Amplitude-squared squeezing in the multiphoton Jaynes-Cummings model: Effect of phases

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The phenomenon of amplitude-squared squeezing is investigated for the one- and two-photon Jaynes-Cummings model. The relative phase between the atomic coherent state and the coherent field state is found to affect this phenomenon with larger amounts of squeezing that can be found by changing the angles (θ) for fixed relative phase.

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I. INTRODUCTION

When the atom is prepared initially in a coherent superposition of its upper and lower levels, and its interacts with a single coherent mode, it has been shown recently that the population inversion and field spectrum show dramatic changes with the change in the relative phase between the atomic dipole and the coherent field [1]. For a special choice of the relative phase, the population inversion essentially remains unaffected (i.e., coherent trapping), contrary to the case of incoherent atomic excitation, where collapses and revivals occur [2]. Such a phase-sensitive system has been proposed as a tool to probe the coherence produced in the field by the atom [3]. It is worth mentioning that when the field is taken into the squeezed vacuum state the model does not have a phase sensitivity in this case due to the lack of coherent coupling between the one-photon transition and the "two-photon" squeezed state.

The squeezed state in quantum optics is characterized by the property that the quantum fluctuations in one of the field quadratures are smaller than those associated with the vacuum [4]. There are a number of nonlinear systems that are capable of producing squeezed fields [5]. One of these systems is the Jaynes-Cummings model (JCM) [6]. This model has been investigated and it was found to produce squeezed states for the field when the input field has a coherent, binomial, or logarithmic distribution, whether the atom starts in its grounds or excited state [7].

The definition for the field quadrature squeezing (or normal squeezing) has been generalized to Nth-order squeezing [8]. The quantum nature of this definition is implied by the fact that it is only significant for even N. Many systems, such as those producing resonance fluorescence, second-harmonic-generation parametric down-conversion, kth-harmonic-generation multiphoton absorption, anharmonic oscillator, and multiphoton JCM, have been analyzed for higher-order squeezing [8,9]. Another definition has been proposed for higher-order squeezing [10]. This type of squeezing, namely amplitude-squared squeezing, arises in a natural way in second-harmonic generation and in a number of nonlinear optical processes [10]. It should be noted that all the squeezed states produced by the different kinds of higher-order squeezing definitions are nonclassical states. The JCM has been analyzed for the amplitude-squared squeezing when the atom starts in its ground or excited state and coherent input for the field [7(a),9(a),11]. It has been found that the amplitude-squared squeezing occurs, which takes its maximum amount and then decreases during the period of collapse in the collapses and revivals phenomenon of the photon number (or population inversion) [11]. Its amount increases by increasing the initial mean photon number (for $|\alpha|^2 \gg 1$).

In this paper we investigate the amplitude-squared squeezing in the JCM when the atom is supposed to be in a coherent superposition of its two states and the field in a coherent state. In particular, we emphasize the role of the relative phase. In Sec II an evolution operator is used to calculate the density operator for the field under the initial conditions specified above. Once this operator is computed, the expectation operator for any field operator can be easily calculated. The required expectation values for the discussion of amplitude-squared squeezing are given in Sec. III. A discussion of the effects of the change in the phases on the amplitude-squared squeezing comprises the final section.

II. THE DENSITY OPERATOR

The multiphoton Jaynes Cummings model (JCM) describes the interaction of a single-mode quantized electromagnetic field with a two-level atom via a k-photon process. The effective Hamiltonian in the rotating-wave approximation (RWA) [12] is given by

$$H=\tfrac{1}{2}\omega_0\sigma_Z+\omega\hat{a}^\dagger\hat{a}+\lambda[(\hat{a}^\dagger)^k\sigma^-+\sigma^+\hat{a}^k] , \tag{1}$$

where $\hat{a}$ ($\hat{a}^\dagger$) and ω are the annihilation (creation) operator and frequency of the radiation field mode, respectively, and λ is the coupling parameter between the atom and the field. The two-level atom with transition frequency ω is described by the Pauli raising and lower operators σ^+,σ^- and the inversion operator σ_Z. For simplicity, we consider the resonant case (i.e., $k\omega=\omega_0$).

