Quantum collapses and revivals in an optical cavity

Tran Quang,* P. L. Knight, and V. Bužek†
Optics Section, Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom
(Received 15 May 1991)

We study the influence of the atomic spontaneous-emission decay on the collapse-revival phenomena in the optical region. We demonstrate that revivals of the atomic inversion are much more sensitive to cavity-field damping than to spontaneous emission. This suggests that the additional dissipation caused by spontaneous emission would not offer an insurmountable obstacle to the experimental observation of optical revivals in high-Q cavities.

PACS number(s): 42.50.Dv, 42.52.+x, 32.80.−t

I. INTRODUCTION

The interaction of an atom with a single-mode cavity field [1–5] characterized by a coupling constant g results, in the absence of dissipation, in interesting nonclassical dynamics [2–5] including collapses and revivals of the atomic inversion [2,3]. On the other hand, the interaction of an atom with a cavity field naturally involves two dissipative processes [6]: cavity damping characterized by a damping rate k and secondly, spontaneous decay at rate γ into a continuum of field modes other than those of the cavity. In the strong-coupling limit (i.e., $g \gg k,\gamma$) the interaction between the atom and the cavity field cannot be described perturbatively, but involves a detailed knowledge of the coupled (composite) atom-field system [1]. In this case one can expect to observe nonclassical effects such as collapses and revivals of the atomic inversion, which depend on the cavity-field photon statistics, but only if dissipative influences are much less than the atom-field coupling.

Recently theoretical [2,3] and experimental [7] investigations have been carried out in the microwave region with Rydberg atoms, for which spontaneous emission is negligible and the coupling constant g is larger than the cavity damping rate ω/Q, where ω is the field frequency and Q the quality factor of the cavity. Under such circumstances the significant source of dissipation is the finite Q of the cavity. It has been shown that while collapses of the atomic inversion are not affected strongly by the cavity damping (if $g \geq k$), revivals are much more sensitive to cavity-field dissipation [3]. Until recently complementary studies in the optical domain have not been made, mainly because the atom–optical-field coupling constant g is usually far less than the cavity decay rate and the spontaneous-emission rate γ. Nevertheless very-high-Q optical cavities have been constructed recently [8], allowing investigations of the dynamics of an atom interacting with a quantized optical cavity field. In particular vacuum Rabi splitting [5], the splitting of spectral lines by the strong atom–vacuum-field coupling, has been observed [9]. In the microwave region it is very difficult to measure directly the cavity-field spectrum, so that quantum properties of the field must be inferred from measurements on the states of the atom involved in the transition. This would not be necessary in the optical region, for here photons can be counted and spectra measured directly. However, in the optical region, spontaneous emission will play an important role in the atom-field dynamics, and will affect the collapse-revival phenomena.

The purpose of the present paper is to study the influence of the atomic spontaneous-emission decay on collapse-revival phenomena in the optical region. We will demonstrate that revivals of the atomic inversion are much more sensitive to the cavity-field damping than to spontaneous emission. This suggests that the additional dissipation caused by spontaneous emission would not offer an insurmountable obstacle to the experimental observation of optical revivals in high-Q cavities. After presenting our results we discuss the physical origin of the asymmetry between cavity and atomic damping in atom-field evolution.

II. FORMULATION OF THE PROBLEM

Let us consider two levels of a single atom resonantly coupled to a single Fabry-Pérot cavity mode. In this system at least two different dissipation mechanisms can be identified. First, the cavity mode loses photons through the cavity mirrors. Second, in an open-cavity geometry the excited atom spontaneously emits photons into noncavity modes. The atomic spontaneous emission out of the optical cavity occurs at a rate γ. This atomic-decay rate to modes other than the privileged cavity mode is in general different to the free-space rate [6]. Transition losses through the cavity mirrors occurs at a rate $2k$. Generally dissipation through the cavity mirrors can be modeled via the coupling of the cavity mode to a reservoir of external electromagnetic vacuum modes [10]. In the Markovian approximation one can trace over the reservoir modes, which allows us to include dissipation into the master equation for the reduced system-density operator ρ (see, for instance, Refs. [11–13]). The resulting interaction-picture master equation in the dipole approximation for the reduced atom-field density operator ρ is

$$\frac{\partial}{\partial t}\rho = -ig[\sigma_- a^\dagger + \sigma_+ a, \rho] + k(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) , \tag{1}$$

where a and $a^\dagger$ are annihilation and creation operators of photons in the cavity mode and σ_+ and σ_- are the Pauli atomic raising and lowering operators. The coupling strength g between the atom and the cavity mode can be expressed in terms of the free-space spontaneous-emission rate [12].

