Lasing without inversion in dressed-state lasers

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We study the operational properties of the lasing field generated from strongly driven two-level atoms in a ring cavity by including the field saturation to all orders. Lasing without inversion between bare atomic states can occur for both off-resonant– and resonant-pump fields. In the case of an off-resonant–pump field, lasing occurs at one of the Rabi sidebands, for which there exists population inversion between the relevant dressed-atom–field states. In the case of a resonant-pump field, lasing can occur near either of two Rabi sidebands, for which there is no population inversion between the dressed states; here the gain is due to the atomic coherence between the dressed states. While the operation of the off-resonantly–pumped dressed-state laser is similar to that of an ordinary laser, the operation of the resonantly pumped dressed-state laser exhibits some interesting features, such as (1) mode pushing and (2) larger laser intensity coming from smaller linear gain as the detuning is changed. In order to show such features, the calculations have to go beyond the secular approximation.

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I. INTRODUCTION

Recently there have been considerable interest in studying lasing action without population inversion [1–11]. Traditionally, the concept of lasing action in a cavity is introduced and understood in terms of the population inversion between two transition levels in the active medium. When more than two atomic levels are involved in the lasing transition, however, it is possible to achieve lasing without population inversion through interference between different channels [1,2,4,5]. If only two atomic levels are involved in the lasing transition, such interference is impossible. However, one can still achieve lasing without population inversion in a two-level system by using initial atomic coherence between the upper and lower atomic levels [3]. Such a two-level laser is the simplest system for lasing without inversion.

Another kind of system that can exhibit lasing without population inversion is a sample of strongly driven two-level atoms placed within an optical cavity [6–12]. It is well known that the steady-state probe spectrum from a sample of two-level atoms driven by an off-resonant, strong pump field exhibits one emission peak at one Rabi sideband and one absorption peak at another Rabi sideband [13,14]. The lasing action studied in Refs. [7–10] and [12] corresponds to the gain at the emission peak. Since there is no population inversion between the upper and lower atomic states, one obtains lasing without population inversion in this case. Physically, the lasing action coming from the strongly driven two-level atoms can be best understood and explained in terms of the dressed-atom–field states [15], which are the eigenstates of the atoms plus the strong pump field. The lasing actually occurs between such dressed states for which a population inversion exists. In other words, in the dressed-atom picture (DAP), lasing occurs with population inversion. In fact, the cavity frequency is tuned to be resonant with the dressed states, not the bare atomic states in Refs. [7–10]. When a strong pump field is on resonance with a sample of two-level atoms, only the emission peak appears in the transient probe spectrum provided the two-level atoms are prepared initially in a pure dressed state [16]. The dressed-state laser discussed in Ref. [6] corresponds to this situation.

In this paper, we develop a nonlinear theory for lasing action from a sample of driven two-level atoms in a ring cavity, and study the operation of the dressed-state laser. Both off-resonant– and resonant-pump fields of arbitrary strength are considered. For the resonant-pump field, there is no population inversion between the two bare atomic states as in the case of an off-resonant pump. However, in contrast to the off-resonant–pump case, there is no population inversion between the relevant dressed states for a resonant-pump field. We find that one can still obtain lasing action with a resonant-pump field and show that the lasing is due to the atomic coherence between the actual transition (i.e., dressed) states, similar to the situation found in Ref. [3]. Comparing the resonant-pump case with the off-resonant–pump one, we find that, while the operation of the field in the off-resonant–pump case is quite similar to that of an ordinary laser, the resonant-pump case exhibits some interesting characteristics. They include (1) mode pushing instead of the usual mode pulling, and (2) the fact that detuning giving rise to maximum linear gain does not produce the maximum laser intensity. To demonstrate these characteristics, our calculations in the resonant-pump case are carried out beyond the secular approximation.

This paper is organized as follows. In Sec. II we develop a general formalism for the dressed-state laser. In Sec.

III we study the off-resonant–pump case within the secular approximation, and in Sec. IV we examine the resonant-pump case beyond the secular approximation. Finally, Sec. V contains a discussion

II. GENERAL FORMALISM

We consider a system of N two-level atoms interacting with an external pump field and a cavity field. A two-level atom, say the jth one, is represented by its ground state $|1^j\rangle$, an excited state $|2^j\rangle$, the atomic transition frequency ω_{21}, and its position $\mathbf{r}_j$. The external pump field is treated classically in our calculations and is characterized by its frequency ω_L and wave vector $\mathbf{k}_L$. The cavity field, with the passive cavity frequency ω_c and the wave vector $\mathbf{k}_c$, can be treated as quantized: a and $a^\dagger$ are, respectively, the annihilation and creation operators for the cavity mode, which satisfy the commutation relation $[a,a^\dagger]=1$. The total Hamiltonian H for the atoms and the cavity field can be written as

$$H/\hbar=\omega_c a^\dagger a+\sum_j \tfrac{1}{2}\omega_{21}\sigma_z^j-\sum_j \tfrac{1}{2}(g e^{i\mathbf{k}_c\cdot\mathbf{r}_j}\sigma^{j\dagger}a+g^* e^{-i\mathbf{k}_c\cdot\mathbf{r}_j}a^\dagger\sigma^j)-\sum_j \tfrac{1}{2}(\chi e^{i(\mathbf{k}_L\cdot\mathbf{r}_j-\omega_L t)}\sigma^{j\dagger}+\chi^* e^{-i(\mathbf{k}_L\cdot\mathbf{r}_j-\omega_L t)}\sigma^j) , \quad (2.1)$$

where $\sigma^j=|1^j\rangle\langle 2^j|$, $\sigma^{j\dagger}=|2^j\rangle\langle 1^j|$, and $\sigma_z^j=|2^j\rangle\langle 2^j|-|1^j\rangle\langle 1^j|$ are the atomic lowering, raising, and population-difference operators for the jth atom, respectively, g is the coupling constant between the atoms and the cavity mode, and χ is the Rabi frequency between the atoms and the external pump field.