We consider the atom injected into the field in a coherent superposition of its excited and ground states, i.e., in the state $|\theta,\Phi\rangle$ where [1]

$$|\theta,\phi\rangle=\cos(\theta/2)|e\rangle+e^{-i\Phi}\sin(\theta/2)|g\rangle . \tag{2}$$

The states $|e\rangle$ and $|g\rangle$ denote the excited and ground states of the two-level atom, respectively.

The density operator for the system at time $t=0$ is assumed to be decoupled, thus it is given by $\rho(0)=\rho_F(0)\otimes\rho_A(0)$, where $\rho_F(0)$ and $\rho_A(0)$ describe the initial values for the field and the atomic density operator, respectively. Thus by taking the state (2) to describe the atom we get $\rho_A(0)=|\theta,\Phi\rangle\langle\theta,\Phi|$.

The evolution operator $U(t)=\exp(-iHt)$ can be calculated easily [7a,1,12] and at any time $t>0$ the field density operator for the system is given by

$$\begin{aligned}\rho_F(t)&=\mathrm{Tr}_{\mathrm{atom}}(U(t)\rho(0)U^+(t))\\&=\sum_{l,m}p_{ml}e^{-i(m-l)\omega t}\Bigg[\sin^2(\theta/2)[\cos(\lambda t\mu_l)\cos(\lambda t\mu_m)|m\rangle\langle l|\\&\qquad+\sin(\lambda t\mu_l)\sin(\lambda t\mu_m)|m-k\rangle\langle l-k|]\\&\qquad+\cos^2(\theta/2)[\cos(\lambda t\mu_{l+k})\cos(\lambda t\mu_{m+k})|m\rangle\langle l|\\&\qquad+\sin(\lambda t\mu_{l+k})\sin(\lambda t\mu_{m+k})|m+k\rangle\langle l+k|]\\&\qquad+\frac{i}{2}\sin(\theta)e^{i(\Phi-k\omega t)}[\sin(\lambda t\mu_l)\cos(\lambda t\mu_{m+k})|m\rangle\langle l-k|\\&\qquad-\cos(\lambda t\mu_l)\sin(\lambda t\mu_{m+k})|m+k\rangle\langle l|]\\&\qquad+\frac{i}{2}\sin(\theta)e^{-i(\Phi-k\omega t)}[\sin(\lambda t\mu_{l+k})\cos(\lambda t\mu_m)|m\rangle\langle l+k|)\\&\qquad-\cos(\lambda t\mu_{l+k})\sin(\lambda t\mu_m)|m-k\rangle\langle l|]\Bigg] ,\end{aligned} \tag{3}$$

where $\mu_s^2=s!/(s-k)!$. When we set $\theta=0$ ($\theta=\pi$), we get the case of the atom in its excited (ground) state that is discussed in Ref. [12].

Let the field be initially in a coherent state $|\alpha\rangle$ given by

$$|\alpha\rangle=e^{-N/2}\sum_n\frac{\alpha^n}{\sqrt{n!}}|n\rangle , \quad \alpha=\sqrt{N}e^{i\Psi} \tag{4}$$

with the average photon number $N=|\alpha|^2$. The initial density operator $\rho_F(0)=|\alpha\rangle\langle\alpha|$ with its matrix element $\langle n|\rho_F(0)|l\rangle=p_{nl}$.

Now the expectation values for any function $F(\hat{a},\hat{a}^\dagger)$ can be computed in the usual manner. Hence, many features can be discussed, such as the phenomenon of amplitude-squared squeezing.

