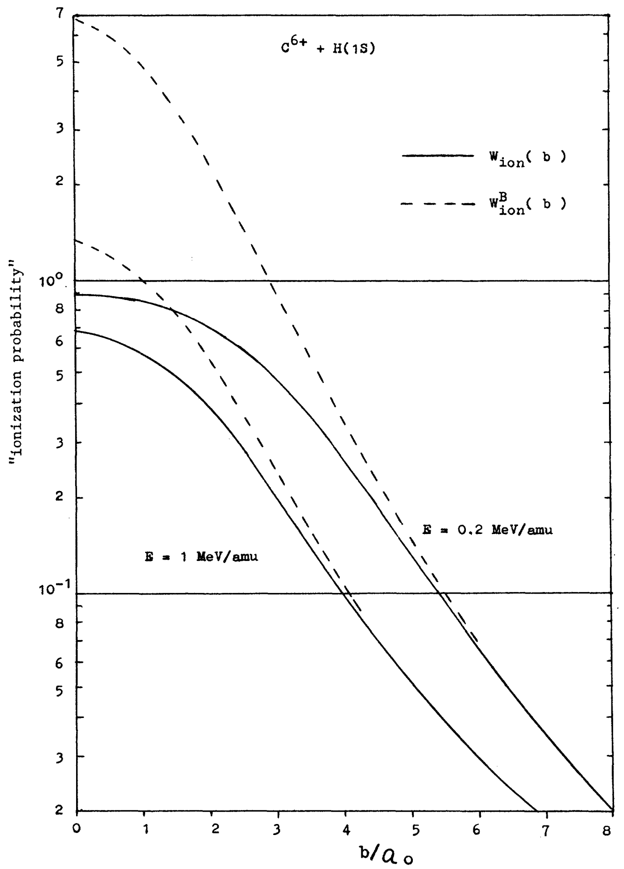

FIG. 1. Born (— — —) and sudden Born (——) ionization probabilities for a multicharged-ion–atom collision.

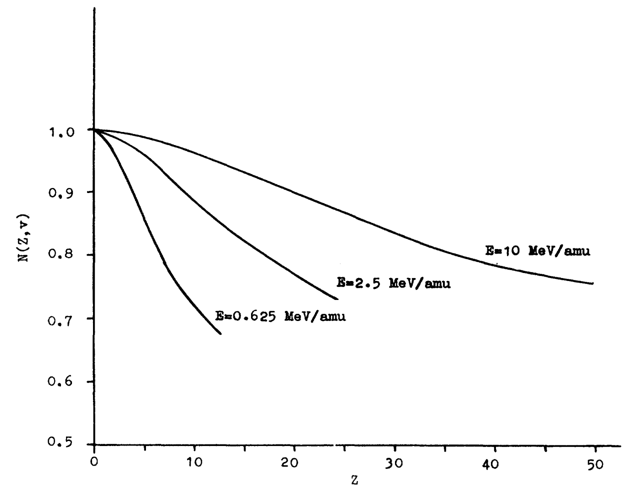

FIG. 2. Renormalization coefficients $N(Z,v)$ in average energy loss for a multicharged-ion-atom collision.

$w^e_{\text{ion}}(\mathbf{b})$. Such an account proves the more exact scaling for the whole nonrelativistic specific energy range:

$$\sigma_{\text{ion}}=f(Z,v)\sigma^B_{\text{ion}}\approx\frac{\sigma^B_{\text{ion}}}{L(\eta)}\ , \tag{36}$$

$$L(\eta)=1+0.076\eta+0.32\eta^2-0.033\eta^3\ . \tag{37}$$

At the projectile energies $E=0.5-50$ MeV/amu and $0\le\eta\le 2.5$ the function $L(\eta)$ reproduces the calculated ionization cross sections with an accuracy to 5%.

V. NORMALIZED ENERGY LOSS

According to Bethe [1], who derived the stopping power in the first-order Born approximation, the energy loss of a charge partical penetrating matter is proportional to the square of its charge. This approximation breaks down over the range $\eta \gtrsim 1$. It is the purpose of this section to investigate the renormalization effects in energy loss due to the unitarity of the exponential operator in Eqs. (27) and (34). For distinguishing this effect it is sufficient to consider the particular case of the nonrelativistic bare-multicharged-ion energy loss.

The average electronic energy loss of a fast multicharged ion in a single collision with an atom is

$$\Delta E(\mathbf{b})=\sum_f \hbar\Omega W_{fi}(\mathbf{b})\ . \tag{38}$$

It can be considered separately for low (l) and high (h) excitations. For low excitation the energy loss $\Delta E_l(\mathbf{b})$ should be calculated by the method as stated above. To avoid the complicated calculations one can solve this problem in the following manner. Dettmann [19] has reported the mean frequency estimation of the energy loss for low excitation $\Delta E_l(\mathbf{b})$ for fast protons. For the collision with a fast multiply charged ion an analogous estimation gives

$$\Delta E_l=\int d^2b\ \Delta E_l(\mathbf{b})\approx\hbar\overline{\Omega}\int d^2b\ \overline{W}^e_{\text{inel}}(\mathbf{b})\ . \tag{39}$$

The energy loss for high excitations,

$$\Delta E_h=\int d^2b\ \Delta E_h(\mathbf{b})\ , \tag{40}$$

may be estimated by the standard method of quantum electrodynamics (see, e.g., [20]):

$$\Delta E_h\approx\frac{2\pi Z^2e^4}{mv^2}\left[\ln\left[\frac{2mv^2\gamma^2}{I}\right]-\frac{v^2}{c^2}\right]\ , \tag{41}$$

where I is of the order of a few times $|\hbar\omega_i|$.

The nonrelativistic average energy loss per collision for the excitation of atomic hydrogen by a fast multicharged fully stripped ion is

$$\Delta E=\Delta E_l+\Delta E_h=N(Z,v)\frac{4\pi Z^2e^4}{mv^2}\ln\left[\frac{2mv^2}{I_0}\right]\ , \tag{42}$$

where $I_0\approx 0.55me^4/\hbar^2$. At the projectile energies $E=0.625$, 2.5, and10 MeV/amu the functions $N(Z,v)$ are displayed in Fig. 2.

In the first order of ordinary perturbation theory (i.e., for small values of η)

$$N(Z,v) \approx 1 \ , \quad \Delta E \approx \Delta E_{\text{Bethe}} \ . \tag{43}$$

It is very important that the scaling relation for the $N(Z,v)$ occurs. For $0 \leq \eta \leq 2.5$ as the projectile energy ranging from $E=0.5$ to 50 MeV/amu this scaling relation may be presented as follows:

$$N(Z,v) \approx Y(\eta) = \frac{\exp[-(v_0/v)(0.2\eta + 0.05\eta^2)]}{1 + 0.013\eta + 0.097\eta^2 - 0.025\eta^3} \tag{44}$$

where $v_0 = e^2/\hbar$. The function $Y(\eta)$ reproduces the average energy loss within an accuracy of 5%.

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