Amplitude-squared squeezing of radiation in some lossless models

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An analysis of the amplitude-squared squeezing is presented for the lossless anharmonic-oscillator, Jaynes-Cummings, and intensity-dependent coupling Jaynes-Cummings models using coherent and squeezed inputs. Periodic revivals of the amplitude-squared squeezing are shown to exist in the anharmonic-oscillator and intensity-dependent coupling Jaynes-Cummings models for any value of the initial average photon number $\bar{n}$. The Jaynes-Cummings model, on the other hand, displays this characteristic for small $\bar{n}$ only. The effect of the squeezing angle on the amplitude-squared squeezing is studied. A comparison with the normal-order squeezing is presented.

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I. INTRODUCTION

The generation and detection in the laboratory [1,2] of squeezed states [3] of the electromagnetic field has spurred a great deal of activity [4] with the aim to discover systems that produce squeezed output from a coherent input, on one hand, and to explore changes in squeezing of a squeezed input as the result of some interaction, on the other. It is known [5] that squeezed states of the electromagnetic field can be obtained in the Jaynes-Cummings model (JCM) if one works with a field initially in a coherent state. This kind of squeezing, also known as the normal-order or second-order squeezing, has been shown to exist in many models including the multiphoton JCM [6], the intensity-dependent coupling JCM [7] (IDC-JCM) and the anharmonic-oscillator model [8,9] (AOM).

More recently, attention has begun to turn to generalizations of the second-order squeezing to higher orders. The first such attempt was published by Hong and Mandel [10] in which they discussed applications of higher-order squeezing to harmonic generation, degenerate parametric down conversion, and resonance fluorescence. Later, it was shown [11] for the AOM that with a coherent field input the degree of Hong-Mandel squeezing increases with the order of squeezing. In the meanwhile, a different kind of generalization to higher-order squeezing was proposed by Hillery [12]. Known as the amplitude-squared (AS) squeezing, it has attracted considerable attention recently [13,14].

The interest in the AOM derives [15] from its applicability to the description of nonlinear interactions of a medium. It is an exactly solvable model and can provide important insight into the dynamics of a nonlinear system even in its simplest form. The post-interaction behavior of the normal-order squeezing in this model reveals revivals [16] of squeezing and enhancement [17] over the initial squeezing due to interaction with the nonlinear medium. In the same model, Kitagawa and Yamamoto [18] have demonstrated the generation of states with squeezed number fluctuation. Similarly, the JCM [19] and IDC-JCM [20] are exactly solvable and have been known to exhibit a number of interesting nonclassical features. Being three of the richest models of quantum optics, we devote the present paper to a study of their AS squeezing behavior. Generally employing squeezed inputs, we focus attention on the dependence of this behavior on the input field intensity. We also examine its dependence on the squeezing direction in the AOM as a generic example. We assume a high-Q cavity so that losses can be ignored.

The plan of the paper is as follows. In Sec. II we briefly introduce the AS squeezing. In Sec. III we compute the AS squeezing functions for the AOM and discuss numerical results for a number of initial conditions. Section IV comprises the analytical and numerical results as well as their discussion for the JCM and IDC-JCM. A comparison of the results for the three models is presented in Sec. V.

II. AMPLITUDE-SQUARED SQUEEZING

Let a and $a^\dagger$ denote the annihilation and creation operators of a single-mode cavity field of frequency ω. It is convenient to work with the slowly varying operators A and $A^\dagger$ defined by

$$A=e^{i\omega t}a\ ,\quad A^\dagger=e^{-i\omega t}a^\dagger\ . \tag{1}$$

To compute the AS squeezing we introduce the quadrature operators

$$X_1=(A^2+A^{\dagger 2})/2\ ,\quad X_2=(A^2-A^{\dagger 2})/2i\ , \tag{2}$$

which satisfy the commutation relation

$$[X_1,X_2]=i(2N+1)\ ,\quad N=a^\dagger a\ . \tag{3}$$

The corresponding uncertainty relation reads

$$(\Delta X_1)(\Delta X_2)\geq\langle N\rangle+\tfrac{1}{2}\ , \tag{4}$$

with ΔX_i^2 denoting the respective variances. If the field is in a coherent state, it is easily checked that

$$\Delta X_i^2=\langle N\rangle+\tfrac{1}{2}\ ,\quad i=1 \text{ or } 2\ . \tag{5}$$

A state is AS squeezed in variable X_i if its variance satisfies

$$\Delta X_i^2 < \langle N \rangle + \tfrac{1}{2} . \tag{6}$$

Let us introduce the quantities

$$A_0 = \langle A^{\dagger 2} A^2 \rangle , \quad A_1 = \langle A^2 \rangle , \quad A_2 = \langle A^4 \rangle , \tag{7}$$

so that the normal-ordered variances

$$S_i = \langle :\Delta X_i^2: \rangle \tag{8}$$

are given by

$$S_i = \tfrac{1}{4}[\pm(A_2 + A_2^*) + 2A_0 \mp (A_1 \pm A_1^*)^2] . \tag{9}$$

In terms of S_i, the AS squeezing is defined by

$$S_i < 0 , \quad i = 1 \text{ or } 2 . \tag{10}$$

Note that S_i are bounded below by $-(\langle N \rangle + \frac{1}{2})$.

III. THE ANHARMONIC-OSCILLATOR MODEL

A. Description of the model

A commonly used form of the AOM Hamiltonian is given by

$$H = \hbar\omega a^\dagger a + \tfrac{1}{2}\hbar\lambda a^{\dagger 2} a^2 , \tag{11}$$

where λ is the nonlinearity parameter which is real. We assume that there are no losses present.