From the master equation (1) we can derive the equations of motion for the matrix elements of the reduced field-density operator, defined as

$$\rho_{ij} = \langle i|\rho|j\rangle , \quad i,j=1,2$$

where $|i\rangle$ corresponds to either the lower ($i=1$) or the upper ($i=2$) state of the atom. The equations of motion for ρ_{ij} read

$$\dot\rho_{11} = ig(\rho_{12}a - a^\dagger\rho_{21}) + \gamma\rho_{22} + kL(\rho_{11}) , \tag{2}$$

$$\dot\rho_{22} = ig(\rho_{21}a^\dagger - a\rho_{12}) - \gamma\rho_{22} + kL(\rho_{22}) , \tag{3}$$

$$\dot\rho_{12} = ig(\rho_{11}a^\dagger - a^\dagger\rho_{22}) - \frac{\gamma}{2}\rho_{12} + kL(\rho_{12}) , \tag{4}$$

$$\dot\rho_{21} = ig(\rho_{22}a - a\rho_{11}) - \frac{\gamma}{2}\rho_{21} + kL(\rho_{21}) , \tag{5}$$

where

$$L(\rho) = k(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) . \tag{6}$$

We note that the atomic matrix elements ρ_{ij} $(i,j=1,2)$ are still operators with respect to the photon field. Now instead of the four matrix elements ρ_{ij} $(i,j=1,2)$ we introduce the following Hermitian operators:

$$\rho^{(1)} = \rho_{22} + \rho_{11} , \tag{7}$$

$$\rho^{(2)} = \rho_{22} - \rho_{11} , \tag{8}$$

$$\rho^{(3)} = \frac{i}{2}(a\rho_{12} - \rho_{21}a^\dagger) , \tag{9}$$

$$\rho^{(4)} = \frac{i}{2}(\rho_{12}a - a^\dagger\rho_{21}) . \tag{10}$$

It can be easily seen that the operator

$$\rho^{(1)} = \mathrm{Tr}_A(\rho) = \rho_{22} + \rho_{11} \tag{11}$$

is equal to the reduced-density operator of the field. Moreover the trace over the photon variables of the operator $\rho^{(2)}$, i.e.,

$$W = \mathrm{Tr}_{AF}[(\sigma_{22} - \sigma_{11})\rho] = \mathrm{Tr}_F(\rho^{(2)}) , \tag{12}$$

is equal to the population inversion of the atom. From Eqs. (2)–(5) we obtain the equations of motion for the operators $\rho^{(i)}$ in the form

$$\dot\rho^{(1)} = -2g\rho^{(3)} + 2g\rho^{(4)} + kL(\rho^{(1)}) , \tag{13}$$

$$\dot\rho^{(2)} = -2g\rho^{(3)} - 2g\rho^{(4)} - \gamma(\rho^{(1)} + \rho^{(2)}) + kL(\rho^{(2)}) , \tag{14}$$

$$\dot\rho^{(3)} = \frac{g}{4}(\rho^{(1)}aa^\dagger + aa^\dagger\rho^{(1)} - 2a\rho^{(1)}a^\dagger) + \frac{g}{4}(\rho^{(2)}aa^\dagger + aa^\dagger\rho^{(2)} + 2a\rho^{(2)}a^\dagger) - \left[k + \frac{\gamma}{2}\right]\rho^{(3)} + kL(\rho^{(3)}) , \tag{15}$$

$$\dot\rho^{(4)} = \frac{g}{4}(2a^\dagger\rho^{(1)}a - a^\dagger a\rho^{(1)} - \rho^{(1)}a^\dagger a) + \frac{g}{4}(2a^\dagger\rho^{(2)}a + a^\dagger a\rho^{(2)} + \rho^{(2)}a^\dagger a) + \left[k - \frac{\gamma}{2}\right]\rho^{(4)} - 2k\rho^{(3)} + kL(\rho^{(4)}) . \tag{16}$$

Here we note that, in contrast to Eqs. (2)–(5), Eqs. (13)–(16) contain only bilinear combinations of photon operators in the form $\rho^{(i)}a^\dagger a, a\rho^{(i)}a^\dagger, \ldots,$ which means that in the photon-number representation, equations for the diagonal matrix elements $\rho_{nn}^{(i)}$ and for the off-diagonal elements $\rho_{nm}^{(i)}$ are decoupled. This significantly reduces the complexity of numerical calculations, because instead of integrating N^2 coupled differential equations (where N is the upper limit for photon numbers, $N \gg 1$, considered in the numerical evaluation of sums over photon-number distributions) we have to integrate only $4N$ coupled equations.