Using the Heisenberg equation of motion and Hamiltonian (2.1), we obtain the following equations of motion for the cavity field and for the atoms after taking expectation values:

$$\langle \dot a\rangle=-(\tfrac{1}{2}\gamma_c+i\omega_c)\langle a\rangle+\tfrac{1}{2}i\sum_j g^* e^{-i\mathbf{k}_c\cdot\mathbf{r}_j}\langle\sigma^j\rangle , \quad (2.2a)$$

$$\langle\dot\sigma^j\rangle=-(\gamma_{21}+i\omega_{21})\langle\sigma^j\rangle-\tfrac{1}{2}ige^{i\mathbf{k}_c\cdot\mathbf{r}_j}\langle\sigma_z^j a\rangle-\tfrac{1}{2}i\chi e^{i(\mathbf{k}_L\cdot\mathbf{r}_j-\omega_L t)}\langle\sigma_z^j\rangle , \quad (2.2b)$$

$$\langle\dot\sigma_z^j\rangle=-\gamma_2(\langle\sigma_z^j\rangle+1)+ige^{i\mathbf{k}_c\cdot\mathbf{r}_j}\langle\sigma^{j\dagger}a\rangle-ig^*e^{-i\mathbf{k}_c\cdot\mathbf{r}_j}\langle a^\dagger\sigma^j\rangle+i\chi e^{i(\mathbf{k}_L\cdot\mathbf{r}_j-\omega_L t)}\langle\sigma^{j\dagger}\rangle-i\chi^* e^{-i(\mathbf{k}_L\cdot\mathbf{r}_j-\omega_L t)}\langle\sigma^j\rangle . \quad (2.2c)$$

In writing Eqs. (2.2) we have included the field and atomic relaxation terms; γ_c is the cavity (intensity) loss rate, γ_{21} is the atomic coherence decay rate, and γ_2 is the upper-level–to–lower-level population decay rate. Since the two-level atomic system under consideration is a closed system, we have used the closure theorem $|1^j\rangle\langle 1^j|+|2^j\rangle\langle 2^j|=1$ in obtaining Eq. (2.2c). To remove the fast oscillating terms in Eqs. (2.2), we introduce a rotating frame through the relations

$$a(t)=\tilde a(t)e^{-i\nu_c t} , \quad (2.3a)$$

$$\sigma^j(t)=\tilde\sigma^j(t)e^{i(\mathbf{k}_L\cdot\mathbf{r}_j-\omega_L t)} , \quad (2.3b)$$

where ν_c is the cavity oscillation frequency. After substituting Eqs. (2.3) into Eqs. (2.2), we get the equations of motion for the mean values of the cavity mode and of the atoms in the rotating frame,

$$\frac{d}{dt}\langle\tilde a\rangle=-[\tfrac{1}{2}\gamma_c+i(\omega_c-\nu_c)]\langle\tilde a\rangle+\tfrac{1}{2}i\sum_j g^* e^{i(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}\langle\tilde\sigma^j\rangle , \quad (2.4a)$$

$$\frac{d}{dt}\langle\tilde\sigma^j\rangle=-(\gamma_{21}+i\Delta)\langle\tilde\sigma^j\rangle-\tfrac{1}{2}ige^{-i(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}\langle\sigma_z^j\tilde a\rangle-\tfrac{1}{2}i\chi\langle\sigma_z^j\rangle , \quad (2.4b)$$

$$\frac{d}{dt}\langle\sigma_z^j\rangle=-\gamma_2(\langle\sigma_z^j\rangle+1)+ige^{-i(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}\langle\tilde\sigma^{j\dagger}\tilde a\rangle-ig^*e^{i(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}\langle\tilde a^\dagger\tilde\sigma^j\rangle+i\chi\langle\tilde\sigma^{j\dagger}\rangle-i\chi^*\langle\tilde\sigma^j\rangle , \quad (2.4c)$$

where $\delta=\nu_c-\omega_L$, $\mathbf{K}=\mathbf{k}_c-\mathbf{k}_L$, and the atom–pump-field detuning $\Delta=\omega_{21}-\omega_L$.

Our purpose in this paper is to calculate the expectation value $\langle\tilde a\rangle$ for the cavity mode. Thus we need to obtain the atomic coherence $\langle\tilde\sigma^j\rangle$, which could be calculated through Eqs. (2.4b) and (2.4c). However, Eqs. (2.4) are not a closed set of equations in their present form.

Equations (2.4) are reduced to a closed set of equations if we make the semiclassical approximation $\langle\tilde\sigma^{j\dagger}\tilde a\rangle\approx\langle\tilde\sigma^{j\dagger}\rangle\langle\tilde a\rangle$, $\langle\tilde a^\dagger\tilde\sigma^j\rangle\approx\langle\tilde a^\dagger\rangle\langle\tilde\sigma^j\rangle$, which is valid when $|\langle\tilde a\rangle|\gg 1$. By introducing a complex Bloch vector $\underline{R}^j$ for the jth atom,

$$\underline{R}^j=\begin{bmatrix}\langle\tilde\sigma^{j\dagger}\rangle\\ \langle\tilde\sigma^j\rangle\\ \langle\sigma_z^j\rangle\end{bmatrix}=\begin{bmatrix}\tilde\rho_{12}^j\\ \tilde\rho_{21}^j\\ \rho_{22}^j-\rho_{11}^j\end{bmatrix} , \quad (2.5)$$

we can put the optical Bloch equations, consisting of Eq. (2.4b), its complex conjugate, and Eq. (2.4c), into a matrix form:

$$\dot{\underline{R}}^j=(\underline{L}+\underline{S}^j)\underline{R}^j-\underline{\gamma} . \quad (2.6)$$