Since $a^\dagger a$ is a constant of the motion, integration of the Heisenberg equation of motion with the Hamiltonian (11) yields the result

$$a(t) = (\exp\{-it[\omega + \lambda a^\dagger(0) a(0)]\}) a(0) . \tag{12}$$

At a later point we shall also briefly comment upon another form of the anharmonic-oscillator Hamiltonian, namely,

$$H = \hbar\omega a^\dagger a + \tfrac{1}{2}\hbar\lambda (a^\dagger a)^2 . \tag{13}$$

B. Evaluation of the variables A_0, A_1, and A_2

Using Eq. (12), we can evaluate the quantities A_0, A_1, and A_2 of Eqs. (7). We assume that the field is initially in a general squeezed state so that its wave function at $t=0$ can be written as

$$|\psi(0)\rangle = \sum_{n=0}^{\infty} Q_n |n\rangle , \tag{14}$$

where

$$Q_n = (n!\mu)^{-1/2} (\nu e^{i\phi}/2\mu)^{n/2} H_n(x) \times \exp[-\tfrac{1}{2}|\beta|^2 + (\nu e^{-i\phi}/2\mu)\beta^2] , \tag{15}$$

and

$$\mu = \cosh r , \quad \nu = \sinh r , \quad x = \beta/(2\mu\nu e^{i\phi})^{1/2} , \\ \alpha = |\alpha| e^{i\theta} , \quad \beta = \mu\alpha + \nu e^{i\phi}\alpha^* . \tag{16}$$

Here r is the squeeze parameter, θ the direction of coherent excitation, and ϕ the direction of squeezing relative to the direction of coherent excitation. The initial average photon number $\bar{n}$ is given by $\bar{n} = |\alpha|^2 + \nu^2$. Using Eq. (14) we have

$$A_0 = (e^{-\epsilon}/\mu) \sum_{n=0}^{\infty} n(n-1)(z_0^n/2^n n!) H_n(x^*) H_n(x) ,$$

$$A_1 = (z_0/2\mu) e^{i(\phi-\tau)-\epsilon} \sum_{n=0}^{\infty} (z_1^n/2^n n!) H_n(x^*) H_{n+2}(x) , \tag{17}$$

$$A_2 = (z_0^2/4\mu) e^{2i(\phi-3\tau)-\epsilon} \sum_{n=0}^{\infty} (z_2^n/2^n n!) H_n(x^*) H_{n+4}(x) .$$

Here

$$z_0 = \nu/\mu , \quad z_1 = z_0 e^{-2i\tau} , \quad z_2 = z_0 e^{-4i\tau} , \\ \epsilon = |\alpha|^2[1 + z_0 \cos(2\theta - \phi)] . \tag{18}$$

The sums over n in Eqs. (17) can be evaluated by making use of the well-known relations involving the Hermite polynomials [21]

$$\sum_{n=0}^{\infty} (z^n/2^n n!) H_n(x) H_n(y) = (1-z^2)^{-1/2} e^{[2xyz-(x^2+y^2)z^2]/(1-z^2)} |z| < 1 , \tag{19}$$

$$H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0 , \tag{20}$$

$$H_n'(x) = 2nH_{n-1}(x) . \tag{21}$$

After a somewhat tedious calculation we find that

$$A_0 = (e^{-\epsilon}/\mu)[z_0^2(d^2G_0/dz_0^2)] , \tag{22}$$

$$A_1 = (z_0/2\mu) e^{i(\phi-\tau)-\epsilon} F_1 , \tag{23}$$

$$A_2 = (z_0^2/2\mu) e^{2i(\phi-3\tau)-\epsilon} \times ((2x^2-3)F_2 - z_2(dF_2/dz_2) - \{4x^2[2+z_2(d/dz_2)] + 2x[2+z_2(d^2/dx dz_2)]\}G_2) , \tag{24}$$

where $\tau = \lambda t$, and

$$F_i = F_i(\tau) = [G_i/(1-z_i^2)^2][2(x - z_i x^*)^2 - (1-z_i^2)] , \tag{25}$$

$$G_i = G_i(\tau) = (1-z_i^2)^{-1/2} e^{[2|x|^2 z_i - (x^2 + x^{*2}) z_i^2]/(1-z_i^2)} . \tag{26}$$

The corresponding expressions of the A_i for the squeezed vacuum and coherent inputs can be recovered by going to the limits $|\alpha|^2 \to 0$ and $r \to 0$, respectively. We thus find that for the squeezed vacuum input,

$$A_0 = \nu^2(1+3\nu^2) , \tag{27}$$

$$A_1 = (z_0/\mu) e^{i(\phi-\tau)} (1-z_1^2)^{-3/2} , \tag{28}$$

$$A_2 = 3(z_0^2/\mu) e^{i2(\phi-3\tau)} (1-z_2^2)^{-5/2} . \tag{29}$$

For the coherent input, we find

$$A_0=(\bar{n})^2 \,, \tag{30}$$

$$A_1=\bar{n}e^{\bar{n}[(\cos 2\tau)-1]}[\cos(\tau+\bar{n}\sin 2\tau)-i\sin(\tau+\bar{n}\sin 2\tau)] \,, \tag{31}$$

$$A_2=(\bar{n})^2 e^{\bar{n}[(\cos 4\tau)-1]} \times[\cos(6\tau+\bar{n}\sin 4\tau)-i\sin(6\tau+\bar{n}\sin 4\tau)] \,. \tag{32}$$

The functions $S_i(\tau)$ of Eq. (9) can be obtained from the values of A_i. Before proceeding further we note that $S_i(\tau+\pi)=S_i(\tau)$ for the three cases considered. Thus, if the squeezing functions exhibit squeezing they will do so periodically. We note also that the S_i are related for different values of ϕ. For example, for a general squeezed input, $S_i(\phi=0)=S_i(\phi=2\pi)$ while with a squeezed vacuum input $S_i(\phi=0)=S_i(\phi=\pi)$. In the latter case one has also the relation $S_{1,2}(\phi=0)=S_{2,1}(\phi=\pi/2)$. In Sec. III C we present and discuss the numerical results obtained.

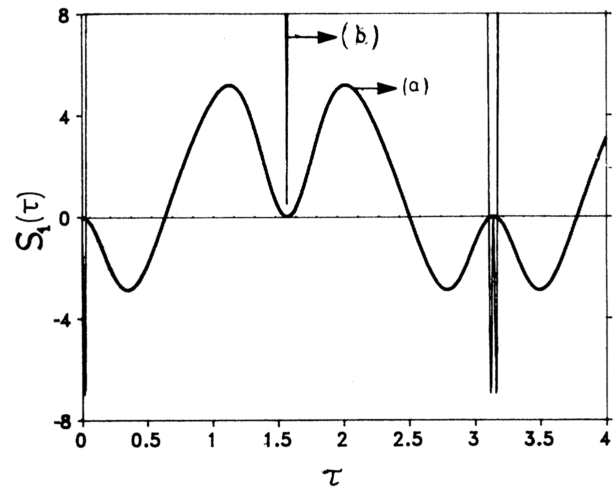

FIG. 1. The squeezing function $S_1(\tau)$ of the AOM with the field initially in a coherent state. (a) $S_1\times 10^2$ for $\bar{n}=0.25$; (b) S_1 for $\bar{n}=16$. $S_1(\tau)<0$ implies amplitude-squared squeezing.