III. COLLAPSE AND REVIVAL PHENOMENA

From Eqs. (13)–(16) we can derive a system of coupled equations for the diagonal matrix elements $\rho_{nn}^{(i)} \equiv P_n^{(i)}$ $(i=1,\ldots,4;\ n=0,1,\ldots)$

$$\dot P_n^{(1)} = -2gP_n^{(3)} + 2gP_n^{(4)} + 2k[(n+1)P_{n+1}^{(1)} - nP_n^{(1)}] , \tag{17}$$

$$\dot P_n^{(2)} = -2gP_n^{(3)} - 2gP_n^{(4)} - \gamma P_n^{(1)} - \gamma P_n^{(2)} + 2k[(n+1)P_{n+1}^{(2)} - nP_n^{(2)}] , \tag{18}$$

$$\dot P_n^{(3)} = \frac{g}{2}(n+1)(P_n^{(1)} - P_{n+1}^{(1)} + P_n^{(2)} + P_{n+1}^{(2)}) - \left[k + \frac{\gamma}{2}\right]P_n^{3)} + 2k[(n+1)P_{n+1}^{(3)} - nP_n^{(3)}] , \tag{19}$$

$$\dot P_n^{(4)} = \frac{g}{2}n(P_{n-1}^{(1)} - P_n^{(1)} + P_{n-1}^{(2)} + P_n^{(2)}) + \left[k - \frac{\gamma}{2}\right]P_n^{(4)} - 2kP_n^{(3)} + 2k[(n+1)P_{n+1}^{(4)} - nP_n^{(4)}] . \tag{20}$$

This system of equations can be integrated numerically, here by the fourth-order Runge-Kutta method. In the case when the atom is initially in the excited state, the initial conditions for $P_n^{(i)}$ are

$$P_n^{(1)}(0) = P_n^{(2)}(0) = P_n^{(f)} , \tag{21}$$

$$P_n^{(3)}(0)=P_n^{(4)}(0)=0 , \tag{22}$$

where $P_n^{(f)}$ is the initial photon-number distribution of the cavity field. The atomic population inversion (12) can be written as

$$W(t)=\sum_{n=0}^{\infty} P_n^{(2)}(t) . \tag{23}$$

In what follows we study in detail the influence of both damping mechanisms on the time evolution of the atomic inversion (23). Using the numerical solutions for the diagonal elements $P_n^{(2)}(t)$ we plot in Figs. 1 and 2 the atomic inversion versus the scaled time gt. The atom is initially in the excited state $[W(t=0)=1]$ and the cavity field is in a coherent state with an intensity $\bar{n}$ equal to 10 (Fig. 1) and 20 (Fig. 2). The time evolution of the atomic inversion is studied for various values of the cavity damping k and the spontaneous decay rate γ (both decay rates are measured in units of g).

From the figures we see that the collapse of the initial Rabi oscillations is weakly affected either by spontaneous emission or by cavity damping. In the quiescent period following the first collapse the inversion remains in a quasisteady state and slowly decays towards $|-1\rangle$ (i.e., the atom decays into the lower state). Unlike the collapse, the revivals are strongly affected by spontaneous emission and by cavity decay. Comparing various curves in Figs. 1 and 2, we note the following: (i) The higher the spontaneous-emission rate γ and the cavity-decay rate k, the smaller the amplitude of the revivals of the Rabi oscillations. (ii) For fixed values of the spontaneous-emission rate γ and the cavity-decay rate k, the amplitude of the revivals is proportional to the inverse value of the intensity of the initial coherent field; that is, the higher the $\bar{n}$ the smaller the amplitude of the revivals. (iii) Finally we turn our attention to the fact that the amplitude of the revivals is much more sensitive to the value of the cavity-decay rate than to the value of the spontaneous-emission rate [compare, for instance, Figs. 1(b) and 1(c)]. This of course is true for $\bar{n} \gg 1$. For small intensities of the initial coherent field both damping mechanisms affect the time evolution of the atomic inversion in the same way.