III. AMPLITUDE-SQUARED SQUEEZING

We define the operators that represent the real and imaginary parts of the square of the amplitude [10]

$$d_1=\tfrac{1}{2}[\hat{A}^2+(\hat{A}^\dagger)^2] , \quad d_2=\frac{1}{2i}[\hat{A}^2-(\hat{A}^\dagger)^2] , \tag{5}$$

where $\hat{A}=\hat{a}e^{i(\omega t-\Psi)}$ is a slowly varying operator. These operators satisfy the commutation relation

$$[d_1,d_2]=i(2\hat{A}^\dagger\hat{A}+1) , \tag{6}$$

and as a result they satisfy the uncertainty relation

$$\Delta d_1\Delta d_2\geq\langle(\hat{A}^\dagger\hat{A}+\tfrac{1}{2})\rangle . \tag{7}$$

When either of them satisfy the condition

$$(\Delta d_j)^2<\langle(\hat{A}^\dagger\hat{A}+\tfrac{1}{2})\rangle , \quad j=1,2 \tag{8}$$

we say that the field is in an amplitude-squared squeezed state. Condition (8) can be rewritten as S_1 and S_2, which are cast in terms of the expectation values of the field operator in the form

$$\begin{aligned}S_1&=(\Delta d_1)^2-\langle(\hat{A}^\dagger\hat{A}+\tfrac{1}{2})\rangle\\&=\tfrac{1}{4}\{2\langle(\hat{A}^\dagger\hat{A})^2\rangle-2\langle\hat{A}^\dagger\hat{A}\rangle+\langle\hat{A}^4\rangle+\langle(\hat{A}^\dagger)^4\rangle\\&\qquad-[\langle\hat{A}^2\rangle+\langle(\hat{A}^\dagger)^2\rangle]^2\}\end{aligned} \tag{9}$$

and

$$S_2=(\Delta d_2)^2-\langle(\hat{A}^\dagger\hat{A}+\tfrac{1}{2})\rangle =\frac{1}{4}\{2\langle(\hat{A}^\dagger\hat{A})^2\rangle-2\langle\hat{A}^\dagger\hat{A}\rangle-\langle\hat{A}^4\rangle-\langle(\hat{A}^\dagger)^4\rangle +[\langle\hat{A}^2\rangle-\langle(\hat{A}^\dagger)^2\rangle]^2\} \tag{10}$$

and squeezing occurs when $S_{1,2}<0$.

The expectation values of the field operator can be obtained by using Eq. (3). For example, the time evolution $(\hat{A}^\dagger)^S\hat{A}^r$ is given by

$$\begin{aligned}\langle(\hat{A}^\dagger)^S\hat{A}^r\rangle=|\alpha|^{(s+r)}\sum_{n=s}p_{n-s}\Bigg[&\cos^2(\theta/2)\Bigg[\cos(\lambda t\mu_{n+k})\cos(\lambda t\mu_{n+r-s+k}) +\mu_{n+k}\mu_{n+r-s+k}\sin(\lambda t\mu_{n+r-s+k})\frac{\sin(\lambda t\mu_{n+k})}{\mu_{n-s+k}^2}\Bigg]\\ &+\sin^2(\theta/2)\Bigg[\cos(\lambda t\mu_n)\cos(\lambda t\mu_{n+r-s})+\mu_{n-s}^2\sin(\lambda t\mu_n)\frac{\sin(\lambda t\mu_{n+r-s})}{\mu_n\mu_{n+r-s}}\Bigg]\\ &+\frac{i}{2}|a|^{-k}\sin(\theta)\exp(i\beta_k)\Bigg[\mu_{n-s}^2\cos(\lambda t\mu_{n+r-s})\frac{\sin(\lambda t\mu_n)}{\mu_n} -\mu_{n+r-s}\sin(\lambda t\mu_{n+r-s})\cos(\lambda t\mu_n)\Bigg]\\ &+\frac{i}{2}|a|^{k}\sin(\theta)\exp(-i\beta_k)\Bigg[\mu_{n+k}\cos(\lambda t\mu_{n+r-s+k})\frac{\sin(\lambda t\mu_{n+k})}{\mu_{n-s+k}^2} -\cos(\lambda t\mu_{n+k})\frac{\sin(\lambda t\mu_{n+r-s+k})}{\mu_{n+r-s+k}}\Bigg]\Bigg],\end{aligned} \tag{11}$$

where $p_n=p_{nn}$, and $\beta_k=(\Phi-k\Psi)$ is the relative phase.

