Establishment of an entangled atom-field state in the Jaynes-Cummings model

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(Received 21 May 1991)

We use the eigenstates of the reduced density operators to give an explicit form of the entangled atom-field state in the Jaynes-Cummings model. We relate the expansion coefficients to physical quantities and discuss the effect of atomic coherence on this entangled state.

PACS number(s): 42.50.−p, 03.65.−w, 42.52.+x

I. INTRODUCTION

The Jaynes-Cummings model (JCM) of quantum optical resonance is an important fundamental theoretical model of the interaction between two dissimilar quantum systems. The model consists of a single quantized field mode interacting with a single two-level atom [1]. Although this model is of obvious theoretical significance its interest is not solely confined to theoretical speculation. Recent pioneering experiments have demonstrated this most fundamental of interactions in the laboratory [2]. This deceptively simple model has long been a testing ground for theoretical concepts and it is only a matter of time before some of the more complex features of the model are subjected to experimental tests. One of the most interesting features of the model is the correlation which develops between the atom and field during the interaction. This correlation is responsible for the interesting properties observed in the evolution of the micromaser, where information concerning the field is inferred by measurement of atomic properties. In this short paper we give, explicitly, the form of the entangled atom-field state as it evolves under the Jaynes-Cummings Hamiltonian. It is the establishment of entanglement between the atom and field states which leads to the strong correlation between the atomic and field properties.

The JCM (within the rotating-wave approximation) is one of the few exactly soluble models in quantum mechanics. Its apparent simplicity belies the fact that the evolution of physical quantities in the model, such as the atomic inversion, display an extraordinary complexity. Of particular interest are the so-called "collapse" and "revival" regions [3] which give a clear, unambiguous signature of the quantum nature of a coherent field [4]. At first sight one may expect that the field and atom most closely return to their initial states at the peak of the revival. However, the work of Phoenix and Knight [5] and the more recent work of Gea-Banacloche [6] has shown that this expectation is incorrect and that the atom and field most closely return to pure states during the collapse region. Gea-Banacloche has given an approximate form for the atomic state at this time and has shown that this state is reached *regardless* of the initial conditions [6]. We give here an explicit form for the entangled atom-field state which can be used to determine the atom and the field states without any of the approximations used by Gea-Banacloche.

II. DIAGONALIZATION OF THE REDUCED DENSITY OPERATORS

The Jaynes-Cummings Hamiltonian, on resonance and in the rotating-wave approximation, is given by

$$H=\omega\hat{a}^{\dagger}\hat{a}+\tfrac{1}{2}\omega\hat{\sigma}_3+\lambda(\hat{a}^{\dagger}\hat{\sigma}_-+\hat{a}\hat{\sigma}_+)\ , \tag{2.1}$$

where ω is the atomic transition frequency, λ is the atom-field coupling constant, $\hat{a}$ is the field annihilation operator, $\hat{\sigma}_3$ is the atomic inversion operator, $\hat{\sigma}_\pm$ are the atomic "spin-flip" operators, and we have set $\hbar$ equal to unity for convenience. Using standard techniques it can be shown [5] that this Hamiltonian gives rise to the following time-evolution operator in the interaction picture

$$\hat{U}(t)=\begin{bmatrix} \cos[\lambda t(\hat{a}\hat{a}^{\dagger})^{1/2}] & -i\hat{a}(\hat{a}^{\dagger}\hat{a})^{-1/2}\sin[\lambda t(\hat{a}^{\dagger}\hat{a})^{1/2}] \\ -i\hat{a}^{\dagger}(\hat{a}\hat{a}^{\dagger})^{-1/2}\sin[\lambda t(\hat{a}\hat{a}^{\dagger})^{1/2}] & \cos[\lambda t(\hat{a}^{\dagger}\hat{a})^{1/2}] \end{bmatrix}\ , \tag{2.2}$$

where we have written $\hat{U}$ in the atomic basis. We shall assume that the initial atom-field state is a pure state and that the atom and field are initially uncorrelated with the atom being fully inverted. The initial atom-field state can therefore be written as

$$|\Psi_{af}(0)\rangle=|\phi_f\rangle\otimes|e\rangle\ . \tag{2.3}$$

The time-evolved atom-field state is then given by

$$|\Psi_{af}(t)\rangle=\hat{C}|\phi_f\rangle\otimes|e\rangle+\hat{S}|\phi_f\rangle\otimes|g\rangle\ , \tag{2.4}$$

where we have written the operators

$$\hat{C}=\cos[\lambda t(\hat{a}\hat{a}^\dagger)^{1/2}]\ ,$$
$$\hat{S}=-i\hat{a}^\dagger(\hat{a}\hat{a}^\dagger)^{-1/2}\sin[\lambda t(\hat{a}\hat{a}^\dagger)^{1/2}]\ . \tag{2.5}$$