C. Results and discussion

The AS squeezing for $r=0$ (*coherent* input) and $\bar{n}=0.25$ and 16 is shown in Fig. 1. For both cases, only the in-phase component shows revivals of squeezing in the form of a succession of duplexes of "squeezing holes." With $\bar{n}$ large, the holes become deeper and narrower. For a sufficiently large value of $\bar{n}$ (≥ 100) the duration of squeezing becomes too short for it to be noticeable with our resolution. We refer to it as the revocation [22] of squeezing.

We next consider the case when the input field is in a *squeezed vacuum* state ($\alpha=0$). The results for $r=0.3$ and 0.9 and $\phi=0$ are shown in Fig. 2. Although the photon number is not too large for $r=0.9$ (i.e., $\bar{n}=1.05$), we can already see from Fig. 2(b) a sharp change in the squeezing behavior of S_1 (and of S_2, not shown here) in that the periodic oscillations of squeezing in the two quadratures seen for $r=0.3$ [Fig. 2(a)] have turned into rare excursions to the region below the standard quantum limit (SQL). S_1 which starts out from a positive value shows later a slightly greater amount of squeezing than S_2 (which is initially negative). This happens for both r values. An inspection of Fig. 2(a) reveals the existence of three revivals in one oscillation period of S_1. Already for $r=0.9$ this number reduces to one and the oscillation and revival periods almost coincide.

Finally, we consider the case of an input in a *general squeezed* state. We take r, α, and ϕ nonzero but $\theta=0$, i.e., α real. The initial values of S_i can be obtained from the expressions

$$S_{1,2}(\tau=0)=\tfrac{1}{2}\left[\pm(\alpha^4+3\mu^2\nu^2\cos 2\phi-6\alpha^2\mu\nu\cos\phi)+(\alpha^4+\mu^2\nu^2+2\nu^4-2\alpha^2\mu\nu\cos\phi+4\alpha^2\nu^2)-2\begin{bmatrix}\alpha^2-\mu\nu\cos\phi\\ \mu\nu\sin\phi\end{bmatrix}^2\right] . \tag{33}$$

Before proceeding further, we make a number of comments about this result. Firstly, the value of ϕ will determine whether S_1 or S_2 are less than zero at $\tau=0$. In particular, while the first quadrature may be initially squeezed for $\phi=0$, it cannot be so for $\phi=\pi$. Secondly, the change of ϕ from 0 to $\pi/2$ which interchanges S_1 and S_2 in the case of the squeezed vacuum does not have the same effect here. Thirdly, this result shows that the amount of initial squeezing increases as ϕ increases from 0 to π. Lastly, for $\phi=\pi/2$ only S_1 shows squeezing provided that $\alpha<2^{-1/2}$.

Figure 3 shows how the AS squeezing behavior changes as we change ϕ from 0 through $\pi/2$ to π for small $\bar{n}$. As an example we set $r=0.3$ and $\alpha=0.25$ ($\bar{n}=0.155$). Periodic revivals of squeezing can be seen in all the three cases. Interestingly, S_1 is seen to have two revival periods ($\tau_R=1.0,0.34$) for $\phi=0$ and two ($\tau_R=0.73,0.37$) for $\phi=\pi$, while three such periods ($\tau_R=0.84,0.54,0.65$) for $\phi=\pi/2$. S_2, on the other hand, has one revival period ($\tau_R=2.93$) for $\phi=0$ and two ($\tau_R=0.89,1.73$) for $\phi=\pi/2$ and two ($\tau_R=0.76,0.84$) for $\phi=\pi$. For the time duration considered S_1 remains below the SQL for approximately 27%, 36%, and 41% of the time for the three ϕ values. Similarly, S_2 remains below the SQL for about 7%, 14%, and 23% of the time. That is, the duration of squeezing increases with ϕ. There is, however, no enhancement in the AS squeezing after the interaction is turned on.

The results for $r=0.9$, $\alpha=0.25$ ($\bar{n}=1.116$), and $\phi=0$, $\pi/2$ and π are shown in Fig. 4. These values were chosen with a view to examining the effect of a high squeeze parameter at low intensity. Although periodic revivals of

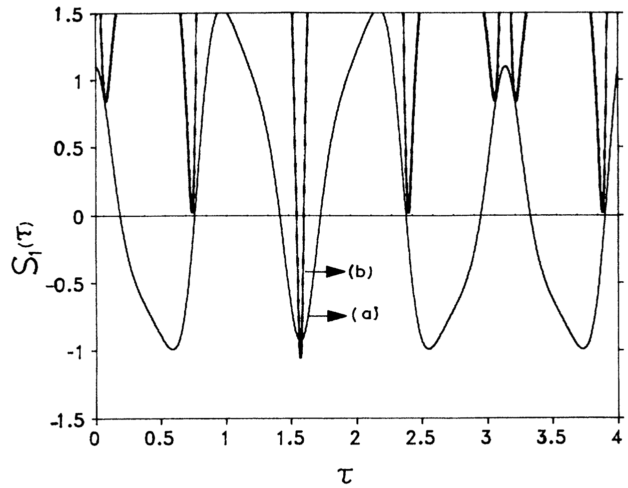

FIG. 2. $S_1(\tau)$ of the AOM with the field initially in a squeezed vacuum state. (a) $S_1 \times 10$ for $r=0.3$ ($\bar{n}=0.093$); (b) S_1 for $r=0.9$ ($\bar{n}=1.05$).

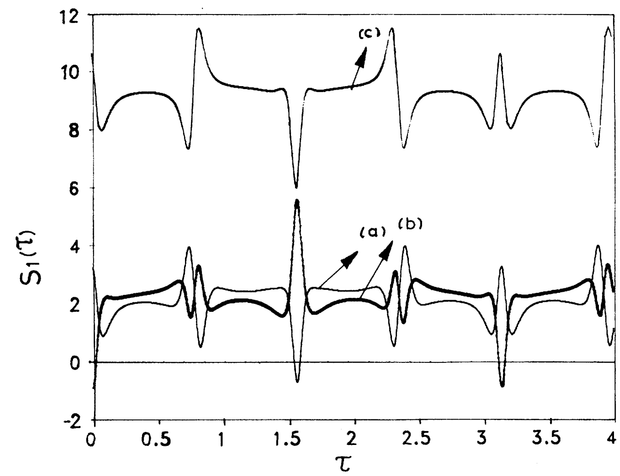

FIG. 4. $S_1(\tau)$ of the AOM with the field initially in a general squeezed state for $r=0.9$, $\alpha=0.25$ ($\bar{n}=1.116$), $\theta=0$. (a) $\phi=0$; (b) $\phi=\pi/2$; (c) S_1+7 for $\phi=\pi$.

the AS squeezing are still there, their frequency has decreased and the duration has become much smaller ($\Delta\tau\cong 0.05$) than that seen in Fig. 3. The revival period equals the oscillation period. Notice that the maximum squeezing obtainable in this case does not exceed the amount of input squeezing.