Here the matrix $\underline{L}$ describes the effects of the pump field (as well as those of the atomic relaxation),

$$\underline{L}=\begin{bmatrix}-\gamma_{21}+i\Delta & 0 & \tfrac{1}{2}i\chi^*\\ 0 & -\gamma_{21}-i\Delta & -\tfrac{1}{2}i\chi\\ i\chi & -i\chi^* & -\gamma_2\end{bmatrix} , \quad (2.7)$$

the matrix $\underline{S}^j$ governs the additional influence of the cavity field,

$$\underline{S}^j=\underline{S}_1 e^{i(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}+\underline{S}_{-1}e^{-i(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}, \quad (2.8a)$$

$$\underline{S}_1=ig^*\langle\tilde{a}^\dagger\rangle\begin{bmatrix}0 & 0 & \frac{1}{2}\\ 0 & 0 & 0\\ 0 & -1 & 0\end{bmatrix}, \quad (2.8b)$$

$$\underline{S}_{-1}=ig\langle\tilde{a}\rangle\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -\frac{1}{2}\\ 1 & 0 & 0\end{bmatrix}, \quad (2.8c)$$

and

$$\underline{\gamma}=\begin{bmatrix}0\\ 0\\ \gamma_2\end{bmatrix}. \quad (2.9)$$

Owing to the time-dependent behavior of the matrix $\underline{S}^j$, the general form of the complex Bloch vector $\underline{R}^j$ can be written as [17,18]

$$\underline{R}^j=\sum_{n=-\infty}^{\infty}\underline{R}^{(n)}e^{in(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}, \quad (2.10)$$

where

$$\underline{R}^{(n)}=\begin{bmatrix}\tilde{\rho}_{12}^{(n)}\\ \tilde{\rho}_{21}^{(n)}\\ \rho_{22}^{(n)}-\rho_{11}^{(n)}\end{bmatrix}. \quad (2.11)$$

In view of Eq. (2.3b), it is easy to see that the atomic coherence $\tilde{\rho}_{21}^{(n)}$ propagates in the direction of $\mathbf{k}_L-n\mathbf{K}=(n+1)\mathbf{k}_L-n\mathbf{k}_c$ with frequency $\omega_L-n\delta=(n+1)\omega_L-n\omega_c$. Upon substitution of Eq. (2.10) into Eq. (2.6), we can obtain an equation of motion for $\underline{R}^{(n)}$, which shows that $\underline{R}^{(n)}$ is independent of the quantity $\mathbf{K}\cdot\mathbf{r}_j$. In other words, $\underline{R}^{(n)}$ is independent of the atomic index j. Since

$$\sum_j e^{-i(n+1)\mathbf{K}\cdot\mathbf{r}_j}=N\delta_{n+1,0}, \quad (2.12)$$

where N is the number of the active atoms in the cavity, we obtain from Eq. (2.4a)

$$\frac{d}{dt}\langle\tilde{a}\rangle=-[\tfrac{1}{2}\gamma_c+i(\omega_c-\nu_c)]\langle\tilde{a}\rangle+\tfrac{1}{2}ig^*N\tilde{\rho}_{21}^{(-1)}, \quad (2.13)$$

which is valid for any value of the detuning δ. Equation (2.13) demonstrates that, in spite of the fact that the pump field may generate many atomic coherence components $\tilde{\rho}_{21}^{(n)}$ that propagate in various directions, only one component, $\tilde{\rho}_{21}^{(-1)}$, that propagates in the $\mathbf{k}_c$ direction can contribute to the buildup and sustenance of the cavity field. This fact is independent of the frequency difference $\delta(\neq 0)$, and whether or not it is one- or two-photon resonance [9].

In what follows we consider the situation where the Rabi frequency $|g\langle\tilde{a}\rangle|$ of the cavity field and the atomic relaxation rates γ_2 and γ_{21} are much smaller than the generalized Rabi frequency of the pump field, i.e., $|g\langle\tilde{a}\rangle|,\gamma_2,\gamma_{21}\ll(|\chi|^2+\Delta^2)^{1/2}\equiv\omega_{BA}$. Such a feature of a strong pump field prompts us to introduce the so-called dressed states, which are the eigenstates of the atom and the pump field [19]. Since the pump field is treated classically in our calculations, we use the semiclassical dressed states [20]. The semiclassical dressed states are

$$|A^j\rangle=\cos(\tfrac{1}{2}\theta)|\tilde{1}^j\rangle+\sin(\tfrac{1}{2}\theta)|\tilde{2}^j\rangle, \quad (2.14a)$$

$$|B^j\rangle=-\sin(\tfrac{1}{2}\theta)|\tilde{1}^j\rangle+\cos(\tfrac{1}{2}\theta)|\tilde{2}^j\rangle, \quad (2.14b)$$

where $|\tilde{1}^j\rangle=|1^j\rangle e^{-i(\mathbf{k}_L\cdot\mathbf{r}_j-\omega_L t)}$, $|\tilde{2}^j\rangle=|2^j\rangle$, and, for simplicity, we have assumed that χ is real and positive, i.e., $\chi>0$. Also

$$\cos\theta=\frac{\Delta}{\omega_{BA}},\quad \sin\theta=\frac{\chi}{\omega_{BA}},\quad 0\le\theta\le\pi. \quad (2.15)$$