The field density operator, $\rho_f=\mathrm{Tr}_a\rho$, evolves according to

$$\rho_f(t)=|C\rangle\langle C|+|S\rangle\langle S|\ , \tag{2.6}$$

where we have written the field states $|C\rangle=\hat{C}|\phi_f\rangle$ and $|S\rangle=\hat{S}|\phi_f\rangle$. An eigenstate of the field-reduced density operator must be of the form

$$|\psi\rangle=\mu|C\rangle+\nu|S\rangle\ , \tag{2.7}$$

so that an eigenvalue π must satisfy the requirement

$$\pi=\left[\langle C|C\rangle+\frac{\nu}{\mu}\langle C|S\rangle\right]=\left[\langle S|S\rangle+\frac{\mu}{\nu}\langle S|C\rangle\right]\ . \tag{2.8}$$

If we make the substitutions [5]

$$\langle C|S\rangle=|\langle C|S\rangle|\exp(i\phi)\ ,$$
$$\Omega=\frac{1}{|\langle C|S\rangle|}(\langle C|C\rangle-\langle S|S\rangle)\ ,$$
$$\theta=\sinh^{-1}(\Omega/2)\ ,$$
$$\mu=\exp(\pm\theta/2)\exp(i\phi/2)\ ,$$
$$\nu=\pm\exp(\mp\theta/2)\exp(-i\phi/2)\ , \tag{2.9}$$

then the eigenvalues and eigenstates of the field-density operator are given by

$$\pi^{(\pm)}=\langle C|C\rangle\pm e^{\mp\theta}|\langle C|S\rangle|$$
$$=\langle S|S\rangle\pm e^{\pm\theta}|\langle C|S\rangle|\ ,$$
$$|\psi_f^{(\pm)}\rangle=\frac{1}{(2\pi^{(\pm)}\cosh\theta)^{1/2}}(e^{(1/2)(i\phi\pm\theta)}|C\rangle \pm e^{-(1/2)(i\phi\pm\theta)}|S\rangle)\ . \tag{2.10}$$

The eigenvalues and eigenstates of the atomic density operator can also be determined by similar methods. We find that the eigenvalues for the field- and atomic-density operators are *identical* so that both density operators have the same eigenvalue spectrum. This is a general theorem for pure states of two-component quantum systems [7], as we demonstrate below. The eigenstates of the atomic reduced density operator are given by

$$|\psi_a^{(\pm)}\rangle=\frac{1}{\sqrt{2\cosh\theta}}(e^{-(1/2)(i\phi\mp\theta)}|e\rangle\pm e^{(1/2)(i\phi\mp\theta)}|g\rangle)\ . \tag{2.11}$$

The eigenvalues can also be described purely in terms of field or atomic operators. The expressions involving field operators are a little more difficult to interpret. The expression for the eigenvalues can be written as

$$\pi^{(\pm)}=\tfrac{1}{2}[1\pm(\langle\hat{\sigma}_3\rangle^2+4\langle\hat{\sigma}_+\rangle\langle\hat{\sigma}_-\rangle)^{1/2}]\ . \tag{2.12}$$

We shall see in Sec. III that these eigenvalues determine the amount of entanglement between the field and atomic states in the JCM and therefore the degree of correlation between the atom and the field.

The atom- and field-density operators have both been diagonalized by the procedure sketched above. One remarkable feature of this technique is that it reveals the fundamental property of the field in the JCM; it can at all times be described by *just two quantum states.* This is a result of a deeper relation concerning the entropies of the atom and field. The entropies of the atom and field are defined through their respective reduced density operators by

$$S_j=-\mathrm{Tr}_j(\rho_j\ln\rho_j) \tag{2.13}$$

and the subscript j is taken to imply either the field, atom, or the complete atom-field system. The entropies of a general two-component quantum system are linked by a remarkable theorem due to Araki and Lieb [8] which states

$$|S_a-S_f|\le S\le S_a+S_f\ . \tag{2.14}$$

The subscripts "a" and "f" here denote any two general quantum systems, which, for our purposes, are the atom and field. The total entropy of the complete a-f system is denoted by S. One immediate consequence of this inequality is that if the total system is prepared in a pure state then the component systems have equal entropies throughout their subsequent evolution. The atom and field entropies in the JCM (with pure state initial conditions) are therefore identical. This has been explicitly demonstrated for both the single-photon JCM [5] and two-photon generalizations of the model [9].