We next consider large values of $\bar{n}$. The results for $r=0.3$ and $\alpha=4$ ($\bar{n}=16.09$) for three values of ϕ are shown in Fig. 5. The $\phi=0$ case shows that the initially present AS squeezing in S_1 (at $t=0$, S_1 equals -7.11) revives periodically but for a very short duration of time ($\Delta\tau\cong 0.046$). As we increase ϕ from 0 to $\pi/2$ the amount of AS squeezing increases and the revival at $\tau=\pi$ splits into two closely spaced revivals, each of approximate duration $\Delta\tau\cong 0.018$. For both values of ϕ, S_2 remains above the SQL. Figure 5(c) shows the case of $\phi=\pi$. S_1 starts out with a positive value but shows AS squeezing later. As with case 5(b), it shows a revival doublet at $\tau=\pi$. In addition, however, it shows a single revival at $\tau=\pi/2$. Unlike the cases 5(a) and 5(b), here S_2 also shows revivals of squeezing. The $r=0.9$ case also exhibits these squeezing characteristics, showing greater AS squeezing for smaller duration.

At this point, it will be instructive to compare our results on the AS squeezing with those on the normal-order squeezing. It is known that the normal-order squeezing can be obtained in the AOM with coherent [8,9] and squeezed vacuum [16,17] inputs. The squeezing functions of the normal-order squeezing being periodic (with period 2π), it will recur periodically. Insofar as the $\bar{n}$ depen-

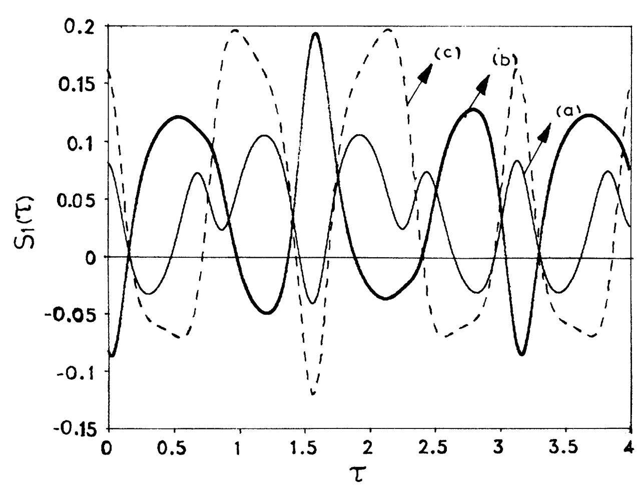

FIG. 3. $S_1(\tau)$ of the AOM with the field initially in a general squeezed state for $r=0.3$, $\alpha=0.25$ ($\bar{n}=0.155$), $\theta=0$. (a) $\phi=0$; (b) $\phi=\pi/2$; (c) $\phi=\pi$.

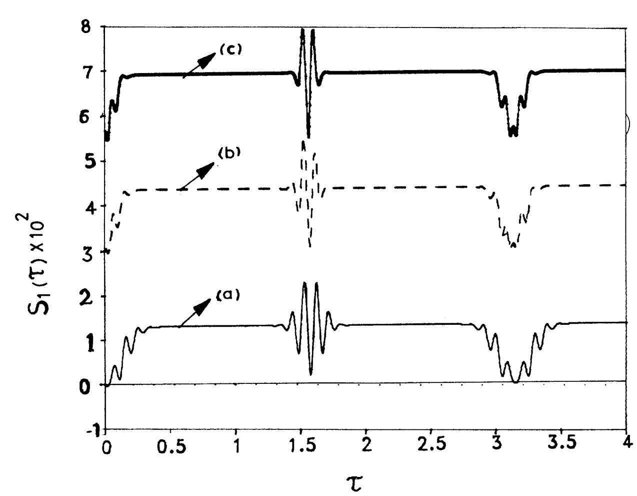

FIG. 5. $S_1(\tau)$ of the AOM with the field initially in a general squeezed state for $r=0.3$, $\alpha=4$ ($\bar{n}=16.115$), $\theta=0$. (a) $\phi=0$; (b) S_1+300 for $\phi=\pi/2$; (c) S_1+550 for $\phi=\pi$.

dence is concerned, not only the occurrence and revocation are hastened with increasing $\bar{n}$ but we have also checked that the frequency of recurrence reduces. To see the influence of ϕ on squeezing, we carried out a computation of the normal-order squeezing with a general squeezed input. It turns out that ϕ has little or no effect on its degree. We may thus conclude that while the $\bar{n}$ dependence of the AS squeezing behavior is similar to that seen in the normal-order squeezing, i.e., the recurrences become fewer and the duration smaller as we increase $\bar{n}$, the ϕ dependence is significantly different. Of course, the periodicity of S_i in the two cases is different too. Interestingly enough, we find that the degree of squeezing is higher in normal-order than in AS squeezing. This is a manifestation of the fact that the AS squeezing is different from the Hong-Mandel [10] squeezing for which Gerry and Rodrigues [11] have shown that the degree of squeezing increases with the order of squeezing.

Before closing this section, we mention that we have also computed the AS squeezing with the Hamiltonian (13). We find that the AS squeezing behavior of the two models is dissimilar for small $\bar{n}$ values but becomes identical when $\bar{n}$ is large.

IV. JAYNES-CUMMINGS MODELS AND INTENSITY-DEPENDENT COUPLING

A. Description of the models

The two models are characterized in the rotating-wave approximation by the Hamiltonians

$$H=\hbar\omega a^{\dagger}a+\tfrac{1}{2}\hbar\omega_0\sigma_3+\hbar g(\sigma_+ a+\sigma_- a^{\dagger}) , \tag{34}$$

and

$$H=\hbar\omega a^{\dagger}a+\tfrac{1}{2}\hbar\omega_0\sigma_3 +\hbar g[\sigma_+ a(a^{\dagger}a)^{1/2}+\sigma_-(a^{\dagger}a)^{1/2}a^{\dagger}] , \tag{35}$$

respectively. Here σ_3 and $\sigma_{\pm}$ are the Pauli spin matrices, ω_0 is the atomic frequency, and g is the atom-field coupling constant. Again, we consider a lossless situation.