Below, we discuss the temporal behavior of S_1 when we take $N=10$, and different values of θ and β_k.

IV. DISCUSSION AND CONCLUSION

From Eq (11) it is noted that choosing the coherent atomic state to represent the initial state for the atom introduces terms that depend on the relative phase between the atomic state [phase (Φ)] and the field coherent state [phase (Ψ)]. For the one-photon model (standard JCM) the population inversion exhibits a phase sensitivity as has been shown recently [1]. Coherent trapping is shown to exist in this very simple model of a two-level atom when (θ) equal $(\pi/2)$ and zero relative phase. Also for the two-photon model, the graphs for the photon number expectation value is shown in Figs. 1 and 2. Figure 1

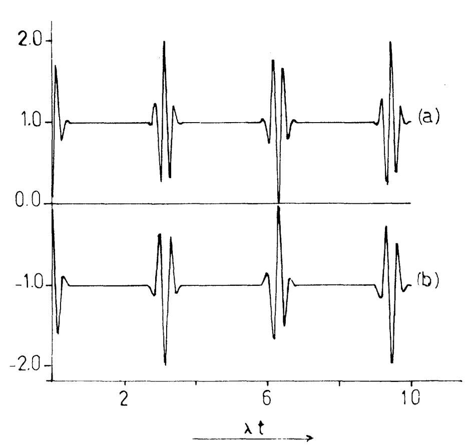

FIG. 1. (a) $\langle\hat{a}^\dagger\hat{a}\rangle-N$ for the two -photon JCM for $\theta=0$; (b) $\langle\hat{a}^\dagger\hat{a}\rangle-N$ the same as in (a) but for $\theta=\pi$.

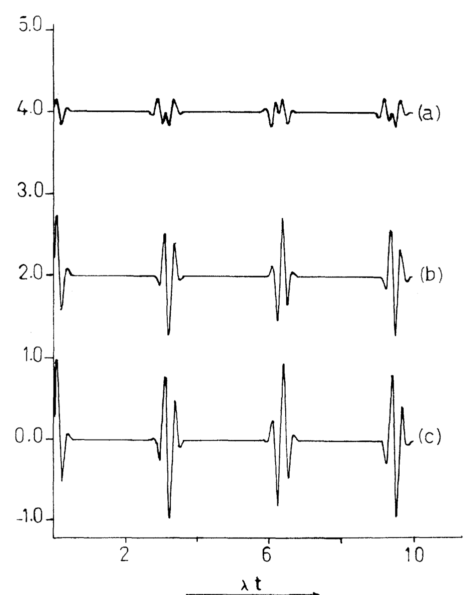

FIG. 2. The time evolution of the photon number $\langle\hat{a}^\dagger\hat{a}\rangle$ for the two-photon JCM, $\theta=\pi/2$, and $N=10$. (a) $\langle\hat{a}^\dagger\hat{a}\rangle-N+4$ for $\beta_2=0$; (b) $\langle\hat{a}^\dagger\hat{a}\rangle-N+2$ for $\beta_2=\pi/4$; (c) $\langle\hat{a}^\dagger\hat{a}\rangle-N$ for $\beta_2=\pi/2$.

shows the collapses and revivals phenomenon for the ground and excited incoherent states of the atom [i.e., $\theta=0$ (excited) shown in Fig. 1(a) and $\theta=\pi$ (ground) shown in Fig. 1(b)]. The periodicity is noted in this case. The effect of the relative phase is shown in Fig. 2 when $\theta=\pi/2$

We note that at β_2 there is almost a coherent trapping and the change in the photon number is almost unnoticed in comparison with the case of the characteristic collapse and revival pictures shown in Fig. 1. As β_2 increases, we note that the collapses and revivals phenomenon starts to appear in a pronounced way and takes its full feature when the relative phase equals $\pi/2$ in this case.