From the above we can conclude that spontaneous emission and cavity damping affect revival phenomena in quite different ways. We recollect here that the revival phenomenon is a purely quantum effect which is due to the quantum (discrete) nature of the cavity field [2]. The collapse of the Rabi oscillations can be associated with the spread in Rabi quantum eigenfrequencies describing the interaction between the atom and the cavity field [2]. This distribution of Rabi frequencies depends on the spread in photon numbers and results in the dephasing of Rabi oscillations and the collapse of the atomic inversion. The revivals of the atomic inversion occur when the Rabi

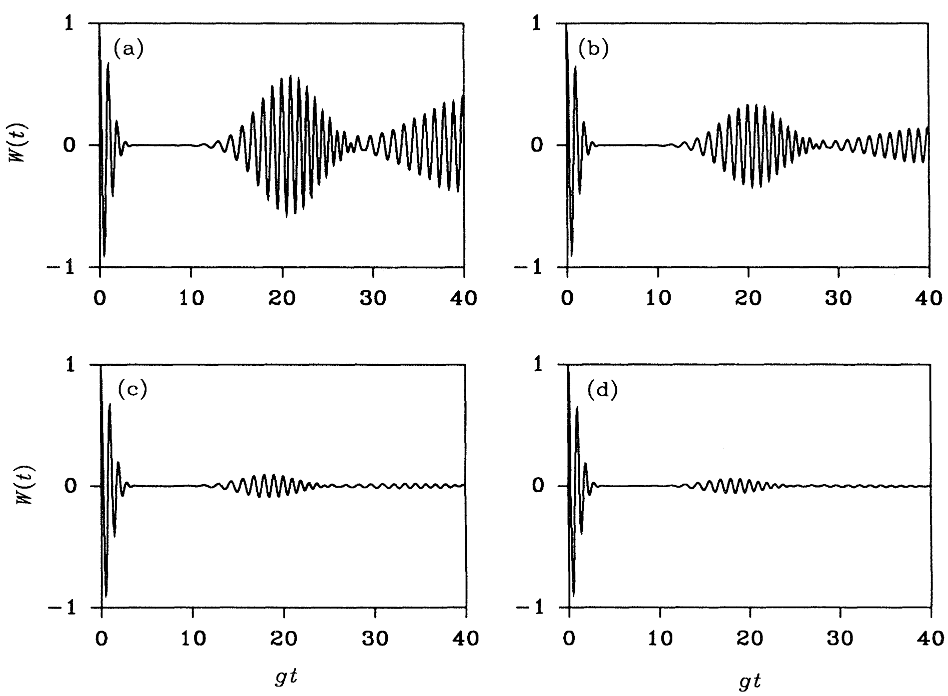

FIG. 1. Time evolution of the atomic inversion $W(t)$ for an initially excited atom interacting with a coherent field with mean photon number $\bar{n}=10$ for (a) $\gamma=k=0$; (b) $k=0$, $\gamma=0.05g$; (c) $k=0.005g$, $\gamma=0$; (d) $k=0.005g$, $\gamma=0.05g$. Comparing (c) and (d) we can easily see the dominant influence of the field damping on the diminishing of the revival phenomena. Namely, in (d) we have taken into account both the field- and atomic-decay mechanisms, but the time evolution of the atomic inversion is almost identical to that in (c) where the atomic decay rate is equal to zero.