The semiclassical JCM in which the field is described as a monochromatic excitation can be solved nonperturbatively to give sinusoidal Rabi oscillations [10] in the atomic-level occupation probability. It is well known that these oscillations are recovered in the JCM if the field is initially prepared in a number state. However, the state which most closely resembles the stable monochromatic excitation of the semiclassical approximation is the coherent state. If the field in the JCM is prepared in such a state the individual number states in the number-state expansion of the coherent state give rise to Rabi oscillations at incommensurate frequencies. These oscillations interfere to produce a complicated beat structure in the evolution of the atomic inversion. We plot the atomic inversion and the entropy of the field (or atom) for various values of the initial mean photon number in Figs. 1–3. We have assumed that the field is initially prepared in a coherent state and that the atom is initially fully inverted. We see from these figures that the atom returns most closely to a pure state sometime during the collapse region. As the photon number is increased this approach occurs nearer to the center of the collapse region. However, this is consistent with the result that this pure state is reached at precisely half of the revival time. The degree to which the atom can be said to be pure at this

point also increases with increasing mean photon number. We note that even for $\bar{n}=75$ the approach to a pure state during the collapse is only about 97%. The increase of the purity of that state with increasing intensity has previously been demonstrated by Gea-Banacloche [6]. The measure of purity chosen by Gea-Banacloche is $1-\mathrm{Tr}_a(\rho_a^2)$ and this quantity is zero for a pure state. The entropy used in this present paper is a more sensitive operational measure of the purity of a quantum state in that it automatically includes higher moments of the density operator. The new and important result of Gea-Banacloche is the demonstration that the atom collapses to a unique pure state *regardless* of the initial atomic conditions. We give, in Sec. III, an explicit expression for these states and show that the atomic "attractor" state can be obtained without any of the approximations used by Gea-Banacloche.

III. THE ENTANGLED ATOM-FIELD STATE

In Sec II we showed that the field- and atomic-density operators can be diagonalized in a two-state basis. We used the entropies of the atom and field to parametrize the purity of the states as they evolve in the JCM. In this section we shall give an explicit form of the entangled atom-field state as it evolves under the Jaynes-Cummings Hamiltonian. In particular, we shall examine the degree of correlation that develops between the atom and field and relate this to the establishment of entanglement between the atom and field states.

In the notation described above the atom-field state evolves according to

$$|\Psi_{af}(t)\rangle=|C\rangle\otimes|e\rangle+|S\rangle\otimes|g\rangle\ . \tag{3.1}$$

However, the $|C\rangle$ and $|S\rangle$ states are not normalized and are slightly cumbersome to work with. It is more convenient, and as we shall see physically more relevant, to work with the eigenstates of the density operators. Rewriting the above state (3.1) in terms of the eigenstates we find the simple and appealing result for the entangled atom-field state in the JCM,

$$|\Psi_{af}(t)\rangle=(\pi^{(+)})^{1/2}|\psi_f^{(+)}\rangle\otimes|\psi_a^{(+)}\rangle +(\pi^{(-)})^{1/2}|\psi_f^{(-)}\rangle\otimes|\psi_a^{(-)}\rangle\ . \tag{3.2}$$