We now introduce the atomic lowering, raising, and population-difference operators in the DAP, $\Sigma^j\equiv|A^j\rangle\langle B^j|$, $\Sigma^{j\dagger}\equiv|B^j\rangle\langle A^j|$, and $\Sigma_z^j\equiv|B^j\rangle\langle B^j|-|A^j\rangle\langle A^j|$. Using Eqs. (2.14) it is easy to verify that the complex Bloch vector in the DAP is related to that in the bare-atom picture (BAP) by the relation

$$\underline{\mathcal{R}}^j\equiv\begin{bmatrix}\langle\Sigma^{j\dagger}\rangle\\ \langle\Sigma^j\rangle\\ \langle\Sigma_z^j\rangle\end{bmatrix}=\begin{bmatrix}\tilde{\rho}_{AB}^j\\ \tilde{\rho}_{BA}^j\\ W^j\end{bmatrix}=\underline{U}(\theta)\underline{R}^j, \quad (2.16)$$

where

$$W^j=\rho_{BB}^j-\rho_{AA}^j$$

is the population difference in the DAP, and

$$\underline{U}(\theta)=\begin{bmatrix}\cos^2(\frac{1}{2}\theta) & -\sin^2(\frac{1}{2}\theta) & \frac{1}{2}\sin\theta\\ -\sin^2(\frac{1}{2}\theta) & \cos^2(\frac{1}{2}\theta) & \frac{1}{2}\sin\theta\\ -\sin\theta & -\sin\theta & \cos\theta\end{bmatrix} \quad (2.17)$$

is a transformation matrix. It follows from Eqs. (2.6) and (2.16) that the matrix form of the optical Bloch equations becomes

$$\dot{\underline{\mathcal{R}}}^j=(\underline{\mathcal{L}}+\underline{\mathcal{S}}^j)\underline{\mathcal{R}}^j-\underline{\Gamma} \quad (2.18)$$

in the DAP, where various matrices in the DAP are related to their counterparts in the BAP by the following relations:

$$\underline{\mathcal{L}}=\underline{U}\,\underline{L}\,\underline{U}^{-1}=\begin{bmatrix}-\Gamma_2+i\omega_{BA} & -\frac{1}{2}\gamma_0\sin^2\theta & -\frac{1}{4}\gamma_0\sin(2\theta)\\ -\frac{1}{2}\gamma_0\sin^2\theta & -\Gamma_2-i\omega_{BA} & -\frac{1}{4}\gamma_0\sin(2\theta)\\ -\frac{1}{2}\gamma_0\sin(2\theta) & -\frac{1}{2}\gamma_0\sin(2\theta) & -\Gamma_1\end{bmatrix}, \quad (2.19)$$

$$\underline{\mathcal{S}}^j=\underline{U}\,\underline{S}^j\underline{U}^{-1}=\underline{\mathcal{S}}_1 e^{i(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}+\underline{\mathcal{S}}_{-1}e^{-i(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}, \quad (2.20a)$$

$$\underline{\mathcal{S}}_1=\underline{U}\,\underline{S}_1\underline{U}^{-1}=ig^*\langle\tilde{a}^\dagger\rangle\begin{bmatrix}\frac{1}{2}\sin\theta & 0 & \frac{1}{2}\cos^2(\frac{1}{2}\theta)\\ 0 & -\frac{1}{2}\sin\theta & \frac{1}{2}\sin^2(\frac{1}{2}\theta)\\ -\sin^2(\frac{1}{2}\theta) & -\cos^2(\frac{1}{2}\theta) & 0\end{bmatrix}, \tag{2.20b}$$

$$\underline{\mathcal{S}}_{-1}=\underline{U}\,\underline{\mathcal{S}}_{-1}\underline{U}^{-1}=ig\langle\tilde{a}\rangle\begin{bmatrix}\frac{1}{2}\sin\theta & 0 & -\frac{1}{2}\sin^2(\frac{1}{2}\theta)\\ 0 & -\frac{1}{2}\sin\theta & -\frac{1}{2}\cos^2(\frac{1}{2}\theta)\\ \cos^2(\frac{1}{2}\theta) & \sin^2(\frac{1}{2}\theta) & 0\end{bmatrix}, \tag{2.20c}$$

$$\underline{\Gamma}=\underline{U}\underline{\gamma}=\gamma_2\begin{bmatrix}\frac{1}{2}\sin\theta\\ \frac{1}{2}\sin\theta\\ \cos\theta\end{bmatrix}, \tag{2.21}$$

with

$$\Gamma_1=\gamma_{21}+\gamma_0\cos^2\theta\ , \tag{2.22a}$$

$$\Gamma_2=\gamma_{21}+\tfrac{1}{2}\gamma_0\sin^2\theta=\tfrac{1}{2}(\gamma_2+\gamma_{21}-\gamma_0\cos^2\theta)\ , \tag{2.22b}$$

$$\gamma_0=\gamma_2-\gamma_{21}\ . \tag{2.22c}$$

In Eq. (2.19), Γ_1 and Γ_2 are, respectively, the population- and coherence-relaxation rates in the DAP. In the above equations, the inverse matrix $\underline{U}^{-1}$ is simply given by

$$\underline{U}^{-1}(\theta)=\underline{U}(-\theta)\ . \tag{2.23}$$

Substitution of Eq. (2.10) into Eq. (2.16) leads to

$$\underline{\mathcal{R}}^j=\sum_{n=-\infty}^{\infty}\underline{\mathcal{R}}^{(n)}e^{in(\delta t-\mathbf{K}\cdot\mathbf{r}_j)}\ . \tag{2.24}$$