The numerical results are shown in Figs 3–8 for the different values of (θ), the relative phase β_k, and $N=10$. In Figs 3 and 4 we have plotted the squeezing parameter S_1 against time (for $0 \leq \lambda t \leq 10$) for one photon $(k=1)$, while in Figs. 5–8 we have plotted the squeezing parameter S_1 against time for the two-photon processes $(k=2)$.

In Figs. 3(a) and 3(b), the two special cases of $\theta=0$ (the excited atomic state) and $\theta=\pi$ (the ground atomic state) [7a,1] are reproduced for later comparisons.

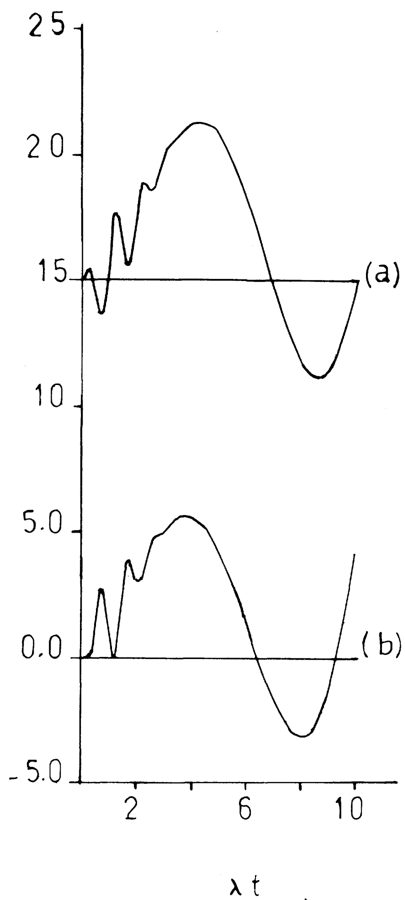

FIG. 3. The time evolution of amplitude-squared squeezing parameter S_1 for the one-photon JCM and for $N=10$. (a) S_1+15 for $\theta=0$ (excited incoherent atomic state); (b) S_1 for $\theta=\pi$ (ground state).

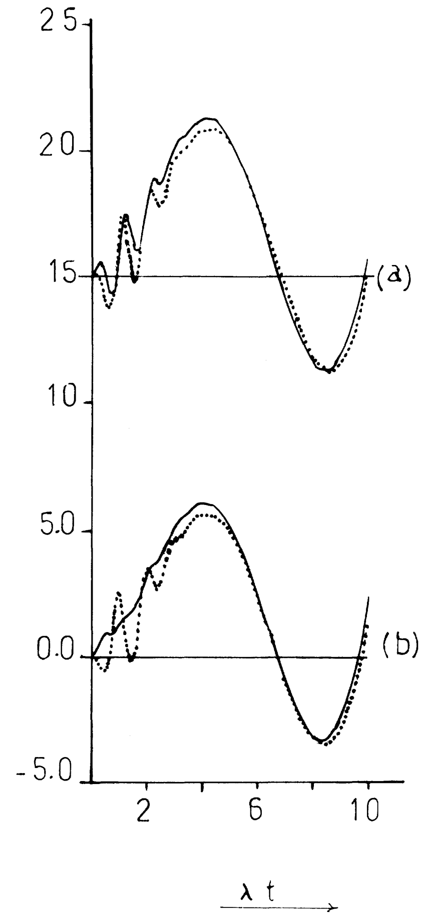

FIG. 4. The temporal behavior of S_1 for $N=10$ and the one-photon JCM. (a) (S_1+15) for $\theta=\pi/4$; ——, for $\beta_1=0$; and · · · ·, for $\beta_1=\pi/2$; (b) S_1 for $\theta=\pi/2$ and the same parameters and notation of (a).