Thus the entanglement, and hence the correlation, be-

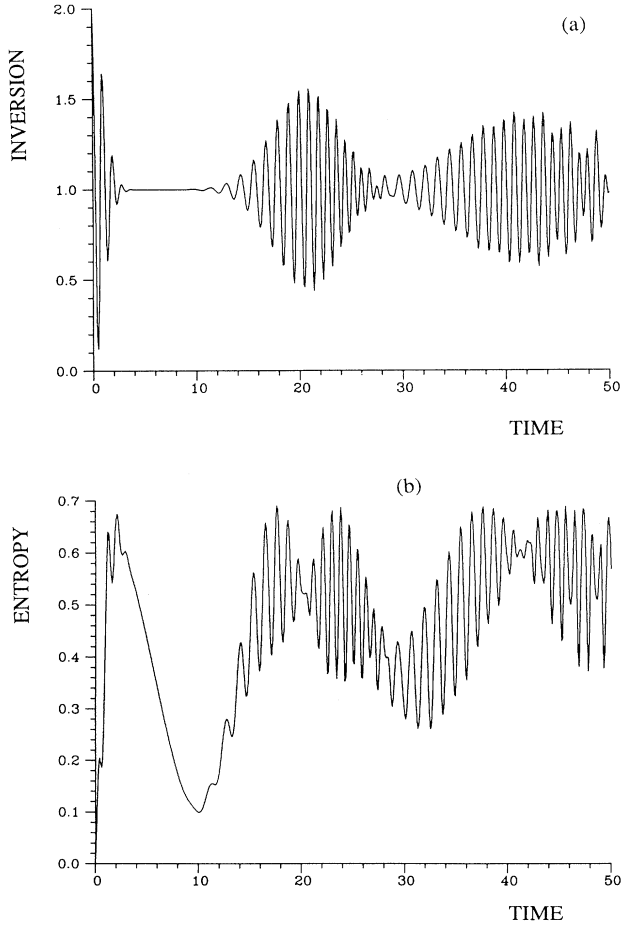

FIG. 1. Plots of (a) the shifted inversion $\langle\hat{\sigma}_3\rangle+1$ and (b) the entropy with the field initially prepared in a coherent state with a mean photon number $\bar{n}=10$. The atom is initially excited and we have set the atom-field coupling constant to unity. Except where noted the time in this and subsequent figures is in seconds.

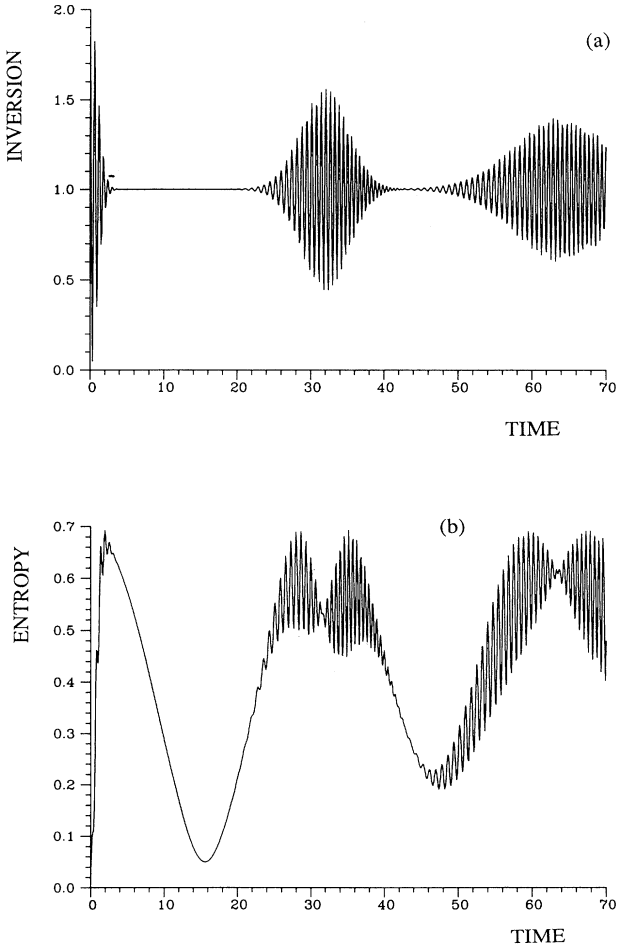

FIG. 2. Same as Fig. 1 with $\bar{n}=25$.

tween the atom and field is determined by the eigenvalues of the density operator. The eigenvalue $\pi^{(-)}$ is plotted in Fig. 4 for the same initial photon numbers considered previously. We see that, to a very good approximation, the entangled state decorrelates during the collapse region and the atom-field state is given approximately by

$$|\Psi_{af}(t_0)\rangle \approx |\psi_f^{(+)}\rangle \otimes |\psi_a^{(+)}\rangle \ . \tag{3.3}$$

The time t_0 is the time at which this pure atomic attractor state is reached and this time has been given by Gea-Banacloche [6], in the high-$\bar{n}$ limit as [6] $t_0=\bar{n}^{1/2}\pi/\lambda$, and is precisely half the time of the peak revival. This is confirmed by Figs. 1–4. These plots also confirm the statement of Gea-Banacloche which shows that during subsequent collapses the atom approaches a pure state at the times $t_q=(2q+1)\bar{n}^{1/2}\pi/\lambda$, with $q=0,1,2,\ldots$, although the approach becomes progressively less perfect.