Here

$$\underline{\mathcal{R}}^{(n)}=\underline{U}(\theta)\underline{R}^{(n)}\ , \tag{2.25}$$

which is similar to Eq. (2.16). The components of $\underline{\mathcal{R}}^{(n)}$ obey the following relations:

$$\rho_{AB}^{(-n)}=(\rho_{BA}^{(n)})^*\ , \tag{2.26a}$$

$$W^{(-n)}=(W^{(n)})^*\ . \tag{2.26b}$$

Using the relation $\tilde{\rho}_{21}^{(-1)}=(0,1,0)\underline{R}^{(-1)}$ and Eq. (2.25), we obtain from Eq. (2.13) the equation of motion for the cavity field expressed in the DAP,

$$\frac{d}{dt}\langle\tilde{a}\rangle=-[\tfrac{1}{2}\gamma_c+i(\omega_c-\nu_c)]\langle\tilde{a}\rangle+\tfrac{1}{2}iNg^*(-\sin^2(\tfrac{1}{2}\theta),\cos^2(\tfrac{1}{2}\theta),-\tfrac{1}{2}\sin\theta)\underline{\mathcal{R}}^{(-1)}\ . \tag{2.27}$$

In the stationary state, $\underline{\mathcal{R}}^{(-1)}$ and all other $\underline{\mathcal{R}}^{(n)}$ are time independent. Substituting Eq. (2.24) into Eqs. (2.18), we obtain a set of recursion relations in the stationary state,

$$in\delta\underline{\mathcal{R}}^{(n)}=\underline{\mathcal{L}}\,\underline{\mathcal{R}}^{(n)}+\underline{\mathcal{S}}_1\underline{\mathcal{R}}^{(n-1)}+\underline{\mathcal{S}}_{-1}\underline{\mathcal{R}}^{(n+1)}-\underline{\Gamma}\delta_{n,0},\quad n=0,\pm1,\pm2,\dots\ . \tag{2.28}$$

Equations (2.27) and (2.28) are the basic equations for calculating the intensity and frequency of the dressed-state laser. In the following we always assume that the generalized Rabi frequency of the pump field is much larger than the Rabi frequency of the cavity field and the atomic relaxation rates, $\omega_{BA}\gg|g\langle\tilde{a}\rangle|,\Gamma_1,\Gamma_2$. Under such conditions we can approximate the matrix $\underline{\mathcal{L}}$ in Eq. (2.19) as a diagonal one by neglecting all off-diagonal elements (all of them are proportional to γ_0). With such an approximation, the recursion relations (2.28) are simplified, yielding for each of the three components of $\underline{\mathcal{R}}^{(n)}$

$$\begin{aligned}[\Gamma_2+i(n\delta-\omega_{BA})]\rho_{AB}^{(n)}=&-\tfrac{1}{2}\gamma_2\delta_{n,0}\sin\theta+\tfrac{1}{2}ig^*\langle\tilde{a}^\dagger\rangle[\rho_{AB}^{(n-1)}\sin\theta+W^{(n-1)}\cos^2(\tfrac{1}{2}\theta)]\\&+\tfrac{1}{2}ig\langle\tilde{a}\rangle[\rho_{AB}^{(n+1)}\sin\theta-W^{(n+1)}\sin^2(\tfrac{1}{2}\theta)]\ ,\end{aligned} \tag{2.29a}$$

$$\begin{aligned}[\Gamma_2+i(n\delta+\omega_{BA})]\rho_{BA}^{(n)}=&-\tfrac{1}{2}\gamma_2\delta_{n,0}\sin\theta-\tfrac{1}{2}ig\langle\tilde{a}\rangle[\rho_{BA}^{(n+1)}\sin\theta+W^{(n+1)}\cos^2(\tfrac{1}{2}\theta)]\\&-\tfrac{1}{2}ig^*\langle\tilde{a}^\dagger\rangle[\rho_{BA}^{(n-1)}\sin\theta-W^{(n-1)}\sin^2(\tfrac{1}{2}\theta)]\ ,\end{aligned} \tag{2.29b}$$

$$(\Gamma_1+in\delta)W^{(n)}=-\gamma_2\delta_{n,0}\cos\theta+ig\langle\tilde{a}\rangle[\rho_{BA}^{(n+1)}\sin^2(\tfrac{1}{2}\theta)+\rho_{AB}^{(n+1)}\cos^2(\tfrac{1}{2}\theta)]$$
$$-ig^*\langle\tilde{a}^\dagger\rangle[\rho_{AB}^{(n-1)}\sin^2(\tfrac{1}{2}\theta)+\rho_{BA}^{(n-1)}\cos^2(\tfrac{1}{2}\theta)] \ . \qquad (2.29c)$$

In Secs. III and IV we study separately the operation of an off-resonantly–pumped and a resonantly pumped dressed-state laser, which correspond to lasing with and without population inversion in the DAP, respectively.

III. OFF-RESONANT–PUMP FIELD

We consider in this section the case in which the external pump field is off-resonant with the two-level atoms, $\Delta\neq 0$, $|\Delta|\gg\gamma_2,\gamma_{21}$. We first examine the result of the pump excitation before the buildup of the cavity field. Setting $\langle\tilde{a}\rangle=0$ in Eqs. (2.29), it is straightforward to see that the components for the atomic population difference and coherence in the DAP are

$$W^{(n)}=\delta_{n,0}\overline{W}, \quad \overline{W}=-\frac{\gamma_2}{\Gamma_1}\cos\theta \ , \qquad (3.1a)$$

$$\rho_{AB}^{(n)}=\delta_{n,0}O(\gamma_2/\omega_{BA}) \ . \qquad (3.1b)$$

When $\Delta>0$ (i.e., $\omega_{21}>\omega_L$), we see that $\overline{W}<0$, i.e., there is more population in the dressed state $|A^j\rangle$ than in the dressed state $|B^j\rangle$. Note that the dressed state $|A^j\rangle$ ($|B^j\rangle$) in the semiclassical DAP corresponds to the lower (upper) one in a doublet of the conventional DAP (see Fig. 1), in which the strong pump field is quantized. Thus, the driven two-level atoms exhibit gain at the lower Rabi sideband ($\delta\approx-\omega_{BA}$) and absorption at the upper Rabi sideband ($\delta\approx\omega_{BA}$). Obviously, lasing action is possible only at the lower Rabi sideband, which involves two dressed states with population inversion between them. Note that there is no population inversion in the BAP, since $\rho_{22}^{(n)}-\rho_{11}^{(n)}=-\delta_{n,0}(\gamma_2/\Gamma_1)\cos^2\theta\leq 0$.