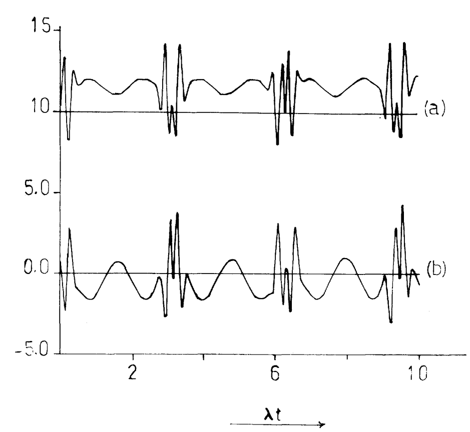

FIG. 5. S_1 for the two-photon JCM and $N=10$. (a) S_1+10 for $\theta=0$; (b) S_1 for $\theta=\pi$.

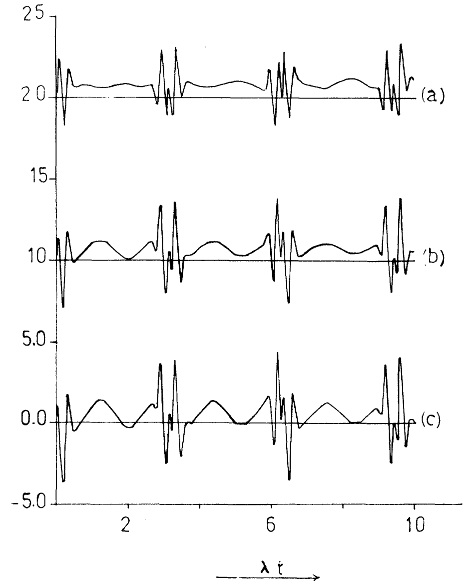

FIG. 6. The time evolution of S_1 for two-photon JCM, $\theta=\pi/4$, and $N=10$. (a) S_1+20 and $\beta_2=0$; (b) S_1+10 and $\beta_2=\pi/4$; (c) S_1 and $\beta_2=\pi/2$.

When $\theta=\pi/4$ [Fig. 4(a)], two cases for β_1, namely 0 and $\pi/2$, are considered. It is observed that at $\beta_1=0$, the amount of squeezing is a little smaller than the case of Fig. 3(a). When $\beta_1=\pi/2$, the curve oscillates. It starts with squeezing, and squeezing occurs a short time later and finally attains its largest value later.

In Fig. 4(b), we have the case $\theta=\pi/2$ and the two above-mentioned values for β_1. The collapses and revivals phenomenon has been discussed earlier [1]. When $\beta_1=0$, we obtain no squeezing on the short-time interval.

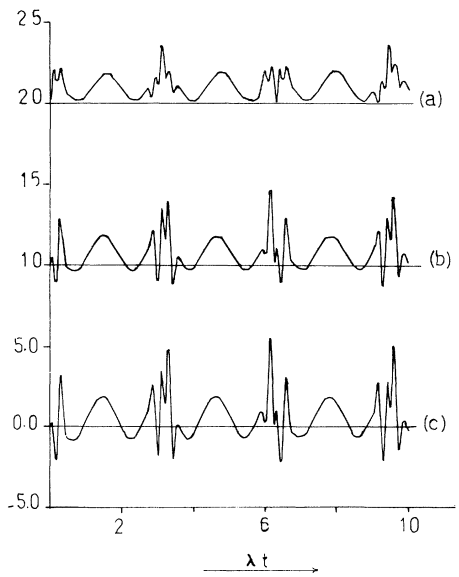

FIG. 7. The same as in Fig. 4, but with $\theta=\pi/2$.