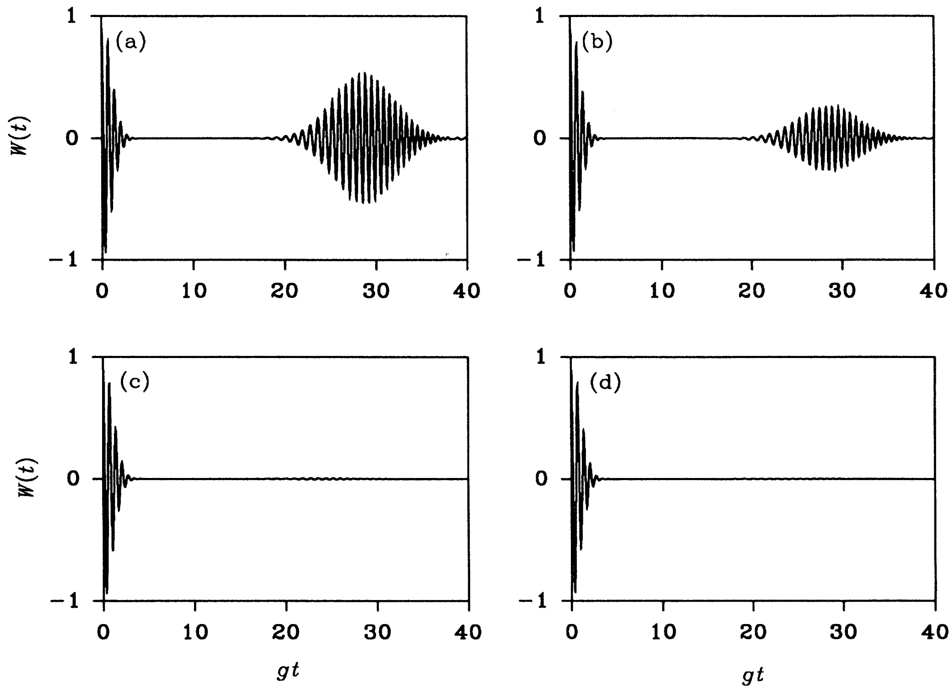

FIG. 2. Same as in Fig. 1 but $\bar{n}=20$.

frequencies with n near the mean $\bar{n}$ become in phase with one other (constructively interfere) provided the atom-field eigenfrequencies are discrete (i.e., distinguishable). The quantum revival disappears when the discrete eigenfrequencies are broadened into a continuous distribution [2]. Generally, increasing dissipation leads to a broadening in the widths of the eigenfrequencies of the Rabi oscillations, i.e., quantum revivals are sensitive to any damping in the atom-field system. Nevertheless, as we see from the figures, the optical revivals are much more robust with respect to the spontaneous emission than to the cavity damping. To explain this feature we write the equation of motion for the photon-number distribution of the cavity field $P_n^{(1)}$. Ignoring the terms proportional to k^2 and γk we can derive from (17) and (18) the following equation:

$$\begin{aligned}\ddot{P}_n^{(1)} = & -\frac{\gamma}{2}\dot{P}_n^{(1)} + 4k[(n+1)\dot{P}_{n+1}^{(1)} - (n-\tfrac{1}{4})\dot{P}_n^{(1)}] \\ & - g^2(n+1)(P_n^{(1)} - P_{n+1}^{(1)} + P_n^{(2)} + P_{n+1}^{(2)}) \\ & + g^2 n (P_{n-1}^{(1)} - P_n^{(1)} + P_{n-1}^{(2)} + P_n^{(2)}) . \end{aligned} \tag{24}$$

From the above it follows that the decay of $P_n^{(1)}$ due to spontaneous emission does not depend on the particular value of n and is proportional to

$$P_n^{(1)} \sim e^{-(\gamma/2)t} . \tag{25}$$

On the other hand, the decay of $P_n^{(1)}$ due to cavity damping is characterized by the *intensity-dependent* rate $k(2n+1)$ (for details see Refs. [3] and [14]),

$$P_n^{(1)} \sim e^{-k(2n+1)t} . \tag{26}$$

If we take into account that the difference between two atom-field eigenfrequencies is (see Ref. [5])

$$(\Delta\omega)_n = g(\sqrt{n+1} - \sqrt{n}) , \tag{27}$$

which for large intensities of the coherent cavity field can be approximated as

$$\overline{(\Delta\omega)} \cong \frac{g}{2(\bar{n})^{1/2}} , \tag{28}$$

then we can derive two necessary conditions for observing revivals of the atomic inversion in an optical cavity:

$$\frac{\gamma}{2} < \frac{g}{2(\bar{n})^{1/2}} \tag{29}$$

and

$$k(2\bar{n}+1) < \frac{g}{2(\bar{n})^{1/2}} . \tag{30}$$

These conditions reflect the fact that revivals can be observed only if the atom-field eigenfrequencies are not broadened into a continuous distribution either by spontaneous emission [condition (29)] or by cavity damping [condition (30)]. Comparing the conditions (29) and (30), we see that the revival phenomena are more fragile with respect to the cavity damping than to spontaneous emission. It is also seen that for a highly occupied cavity mode the revival phenomena are more sensitive to cavity damping. Generally, the higher the intensity of the dissipative cavity mode, the smaller the amplitude of revivals.