We now study the situation when the cavity field starts to build up and limit our discussion to the lower Rabi sideband, $\delta\approx-\omega_{BA}$. By examining Eqs. (2.29) we find that only three components $\rho_{BA}^{(-1)}$, $\rho_{BA}^{(1)}$, and $W^{(0)}$ are resonant with the atomic transition in the DAP and, thus, are large. None of the other components is resonant with a dressed-state transition. Consequently, they are small compared with the three resonant components, since the generalized Rabi frequency ω_{BA} is much larger than the cavity-field Rabi frequency $|g\langle\tilde{a}\rangle|$ and the atomic relaxation rates Γ_1,Γ_2.

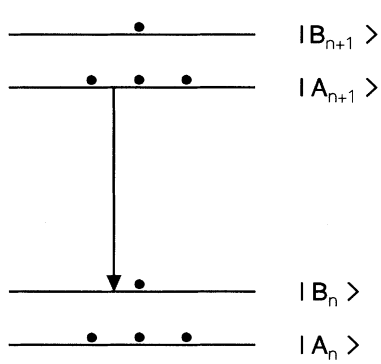

FIG. 1. Energy levels and lasing transition in the conventional dressed-atom picture. The dressed states $|A_n\rangle$ and $|B_n\rangle$ within each doublet are linear combinations [similar to Eqs. (2.14)] of the bare states $|1,n\rangle$ and $|2,n-1\rangle$, where n indicates the photon number of the external pump field. The energy difference between the two dressed states $|B_n\rangle$ and $|A_n\rangle$ within each doublet is $\hbar\omega_{BA}$, whereas that between the centers of adjacent doublets is $\hbar\omega_L$.

Keeping the leading contribution only, the cavity-field equation (2.27) can be simplified to

$$\frac{d}{dt}\langle\tilde{a}\rangle=-[\tfrac{1}{2}\gamma_c+i(\omega_c-\nu_c)]\langle\tilde{a}\rangle+\tfrac{1}{2}i\tilde{g}^*N\rho_{AB}^{(-1)} \ , \qquad (3.2)$$

where $\tilde{g}=-g\sin^2(\tfrac{1}{2}\theta)$. Comparing Eq. (3.2) with Eq. (2.13), we see that g is replaced by $\tilde{g}$ and $\tilde{\rho}_{21}^{(-1)}$ by $\rho_{AB}^{(-1)}$. Thus, $\tilde{g}$ is the effective coupling constant in the DAP (when $\delta\approx-\omega_{BA}$), and the dressed state $|A^j\rangle$ ($|B^j\rangle$) is the upper (lower) transition level. For the purpose of calculating the dressed-state coherence $\rho_{AB}^{(-1)}$ to leading order only, we can make a secular approximation, i.e., neglect all those nonresonant components. The equations of motion for the remaining three resonant components form a closed set of equations,

$$(\Gamma_2-i\tilde{\delta})\rho_{AB}^{(-1)}=\tfrac{1}{2}i\tilde{g}\langle\tilde{a}\rangle W^{(0)} \ , \qquad (3.3a)$$

$$(\Gamma_2+i\tilde{\delta})\rho_{BA}^{(1)}=-\tfrac{1}{2}i\tilde{g}^*\langle\tilde{a}^\dagger\rangle W^{(0)} \ , \qquad (3.3b)$$

$$\Gamma_1W^{(0)}=\Gamma_1\overline{W}-i\tilde{g}\langle\tilde{a}\rangle\rho_{BA}^{(1)}+i\tilde{g}^*\langle\tilde{a}^\dagger\rangle\rho_{AB}^{(-1)} \ , \qquad (3.3c)$$

where

$$\tilde{\delta}=\delta+\omega_{BA}=\nu_c-(\omega_L-\omega_{BA}) \qquad (3.4)$$

is the atom–cavity-field detuning in the DAP.

Solving Eqs. (3.3), we find the atomic population difference in the DAP to be

$$W^{(0)}=\frac{\overline{W}}{1+(I/I_0)(1+\tilde{\delta}^2/\Gamma_2^2)^{-1}} \ , \qquad (3.5)$$

where we have introduced the mean photon number I and the phase ϕ for the cavity field through the relation

$$\langle\tilde{a}\rangle=\sqrt{I}e^{i\phi} \ , \qquad (3.6)$$

and

$$I_0=\frac{\Gamma_1\Gamma_2}{|\tilde{g}|^2} \ . \qquad (3.7)$$

In Eq. (3.5), $I_0(1+\tilde{\delta}^2/\Gamma_2^2)$ is the saturation photon number, which depends on the detuning $\tilde{\delta}$ in the DAP. Equation (3.5) shows that the population inversion between the dressed states $|A^j\rangle$ and $|B^j\rangle$ decreases when the cavity field builds up, similar to what happens in an ordinary laser. Substituting Eqs. (3.3a), (3.5), and (3.6) into Eq.