We denote by $|\pm\rangle$ the excited and ground states of the atom and, for simplicity, consider the resonant case, $\omega=\omega_0$.

B. Evaluation of the variables A_0, A_1, and A_2

For the atom initially in the ground state and the field in a general squeezed state, the state vector of the system at $\tau=0$ is written as

$$|\psi(0)\rangle \equiv \sum_{n=0}^{\infty} Q_n|-,n\rangle , \tag{36}$$

where Q_n is given by Eq. (15). With the Hamiltonian (34), the expression for the field density-matrix elements at $\tau>0$ is given by

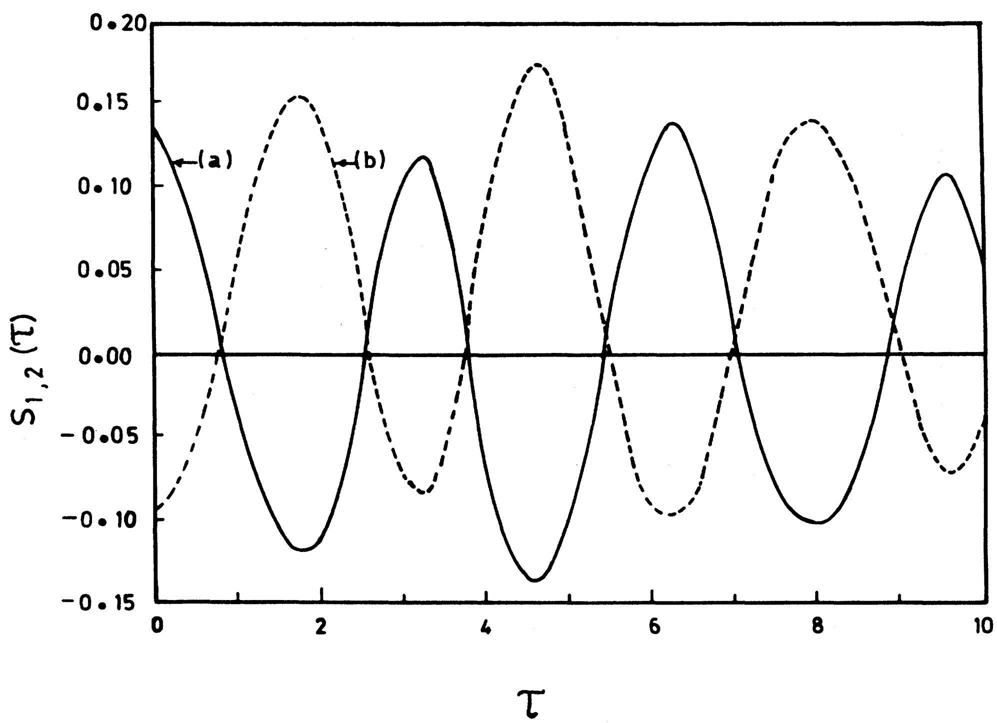

FIG. 6. The squeezing functions of the JCM with the field initially in a squeezed vacuum state for $r=0.3$ ($\bar{n}=0.09$). (a) $S_1(\tau)$; (b) $S_2(\tau)$.

$$\rho_{n,n'}(\tau)=\exp[-i(n-n')\omega t][\cos(\sqrt{n}\,\tau)\cos(\sqrt{n'}\tau)\rho_{n,n'}(0)+\sin(\sqrt{n+1}\tau)\sin(\sqrt{n'+1}\tau)\rho_{n+1,n'+1}(0)]\ . \tag{37}$$

The corresponding expression in the case of a squeezed vacuum field input can be obtained by taking $\alpha\rightarrow 0$,

$$\rho_{n,n'}(\tau)=\exp[-i(n-n')\omega t]\begin{cases}\cos(\sqrt{n}\,\tau)\cos(\sqrt{n'}\tau)\rho_{n,n'}(0)\ , & (n,n'\text{ even})\\ \sin[\sqrt{(n+1)}\tau]\sin[\sqrt{(n'+1)}\tau]\rho_{n+1,n'+1}(0) & (n,n'\text{ odd})\ ,\end{cases} \tag{38}$$

where $\tau=gt$.

The expressions of $\rho_{n,n'}(\tau)$ with the Hamiltonian (35) can be obtained by dropping the square roots in Eqs. (37) and (38). Using these last two equations, we can compute the variables A_0, A_1, and A_2. In the special cases of interest we have for the general squeezed input,

$$A_0=\sum_{n=0}^{\infty}\rho_{n,n}(0)[n(n-1)-2(n-1)\sin^2(\sqrt{n}\,\tau)]\ , \tag{39}$$

$$A_1=\sum_{n=0}^{\infty}\sqrt{(n+1)}\rho_{n,n+2}(0)\{\sqrt{(n+2)}\cos(\sqrt{n}\,\tau)\cos[\sqrt{(n+2)}\tau]+\sqrt{n}\,\sin(\sqrt{n}\,\tau)\sin[\sqrt{(n+2)}\tau]\}\ , \tag{40}$$

$$A_2=\sum_{n=0}^{\infty}\sqrt{(n+1)(n+2)(n+3)}\rho_{n,n+4}(0)\{\sqrt{(n+4)}\cos(\sqrt{n}\,\tau)\cos[\sqrt{(n+4)}\tau]+\sqrt{n}\sin(\sqrt{n}\,\tau)\sin[\sqrt{(n+4)}\tau]\}\ . \tag{41}$$

For the squeezed vacuum input we have

$$A_0=\tanh^2 r\sum_{n=0}^{\infty}[(2n+1)^2/(n+1)]\rho_{2n,2n}(0)\{n+\cos^2[\sqrt{(2n+2)}\tau]\}\ , \tag{42}$$

$$A_1=\tanh r\sum_{n=0}^{\infty}(2n+1)\rho_{2n,2n}(0)\{\cos(\sqrt{2n}\,\tau)\cos[\sqrt{(2n+2)}\tau]+\sqrt{[n/(n+1)]}\sin(\sqrt{2n}\,\tau)\sin[\sqrt{(2n+2)}\tau]\}\ , \tag{43}$$

$$A_2=\tanh^2 r\sum_{n=0}^{\infty}(2n+1)(2n+3)\rho_{2n,2n}(0)\{\cos(\sqrt{2n}\,\tau)\cos[\sqrt{(2n+4)}\tau]+\sqrt{[n/(n+2)]}\sin(\sqrt{2n}\,\tau)\sin[\sqrt{(2n+4)}\tau]\}\ . \tag{44}$$