ACKNOWLEDGMENTS

This work is supported by the Alexander von Humboldt Foundation (Germany) and the U.K. Science and Engineering Research Council. T.Q. acknowledges Professor H. Walther for hospitality at the Max-Planck-Institut für Quantenoptik, where part of this work was done.

---

*Present address: Max-Planck-Institut für Quantenoptik, D-8046 Garching, Germany.

†Permanent address: Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 842 28 Bratislava, Czechoslovakia.

[1] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963); L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975); S. Haroche and J. M. Raimond, in *Advances in Atomic and Molecular Physics,* edited by D. Bates and B. Bederson (Academic, New York, 1985), Vol. 20, p. 347.

[2] J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980); T. von Foerster, J. Phys. A **8**, 95 (1975); P. L. Knight and P. M. Radmore, Phys. Lett. A **90**, 342 (1982); M. Fleischhauer and W. Schleich (unpublished).

[3] S. M. Barnett and P. L. Knight, Phys. Rev. A **33**, 2444 (1986); N. Nayak, R. K. Bullough, B. V. Thompson, and G. S. Agarwal, IEEE J. Quantum Electron. **24**, 1331 (1988); J. Eiselt and H. Risken, Phys. Rev. A **43**, 346 (1991).

[4] P. Meystre and M. S. Zubairy, Phys. Lett. A **89**, 390 (1982); J. R. Kuklinski and J. L. Madajczyk, Phys. Rev. A **37**, 3175 (1988).

[5] J. J. Sanchez-Mondragon, N. B. Narozhny, and J. H. Eberly, Phys. Rev. Lett. **51**, 550 (1983); G. S. Agarwal, *ibid.* **53**, 1732 (1984); V. Bužek and Tran Quang, Opt. Commun. **78**, 274 (1990).

[6] S. Haroche and D. Kleppner, Phys. Today **42**, 24 (1989), and references therein.

[7] D. Meschede, H. Walther, and G. Müller, Phys. Rev. Lett. **54**, 551 (1985); G. Rempe, H. Walther, and N. Klein, *ibid.* **58**, 353 (1987); M. Brune, J. M. Raimond, P. Goy, L. Davidovich, and S. Haroche, *ibid.* **59**, 1899 (1988).

[8] M. G. Raizen, R. J. Thomson, R. J. Brecha, H. J. Kimble, and H. J. Carmichael, in *Quantum Optics V,* edited by J. Harvey and D. F. Walls (Springer-Verlag, Berlin, 1989); R. J. Brecha, L. A. Orozco, M. G. Raizen, Min Xiao, and H. J. Kimble, J. Opt. Soc. Am. B **3**, 2381 (1986).

[9] M. G. Raizen, R. J. Thomson, R. J. Brecha, H. J. Kimble, and H. J. Carmichael, Phys. Rev. Lett. **63**, 240 (1989); Y. Zhu, D. J. Gauthier, S. E. Morin, Q. Wu, H. J. Carmichael, and T. W. Mossberg, *ibid.* **64**, 2499 (1990).

[10] W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973); G. S. Agarwal, *Quantum Statistical Properties of Spontaneous Emission and Their Relation to Other Approaches,* Springer Tracts in Modern Physics Vol. 70 (Springer-Verlag, Berlin, 1974).

[11] H. J. Carmichael, Phys. Rev. Lett. **55**, 2790 (1985).

[12] C. M. Savage, Phys. Rev. Lett. **60**, 1828 (1988); **63**, 1376 (1989); J. Mod. Opt. **37**, 1711 (1990).

[13] C. M. Savage and H. J. Carmichael, IEEE J. Quantum Electron. **24**, 1495 (1988).

[14] We have stressed that the revivals are dependent on the *discreteness* of the atom-field eigenvalues, that is, on the lifetimes of states labeled not only by the degree of atomic excitation but also by the photon number. The relevant field lifetime is *not* that of the cavity field energy, which is, of course $k=\omega/Q$, but of the field state with n excitations, which is $(2n+1)k$. This rapid decay of photon number state components of the atom-field state has been stressed by Barnett and Knight (Ref. [3]). See also M. O. Scully and W. E. Lamb Jr., Phys. Rev. **179**, 368 (1969); Ning Lu, Phys. Rev. A **40**, 1707 (1989); J. Janszky and T. Kobayashi, *ibid.* **41**, 4074 (1990).