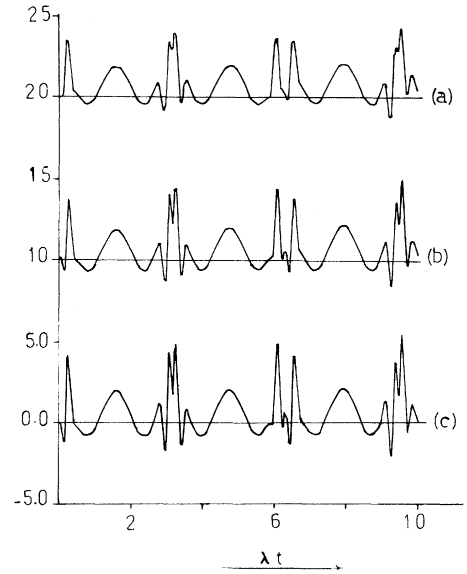

FIG. 8. The same as in Fig. 4, but with $\theta=3\pi/4$.

But when $\beta_1=\pi/2$, we find that the curve oscillates and a little small squeezing occurs in the short time. We notice that when $\theta=\pi/4$ and $\pi/2$, amplitude-squared squeezing for the standard JCM can be increased by increasing the relative phase from 0 to $\pi/2$.

In Figs. 5(a) and 5(b), we display the amplitude-squared squeezing results for the two-photon with the atom initially in the excited and ground states, respectively. We note that the squeezing occurs periodically in the short- and long-time intervals in both cases. In Fig. 5(b) we find that the curve oscillates and large squeezing occurs more than in Fig. 5(a) [7a,12]. It is observed that amplitude-squared squeezing occurs during collapse time for the ground state only.

In Figs. 6(a)–6(c), the angle $\theta=\pi/4$ and different values of the relative phase β_2 (namely 0, $\pi/4$ and $\pi/2$) are considered. We note that the squeezing appears in short-time intervals as well as long-time intervals. In this case it is noted that as $\beta_2=0$ [Fig. 6(a)], the amplitude-squared squeezing is not significant, however it does occur during revival periods [compare with Fig. 2(a)]. As the relative phase increases, the squeezing parameter oscillates and squeezing is attained (albeit for a short period) during the collapse time [see Fig. 6(c)]. It is remarked that the squeezing occurs at periods of $(2\pi/\lambda)$ and as time grows larger squeezing decreases however slightly.

The same values for β_2 and $\theta=\pi/2$ are studied in Figs. 7(a)–7(c). We note that the squeezing is lost for $\beta_2=0$; the case of coherent trapping. While at $\beta_2=\pi/4$ or $\pi/2$, the amplitude-squared squeezing occurs frequently on short- and long-time intervals, but with a small amount in comparison with Fig. 5(b).

The case $\theta=3\pi/4$ is studied in Figs. 8(a)–8(c). We observe that (when $\beta_2=0$), a small amount of amplitude-squared squeezing occurs during collapses and revivals periods. By increasing β_2 to $\pi/4$and $\pi/2$, it is noted that the amount of amplitude-squared squeezing increases. The same remark about the periodicity of $(2\pi/\lambda)$ of amplitude-squared squeezing is applicable in this case also.

In conclusion, the amplitude-squared squeezing for one-photon JCM and fixed value $\theta(\neq 0$ or $\pi)$ can be increased by increasing the relative phase β_1 from 0 to $\pi/2$. It is noted that the maximum amount of the squeezing occurs during the collapse time and $\beta_1=\pi/2$. Also, we remark that for the two-photon JCM and a fixed value of the relative phase β_2, the amount of amplitude-squared squeezing increases and occurs more frequently by increasing (θ) from $\pi/4$ to $3\pi/4$. It is found that the largest amount of amplitude-squared squeezing (two-photon JCM) increases by increasing θ from $\pi/4$ to $3\pi/4$ for the relative phase equal $\pi/2$. It is observed that for the two-photon JCM, the amplitude-squared squeezing phenomenon has a periodicity of $(2\pi/\lambda)$.

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