The atomic eigenstates can be rewritten in terms of the eigenvalues and the inversion so that

$$|\psi_a^{(\pm)}\rangle = \frac{e^{-i\phi/2}}{\sqrt{2}}\left[1\pm\frac{\langle\hat{\sigma}_3\rangle}{\pi^{(+)}-\pi^{(-)}}\right]^{1/2}|e\rangle \pm \frac{e^{i\phi/2}}{\sqrt{2}}\left[1\mp\frac{\langle\hat{\sigma}_3\rangle}{\pi^{(+)}-\pi^{(-)}}\right]^{1/2}|g\rangle \ . \tag{3.4}$$

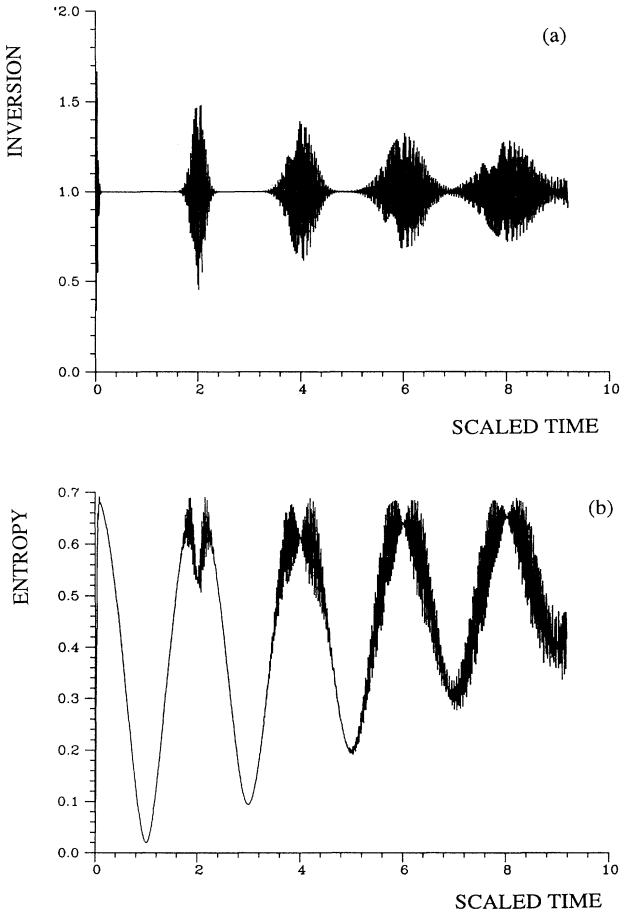

FIG. 3. Same as Fig. 1 but with $\bar{n}=75$. The time has also been scaled into units of half the revival time.

It is easy to show that during the collapse region these eigenstates are given by

$$|\psi_a^{(\pm)}\rangle_{\text{collapse}} = \frac{1}{\sqrt{2}}(e^{-i\phi/2}|e\rangle \pm e^{i\phi/2}|g\rangle) \ . \tag{3.5}$$