(3.2), we find the equation of motion for the laser intensity,

$$\dot{I}=[G(I)-\gamma_c]I \ , \tag{3.8}$$

and that for the laser phase,

$$\dot{\phi}=\nu_c-\omega_c+\tfrac{1}{2}G(I)\tilde{\delta}/\Gamma_2 \ . \tag{3.9}$$

Here

$$G(I)=\frac{\alpha_0}{1+\tilde{\delta}^2/\Gamma_2^2+I/I_0} \tag{3.10a}$$

is the nonlinear gain, and

$$\alpha_0=\frac{N|\tilde{g}|^2(-\overline{W})}{2\Gamma_2} \tag{3.10b}$$

is the linear gain coefficient. Note that, in Eq. (3.10a), the nonlinear gain $G(I)$ includes saturation effects to all orders in the laser intensity I and, in Eq. (3.10b), the positive quantity $-\overline{W}$ is the population difference between the upper transition level $|A^j\rangle$ and the lower level $|B^j\rangle$. The quantity $G(0)=\alpha_0/(1+\tilde{\delta}^2/\Gamma_2^2)$ is the linear gain of the cavity field, and α_0 is simply the linear gain at $\tilde{\delta}=0$. We emphasize that the ratio of the nonlinear gain $G(I)$ to the population inversion $-W^{(0)}$ is independent of the laser intensity I,

$$\frac{G(I)}{-W^{(0)}}=\frac{G(0)}{-\overline{W}} \ , \tag{3.11}$$

which implies that the saturation of the laser field, caused by the decrease of the laser gain, is solely due to the decrease of the population inversion. It is clear from Eqs. (3.8) and (3.10a) that the threshold condition is $G(0)=\gamma_c$. Above threshold $G(0)>\gamma_c$, the laser intensity I will build up from vacuum through spontaneous emission until it reaches its steady-state value I_{SS}. In steady state, $\dot{I}=0$ leads to $G(I_{SS})=\gamma_c$, i.e.,

$$I_{SS}=I_0\left[\frac{\alpha_0}{\gamma_c}-1-\frac{\tilde{\delta}^2}{\Gamma_2^2}\right] . \tag{3.12}$$

For given parameters Γ_2, I_0, α_0, and γ_c, the steady-state laser intensity I_{SS} is a function of the detuning $\tilde{\delta}$ in the DAP and reaches its maximum value at $\tilde{\delta}=0$, as does the linear gain $G(0)$. In other words, when the detuning $\tilde{\delta}$ is changed, the larger the linear gain, the larger the laser intensity.

Also in steady state, the cavity oscillation frequency will take a value such that $\dot{\phi}=0$, which leads to the mode-pulling relation for the off-resonantly–pumped dressed-state laser,

$$\nu_c=\frac{\Gamma_2\omega_c+\frac{1}{2}\gamma_c(\omega_L-\omega_{BA})}{\Gamma_2+\frac{1}{2}\gamma_c} \ . \tag{3.13}$$

This is a center-of-mass formula in which the oscillation frequency ν_c is a weighted average value of the passive cavity frequency ω_c and the atomic transition frequency $\omega_L-\omega_{BA}$ in the DAP. Thus ν_c is always between ω_c and $\omega_L-\omega_{BA}$. Equation (3.13) also predicts a pulling of the oscillation frequency ν_c away from the passive cavity frequency ω_c toward the atomic frequency $\omega_L-\omega_{BA}$ in the DAP, when ω_c and $\omega_L-\omega_{BA}$ do not coincide. When the passive cavity frequency is tuned to exact resonance with the dressed states, i.e., $\omega_c=\omega_L-\omega_{BA}$, there is no mode pulling, i.e., $\nu_c=\omega_c$.

Comparing the results of the off-resonantly–pumped dressed-state laser to those of an ordinary laser, we find that their operations share many common features, provided the longitudinal (i.e., population-difference) and transverse (i.e., coherence) relaxation rates, atomic transition frequency, and detuning are replaced by their counterparts in the BAP.

IV. RESONANT-PUMP FIELD

Having discussed the off-resonant–pump case in Sec. III, we now turn to the resonant-pump case, $\Delta=0$, in this section. All other assumptions are the same as in Sec. III; moreover, we again focus our attention on the lower Rabi sideband $\delta\approx-\omega_{BA}=-\chi\cdot(\chi>0)$. The reason that we have to discuss the resonant-pump case separately is due to the fact that the results of Sec. III based on the secular approximation are not adequate for a resonant-pump field, since $\Delta=0$ leads to zero population difference $\overline{W}=0$ [see Eq. (3.1a)] and, consequently, vanishing gain $G(I)=0$ for any detuning $\tilde{\delta}$ [see Eqs. (3.10)]. In order to obtain the results in the resonant-pump case, we need to go beyond the secular approximation.

As in Sec. III, we first analyze the result of the pump excitation before the cavity field builds up. Setting $\langle\tilde{a}\rangle=0$ in either Eqs. (2.28) or Eqs. (2.29), we find that the populations in the two dressed states are exactly equal,

$$W^{(n)}=0, \tag{4.1a}$$

and, to leading order, the atomic coherences between the dressed states are

$$\rho_{AB}^{(n)}=\delta_{n,0}\overline{\rho}_{AB}, \quad \overline{\rho}_{AB}=-\frac{i\gamma_2}{2\chi} \ . \tag{4.1b}$$

As in the off-resonant–pump case discussed in Sec. III, there is no population inversion in the BAP; however, in contrast to that case, there is no population inversion between the two dressed states now. It is our purpose to show in this section that one can still obtain gain in the case of a resonant-pump field.