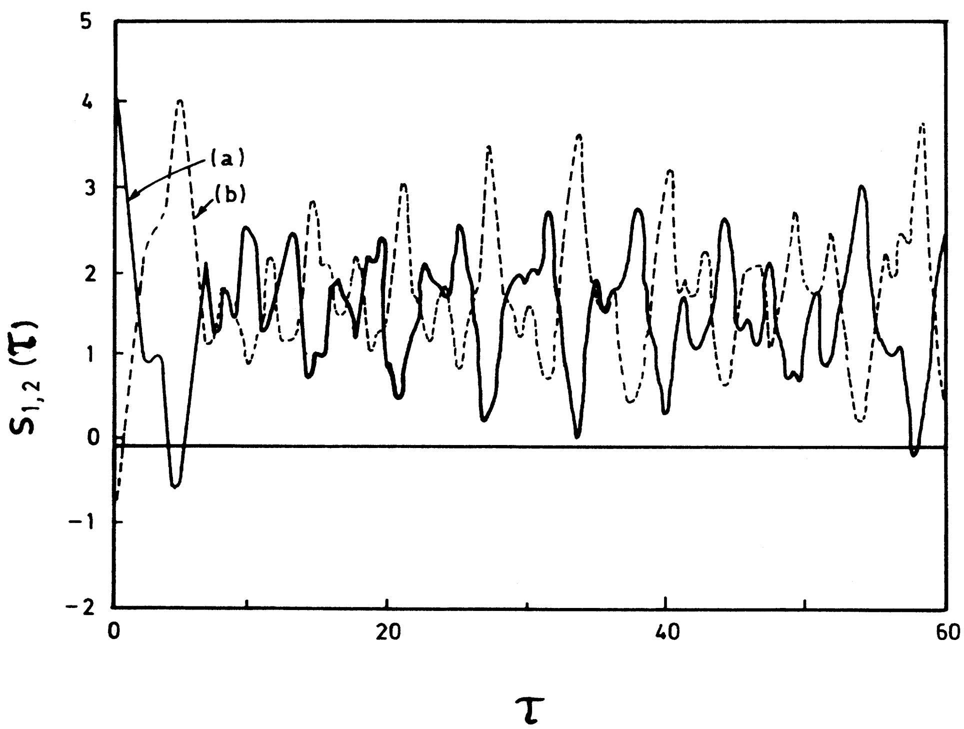

FIG. 7. Same as in Fig. 6 but $r=0.9$ ($\bar{n}=1.05$).

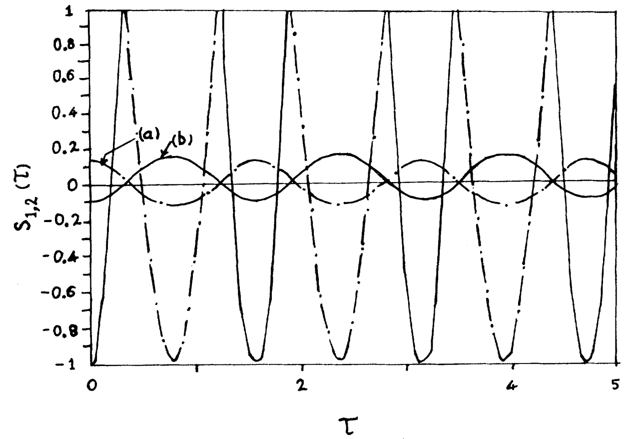

FIG. 8. The squeezing functions of the IDC-JCM with the field initially in a squeezed vacuum state. (a) $S_1(\tau)$ and (b) $S_2(\tau)$ for $r=0.3$ ($\bar{n}=0.09$); (c) $S_1(\tau)$ and (d) $S_2(\tau)$ for $r=0.9$ ($\bar{n}=1.05$).

The case of a coherent input has been considered by Mahran and Obada [13] and Xiaping and Xiping [13], so that we confine ourselves to the discussion of squeezed inputs only.

C. Results and discussion

The time evolution of S_1 and S_2 based on (34) with the field initially in the squeezed vacuum state is shown in Figs. 6 and 7. For $r=0.3$ (small input squeezing) the initially present AS squeezing revives periodically with S_1 and S_2 exchanging noise. However, for $r=0.9$ (large input squeezing) the AS squeezing in S_2 is revoked soon after the interaction is turned on. After a sufficiently long time ($\tau \geq 48$) S_1 also stops going below the SQL. The large $\bar{n}$ values ($\bar{n}=\nu^2$) act to destroy not only normal-order squeezing due to a larger emission rate [23] but also the AS squeezing.

Figure 8 displays the behavior of S_1 and S_2 obtained from the IDC-JCM Hamiltonian (35) for $r=0.3$ and 0.9. We see periodic revivals of AS squeezing both for low and high intensities, though the duration of squeezing reduces with increase in r or $\bar{n}$. In other words, for high $\bar{n}$, the two models show totally different behavior: exact revocation versus strictly periodic recurrence of AS squeezing. To achieve revocation in the IDC-JCM in the sense of Ref. [22] we need extremely large values of $\bar{n}$. Note that for small $\bar{n}$ the maximum AS squeezing achievable in the two models is comparable.