At the beginning of the collapse region the atom is in a

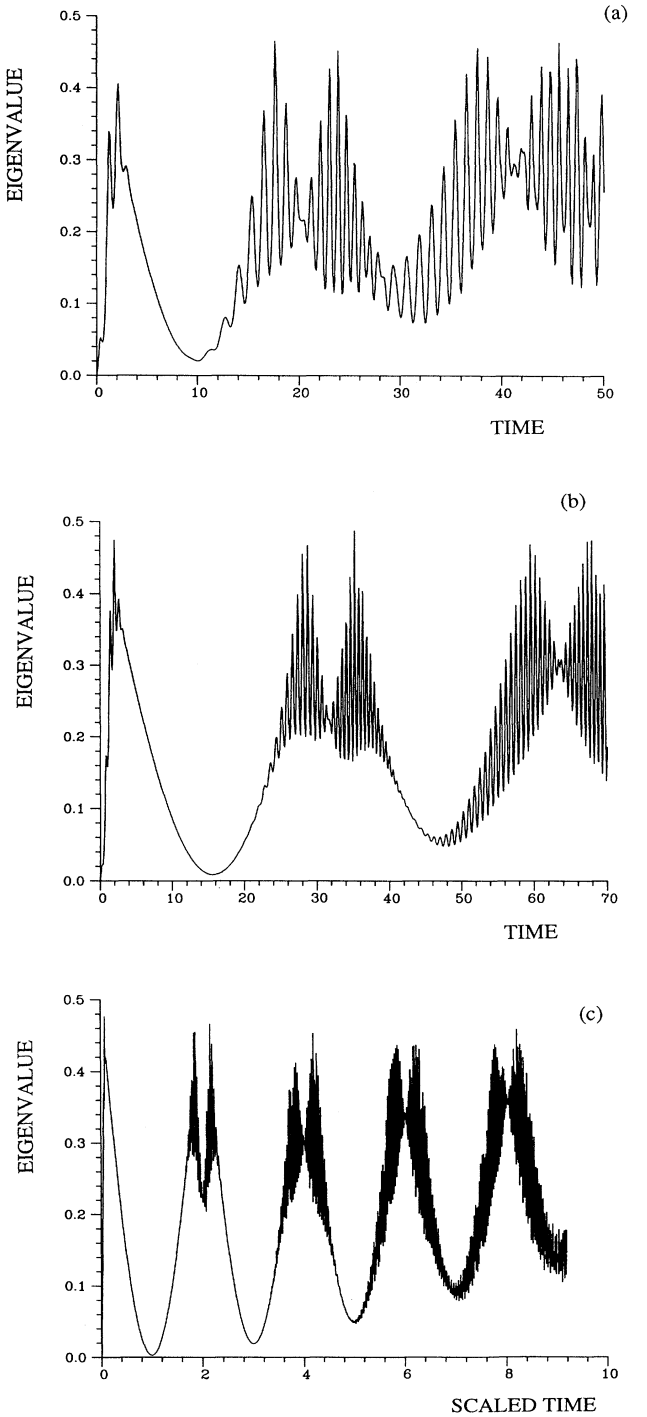

FIG. 4. The eigenvalue $\pi^{(-)}$ for the initial-field photon numbers (a) $\bar{n}=10$, (b) $\bar{n}=25$, (c) $\bar{n}=75$. (c) has been plotted using a time scaled into units of half the revival time.

mixture of the above states. The evolution in the JCM is such, however, that the dynamics eventually force the atom into the state $|\psi_a^{(+)}(t_0)\rangle$. Note that ϕ is, in fact, time dependent and given by the formula

$$\phi(t)=\tan^{-1}\left[\frac{\mathrm{Im}\langle\hat{\sigma}_+\rangle}{\mathrm{Re}\langle\hat{\sigma}_+\rangle}\right] . \tag{3.6}$$

The phases $\phi(t)$ for $\bar{n}=10$ and 50 are plotted in Fig. 5. The phase oscillates between $\pm\pi/2$ being $+\pi/2$ during the collapse region and $-\pi/2$ during the revival. The dipole moment as a function of time exhibits collapses and revivals. The sharp jumps in ϕ occur when the dipole moment goes through zero. As Gea-Banacloche has shown [6] the atom is forced into a *unique* pure state *regardless* of the initial atomic conditions. This state is given by $|\psi_a^{(+)}(t_0)\rangle$ as determined from the above expressions, and we find that, up to a global phase factor, that

$$|\psi_a^{(+)}(t_0)\rangle\approx\frac{1}{\sqrt{2}}(|e\rangle+i|g\rangle) . \tag{3.7}$$

We have thus derived the form of the atomic attractor state without the necessity of approximations previously used [6].

