COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Approximate solution of the hydrogenlike atoms in intense laser radiation"

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(Received 19 December 1990)

Rashid [Phys. Rev. A **40**, 4242 (1989)] proposes an approximate solution for the relativistic hydrogen atom in a laser field. The error he quotes is such that the solution becomes *exact* in the nonrelativistic limit. It is shown here to be in error.

PACS number(s): 31.15.+q, 03.65.Ge, 31.20.Wb, 31.30.Jv

The hydrogen atom in an intense electromagnetic field is the central problem in the field of multiphoton physics. Radhid [1] claims to have given an approximation to the relativistic (Dirac equation) solution to this problem with errors of the order of the "Kibble parameter," which is the free-electron quiver energy divided by the rest energy

$$\varepsilon_k = \frac{e^2E^2}{4m\omega^2} \Big/ mc^2 , \tag{1}$$

where all the parameters have their usual meaning. In the nonrelativistic limit ε_k vanishes and then the expression should be exact. In fact, the contention is incorrect and the procedure is just an attempt at a relativistic generalization of the transformation to the Kramers frame which has become so popular lately.

Rashid's procedure is as follows: An operator $\hat{R}$ [Eq. (3.1)], was defined which takes a plane-wave solution of the Dirac equation into a Volkov state (an electron in a plane-wave field),

$$\psi_{\text{Volk}} = \hat{R}\psi_{\text{free}} . \tag{2}$$

It is then used to construct an approximate solution to the Dirac equation with both Coulomb and plane wave field by

$$\psi_A = \hat{R}\psi_{\text{H}} , \tag{3}$$

where ψ_{H} is the solution in the presence of the Coulomb potential alone, i.e., the textbook hydrogen wave function. Operation on ψ_A with the Dirac operator (with both the Coulomb and plane-wave interaction) then supposedly yields terms of order ε_k.

The error involved in the procedure stems from the fact that $\hat{R}$ is a very complex operator depending upon the 4-momentum *operator* and it has been improperly handled. The relativistic factors and the matrix operators in $\hat{R}$ and ψ_{H} clutter the discussion and make it difficult to follow. It is simpler to go directly to the nonrelativistic limit where the error is much more evident. It can also be handled relativistically in an approximate form [2].

In the nonrelativistic (spin-independent) limit we get for a linearly polarized field

$$\hat{R}_{\text{NR}} = \exp\left\{ -i\left[\boldsymbol{\alpha}_0\cdot\hat{\mathbf{P}}\sin\omega t + U_p\left[t + \frac{\sin2\omega t}{2\omega} \right] \right] \right\} , \tag{4}$$

where in the usual notation

$$\boldsymbol{\alpha}_0 = e\mathbf{E}/m\omega^2, \quad U_p = e^2E^2/4m\omega^2 . \tag{5}$$

The nonrelativistic form of (3), when substituted back into the appropriate Schrödinger equation is not zero. If the equation is multiplied from the left by $\hat{R}_{\text{NR}}^{-1}$ the result is

$$\left[i\frac{\partial}{\partial t} - \left[\frac{\hat{\mathbf{P}}^2}{2m} + V(\mathbf{r}+\boldsymbol{\alpha}_0\sin\omega t) \right] \right] [\psi(r)]_{\text{H}_{\text{NR}}} \neq 0 . \tag{6}$$

The new Hamiltonian operator is the familiar Hamiltonian in the accelerating frame [3] and the wave function $\psi_{\text{H}_{\text{NR}}}$ is not an approximate solution of this equation.

[1] Shahid Rashid, Phys. Rev. A **40**, 4242 (1989).
[2] P. S. Krstic and M. H. Mittleman, Phys. Rev. A **42**, 4037 (1990).
[3] See, for example, M. Pont, N. R. Walet, and M. Gavrila, Phys. Rev. A **41**, 477 (1990).