The results for a general squeezed state field input (with $\phi=0$) in the case of the JCM are shown in Figs. 9 and 10. With a low input intensity (Fig. 9) we do have revivals of AS squeezing but, unlike the case of the

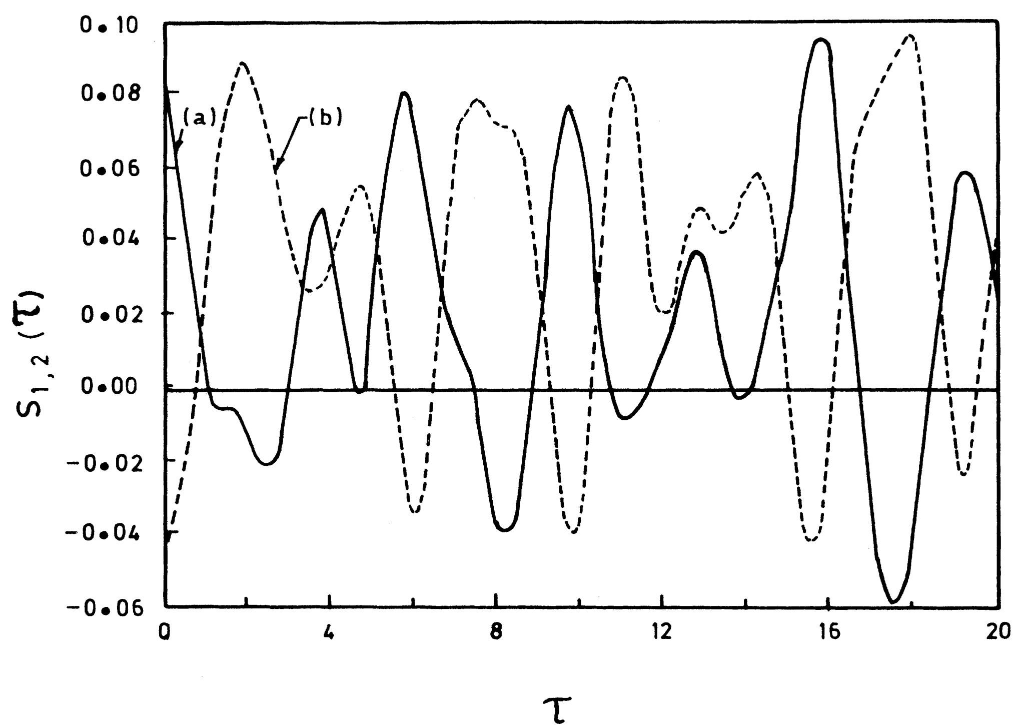

FIG. 9. (a) $S_1(\tau)$ and (b) $S_2(\tau)$ of the JCM with the field initially in a general squeezed state for $r=0.3$ and $\alpha=0.25$.

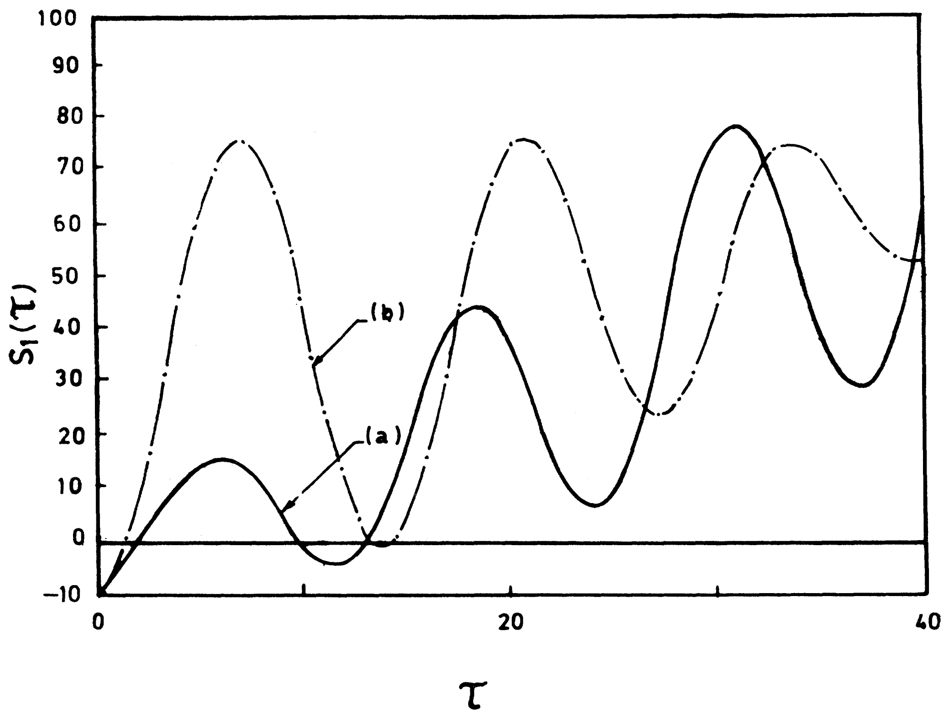

FIG. 10. $S_1(\tau)$ of the JCM with the field initially in a general squeezed state for $\alpha=4$. (a) $r=0.3$; (b) $r=0.9$.

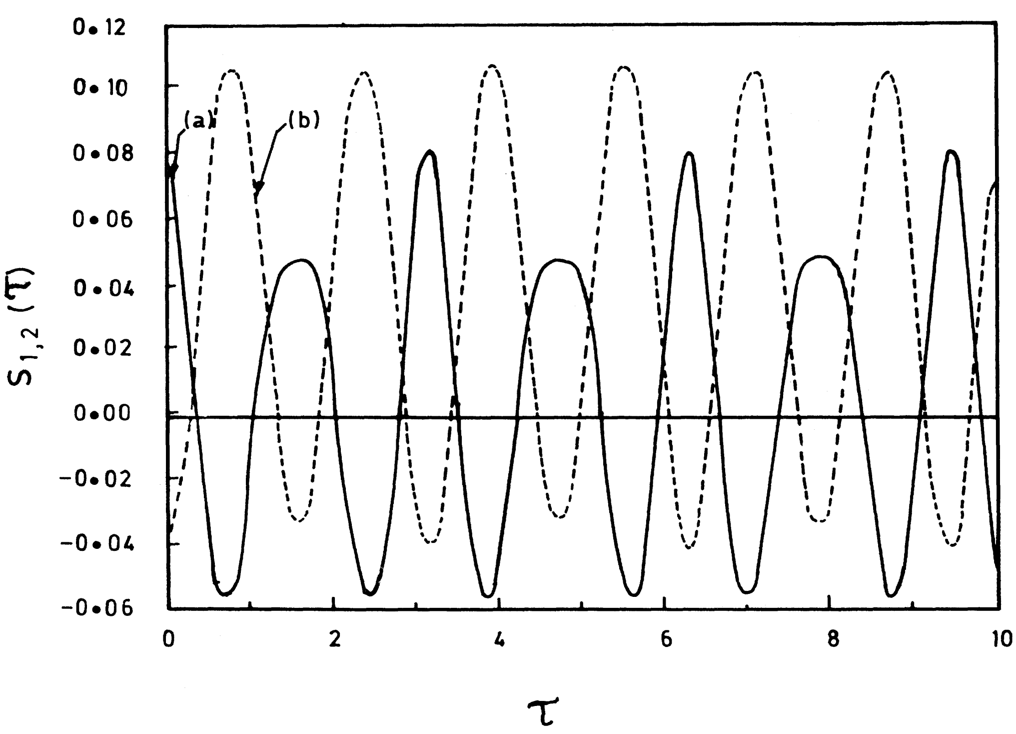

FIG. 11. (a) $S_1(\tau)$ and (b) $S_2(\tau)$ of the IDC-JCM with the parametric values of Fig. 9.