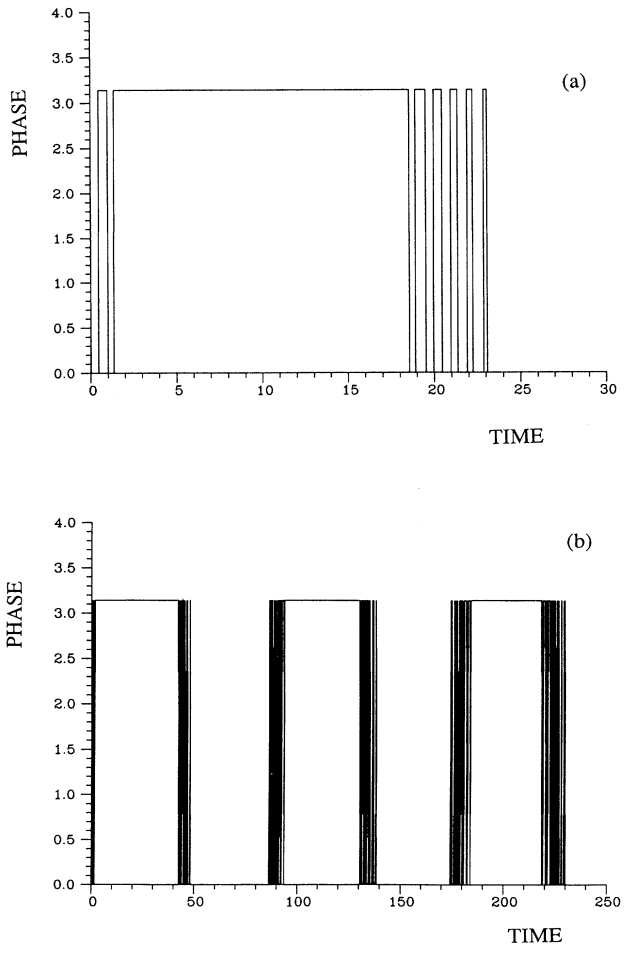

FIG. 5. The shifted phase $\phi(t)+\pi/2$ with the initial conditions (a) $\bar{n}=10$, (b) $\bar{n}=50$.

As has been previously demonstrated [6] *all* initial atomic conditions lead to convergence of the atomic state onto the unique pure state (3.7). Subsequent evolution from this attractor state depends upon the choice of initial condition. It is interesting to examine the stability of the attractor state in terms of phase-space trajectories on the Bloch sphere. It is important to demonstrate that this method is also capable of dealing with the effects of coherence between the atomic levels and we shall examine this issue in Sec. IV.

An observable-independent measure of the amount of correlation between two quantum states is the so-called index of correlation [11,12]. This measure gives the information content of that correlation; failure to measure joint properties of the correlated systems will result in a loss of exactly this amount of information. The index of correlation is given by [11,12]

$$I_c=S_a+S_f-S , \tag{3.8}$$

where the entropies occurring in this expression are defined by (2.13). It has been shown [11,12] that an optimally correlated quantum state of a two-component system is pure in which each of the component systems display maximal, and equal, disorder, subject to any physical constraints. The maximum value that this index can take is $I_c^{\max}=2S_a^{\max}$, where "a" refers to the system with fewer states. Thus in the JCM we have that, for pure initial conditions, $I_c=2S_a$ [of course, we must remember that the Araki-Lieb inequality (2.14) imposes the condition that for an initially pure atom-field state, $S_a=S_f$; we use the atomic entropy here to remind us that the entropy is a two-state quantity under these conditions]. The atom-field state is maximally correlated when $S_a=\ln 2$. The entropy plots in Figs. 1–3 are equivalent to the index of correlation (to within the trivial factor of 2), and can thus be interpreted as giving the evolution of the correlation between the atom and field. We see that there are certain times at which the atom and field are more strongly correlated. At these times the information that can be obtained *about the field* from measurements of atomic properties is correspondingly greater. The index of correlation thus allows one to determine the best cavity "time-of-flight" in micromaser experiments in order to yield maximum information about the field. The eigenstate basis is thus the correct basis in which to determine the overall correlation between the atom and field. It is important to note that any measurement of the field by measurement of the atom will change the state of the field. One could then suppose that sending atoms through a micromaser cavity for a time t_0 would then disturb the field least. However, we would need to make more measurements in order to yield as much information at these times as can be obtained from measurements made on atoms with a different flight time. It is the entanglement which determines at which points maximal information about the field can be obtained from atomic properties, and it is the eigenvalues of the density operators which determine the entanglement. As a final point we note that for long times the atom and field become extremely entangled so that maximal correlations are established as $t\rightarrow\infty$.

IV. THE EFFECT OF ATOMIC COHERENCE

One of the first studies of the effects of atomic coherence in the JCM was by Zaheer and Zubairy [13] who showed that the dynamics of the atom, when prepared in a coherent superposition state, depend upon the relative phase between the field and atomic dipole. Earlier studies had shown that a three-level atom prepared in a coherent superposition of the upper and lower levels could give rise to squeezing in the fluorescence field [14]. The more recent work of Gea-Banacloche [6] has demonstrated that the atomic attractor state is *completely independent of* the initial atomic state. We show here how the methods of the previous sections can be extended to include the case where the atom is initially prepared in a superposition of its upper and lower levels. The extension to mixed states is then trivial.

Let us consider an initial atomic state of the form

$$|\phi_a\rangle = p|e\rangle + q|g\rangle \ , \tag{4.1}$$

with $|p|^2+|q|^2=1$. If we assume that the initial field state is given by $|\phi_f\rangle$ and that the atom and field states are initially uncorrelated, then the time-evolved atom field state is given by

$$|\Psi_{af}(t)\rangle = (p\hat{C}+q\hat{S}')|\phi_f\rangle\otimes|e\rangle + (p\hat{S}+q\hat{C}')|\phi_f\rangle\otimes|g\rangle \ . \tag{4.2a}$$

The primed and unprimed field operators are related by, for example,

$$\hat{C}=\hat{C}(\hat{a},\hat{a}^\dagger), \quad \hat{C}'=\hat{C}(\hat{a}^\dagger,\hat{a}) \ . \tag{4.2b}$$

The primed operators are formed from the unprimed operators by switching the creation and annihilation operators in the unprimed expressions. If we define two new quantum field states by

$$\begin{aligned} |\xi\rangle &= (p\hat{C}+q\hat{S}')|\phi_f\rangle \ , \\ |\eta\rangle &= (p\hat{S}+q\hat{C}')|\phi_f\rangle \ , \end{aligned} \tag{4.3}$$

then all previous results hold provided we make the replacements

$$|C\rangle\rightarrow|\xi\rangle, \quad |S\rangle\rightarrow|\eta\rangle \ . \tag{4.4}$$

Thus, for example, the atomic inversion, with an initial coherent superposition of atomic levels, can be written as

$$\langle\hat{\sigma}_3\rangle = \langle\xi|\xi\rangle - \langle\eta|\eta\rangle \ . \tag{4.5}$$

The dynamics of the new eigenvalues will determine the form of the atomic pure state in the collapse region. The details are essentially similar to the results given in Sec. III.

It is of interest to examine the form of the atomic inversion more closely. We assume that the field is initially prepared in a coherent state of amplitude $\bar{n}\exp(-i\phi_\alpha)$. We shall further use the subscripts ex and g to denote quantities derived from the atom being in the excited and ground states, respectively. If we let p be a real variable and write $q\rightarrow q\exp(i\phi_q)$ then for high $\bar{n}\gg 1$ we can write.

$$\langle\hat{\sigma}_3\rangle \approx (p^2-q^2)\langle\hat{\sigma}_3\rangle_{\rm ex} + \sin(\phi_q-\phi_\alpha)\frac{pq}{\lambda\bar{n}^{1/2}}\frac{d}{dt}\langle\hat{\sigma}_3\rangle_g \ . \tag{4.6}$$

Writing this out explicitly using the Poissonian number statistics for the coherent state we find that this result agrees with that of Zaheer and Zubairy [13]. It is important to note that if the phase difference $\phi_q-\phi_\alpha$ is chosen to be $\pi/2$ then the evolution of the inversion is indistinguishable from that obtained from an initial atomic mixture of the upper and lower levels. It is the phase dependence which is the signature of the coherent superposition. One could then envisage an experiment to test this phase dependence using two micromaser cavities, the first of which prepares the atoms in the pure state (3.7).

V. CONCLUSIONS

The JCM is a fundamental, fully quantized, model of the interaction between a two-level atom and a single-field mode. It is of importance both theoretically and experimentally. Recent work has shown that examination of the inversion alone is not sufficient to give a clear understanding of the properties of this model. We have demonstrated that the entangled atom-field state can be derived and have given the form of this state at all times in terms of the eigenvalues and eigenstates of the reduced density operators. We find that the eigenvalues determine the degree of correlation between the atom and field. The entropy also indicates an optimum cavity flight time for atoms in a micromaser experiment for which to deduce maximal information about the field properties from measurement of the atoms.

ACKNOWLEDGMENT

We would like to thank J. Gea-Banacloche for interesting comments and for sending us copies of a manuscript prior to publication.

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