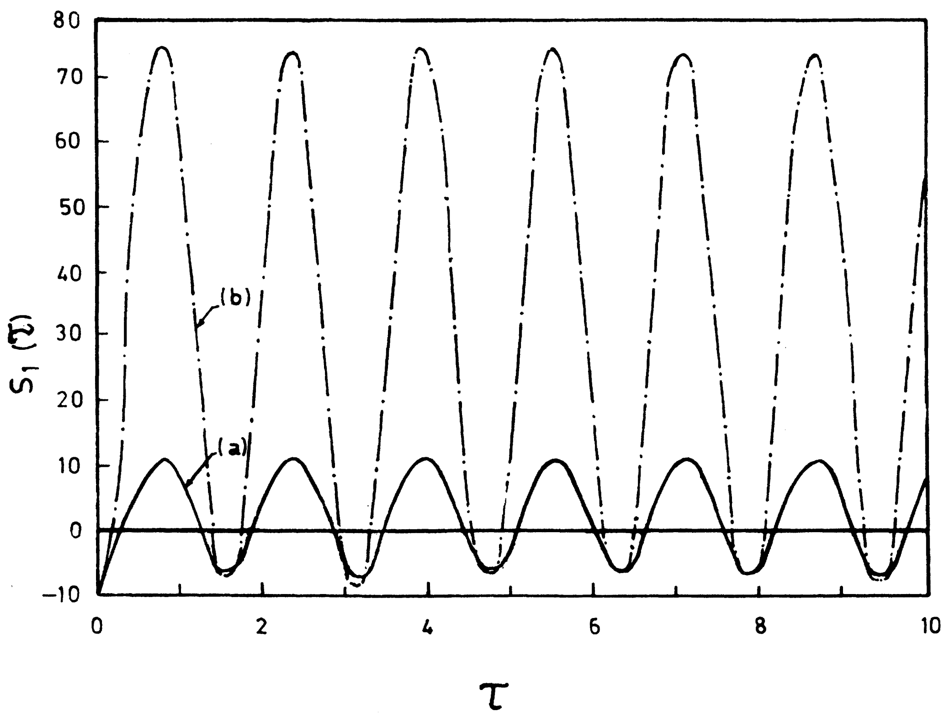

FIG. 12. $S_1(\tau)$ of the IDC-JCM with the parametric values of Fig. 10.

squeezed vacuum, they are not periodic. With the same value of α but $r=0.9$, we found that AS squeezing disappears within a few moments of turning on the interaction and does not return later. For the higher input intensity (Fig. 10), while S_2 remains above the SQL at all times, S_1, which is initially squeezed, desqueezes after awhile but squeezes again once after which it remains above the SQL. It seems then that while larger initial AS squeezing reduces squeezing later, α acts to partially offset this effect.

The corresponding results for the IDC-JCM are shown in Figs. 11 and 12. Again for small $\bar{n}$ (Fig. 11), there are strictly periodic revivals of AS squeezing with two periods for S_1 ($\tau \cong 0.7, 1.07$) and only one for S_2 ($\tau \cong 1.07$). For large $\bar{n}$ (Fig. 12) S_2 remains above the SQL but S_1 squeezes periodically with the period depending on the value of r ($\tau \cong 1$ and 1.2 for $r=0.3$ and 0.9, respectively). We see again that with a small input squeezing we obtain larger AS squeezing later, while a large input leads to reduced AS squeezing.

Let us compare our results with those on the normal-order squeezing with squeezed inputs [23,24]. In the case of the JCM we find that (i) the revival periods in the two types of squeezing are different, (ii) the AS squeezing is much harder to revoke than normal-order squeezing, and (iii) the degree of squeezing in the AS case is higher than the normal-order case.

In the case of IDC-JCM, Buzek [7] has shown that with a squeezed input the normal-order squeezing revives periodically with a period independent of the value of r. We have seen, on the other hand, that the revival periods of AS squeezing depend on r. We also find that the two types of squeezing are equally persistent and that their degrees of squeezing are comparable.

Finally, for the sake of completeness we refer to the work of the authors of Ref. [14]. Kien, Kozierowski, and Quang [14] have studied fourth-order squeezing as given by Hong and Mandel in the multiphoton JCM with a coherent input. Although their analysis mainly focuses on the short time behavior, using low and moderate intensity inputs ($\bar{n} \leq 4$) they point out that this kind of squeezing recurs at the long time scale. Gerry and Moyer [14] give a somewhat brief discussion of higher-order squeezing in one- and two-photon JCM with the field initially in a coherent state. With $\bar{n}=10$, they find that the AS squeezing does occur once but for an extended duration of time. Their observations are in general agreement with the work reported here in which we have used squeezed rather than coherent inputs.

V. SUMMARY AND CONCLUSIONS

In this paper we have examined the AS squeezing behavior of three different models for a broad range of input intensities ranging from low values (low enough to have appreciable degree of AS squeezing greater than or equal to 5%) to high values (high enough to cause revocation [22] of AS squeezing) assuming that our field inputs have squeezing characteristics. For the AOM, we have studied the dependence of our results on the input field intensity and the squeezing direction. We found that as long as the initial average photon number is small, we see genuinely periodic revivals of the AS squeezing in the two quadratures with one or more revival periods. As we increase the value of $\bar{n}$, these oscillations turn into sharp periodic spikes reaching below the SQL. With still larger $\bar{n}$ the squeezing appears for extremely short moments. This effect is more pronounced with a squeezed input than a coherent one. We note also the important fact that an increase in the value of ϕ leads to an increase in the AS squeezing. The AOM more or less mimics the IDC-JCM in many aspects of the AS squeezing.

The JCM radically differs from the AOM and IDC-JCM in the $\bar{n}$ dependence of its squeezing behavior. The AS squeezing is promptly and completely revoked for any but low or moderate input field intensities in contrast to the continued existence of periodic revivals of the other two models.

ACKNOWLEDGMENT

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