We now consider the situation where the cavity field is building up. Since $\delta\approx-\chi$, we find that, as in the off-resonant–pump case, only three components $\rho_{AB}^{(-1)}$, its complex conjugate $\rho_{BA}^{(1)}$, and $W^{(0)}$ are resonant with the atomic transition in the DAP. Because of Eq. (4.1a), however, these three components are of the same order as the nonresonant component $\rho_{AB}^{(0)}$. None of the other components is resonant with a dressed-state transition. Consequently, they are much smaller than the above-mentioned four components. After neglecting all those small components, we find that Eq. (3.2) is still valid here, but Eqs. (2.29) now give us

$$(\Gamma_2-i\tilde{\delta})\rho_{AB}^{(-1)}=\tfrac{1}{2}i\tilde{g}\langle\tilde{a}\rangle(W^{(0)}-2\rho_{AB}^{(0)}) \ , \tag{4.2a}$$

$$(\Gamma_2 - i\chi)\rho_{AB}^{(0)} = -\tfrac{1}{2}\gamma_2 - i\tilde{g}^*\langle \tilde{a}^\dagger \rangle \rho_{AB}^{(-1)} , \tag{4.2b}$$

$$\Gamma_1 W^{(0)} = -i\tilde{g}\langle \tilde{a} \rangle \rho_{BA}^{(1)} + i\tilde{g}^*\langle \tilde{a}^\dagger \rangle \rho_{AB}^{(-1)} , \tag{4.2c}$$

where the dressed-state detuning $\tilde{\delta}$ and the dressed-state coupling constant $\tilde{g}$ are simplified to $\tilde{\delta} = \delta + \chi$, $\tilde{g} = -\frac{1}{2}g$ in the resonant-pump case. Equations (4.2a)–(4.2c) plus the complex conjugate of Eq. (4.2a) form a closed set of equations. In contrast to Eqs. (3.3), this set of equations goes beyond the secular approximation. Solving this set of equations, we find the population difference in the DAP to be

$$W^{(0)} = \frac{2i\rho_{AB}^{(0)}\tilde{\delta}}{\Gamma_2} \frac{I/I_0}{1+\tilde{\delta}^2/\Gamma_2^2 + I/I_0} , \tag{4.3a}$$

and, to leading order, we find the atomic coherence in the DAP to be

$$\rho_{AB}^{(0)} = \bar{\rho}_{AB} . \tag{4.3b}$$

Equation (4.3a) is quite different from Eq. (3.5), its counterpart for an off-resonant pump. Substituting Eqs. (4.2a), (4.3), and (4.1b) into Eq. (3.2) and using relation (3.6), we find that the intensity equation of motion is still of the type of Eq. (3.8) but with a different nonlinear gain $G(I)$, and the phase equation of motion is now

$$\dot{\phi} = \nu_c - \omega_c - G(I)\frac{1+I/I_0}{2\tilde{\delta}/\Gamma_2} . \tag{4.4}$$

For the resonant pump the nonlinear gain is given by

$$G(I) = \frac{2G_{\max}\tilde{\delta}/\Gamma_2}{1+\tilde{\delta}^2/\Gamma_2^2 + I/I_0} , \tag{4.5}$$

where I_0 has been defined in Eq. (3.7), and

$$G_{\max} = \frac{N|\tilde{g}|^2(i\bar{\rho}_{AB})}{2\Gamma_2} = \frac{N|\tilde{g}|^2\gamma_2}{4\Gamma_2\chi} \tag{4.6}$$

is the maximum value of the linear gain $G(0)$ as a function of the detuning $\tilde{\delta}$. As in Sec. III, the nonlinear gain $G(I)$ contains saturation effects to all orders in the field strength. The quantity $G_{\max}$ is also the maximum value of the nonlinear gain $G(I)$ as a function of the mean photon number I and the detuning $\tilde{\delta}$. When the transition is exactly resonant in the DAP ($\tilde{\delta}=0$), there is no gain $G(I)=0$. However, as the cavity frequency is turned away from exact resonance in the DAP such that $\tilde{\delta}>0$, we have linear gain, $G(0)>0$. The threshold condition is $G(0)=\gamma_c$. Above threshold $G(0)>\gamma_c$, the cavity field will build up from vacuum and start lasing. Since the population difference $-W^{(0)}$ between the upper transition level $|A^j\rangle$ and the lower one $|B^j\rangle$ is not positive when $\tilde{\delta}>0$, the gain is due to the dressed-state atomic coherence $\bar{\rho}_{AB}$ [comparing Eq. (4.6) with Eq. (3.10b)]. The steady-state laser intensity is determined by $G(I_{SS})=\gamma_c$, which leads to

$$I_{SS} = I_0 \left[\frac{2G_{\max}\tilde{\delta}}{\gamma_c\Gamma_2} - 1 - \frac{\tilde{\delta}^2}{\Gamma_2^2} \right] . \tag{4.7}$$

For given parameters Γ_2, I_0, $G_{\max}$, and γ_c, both the laser intensity I_{SS} and the linear gain $G(0)$ vary when the dressed-state detuning $\tilde{\delta}$ is changed. While the linear gain $G(0)$ peaks at $\tilde{\delta}=\Gamma_2$, the maximum laser intensity

$$I_{\max} = I_0 \left[\frac{G_{\max}^2}{\gamma_c^2} - 1 \right] \tag{4.8}$$

occurs at a larger value, $\tilde{\delta}=\Gamma_2 G_{\max}/\gamma_c$. In other words, the smaller linear gain at $\tilde{\delta}=\Gamma_2 G_{\max}/\gamma_c$ produces a larger laser intensity than the larger linear gain at $\tilde{\delta}=\Gamma_2$ does. This is in contrast to the situation in the off-resonantly–pumped dressed-state laser as well as to that in an ordinary laser. The reason for this feature of the resonantly pumped dressed-state laser can be understood by noting Eq. (4.5) that, for given I, the maximum value of $G(I)$ as a function of the detuning $\tilde{\delta}$ occurs at $\tilde{\delta}=\Gamma_2(1+I/I_0)^{1/2}$. This curve on the I-$\tilde{\delta}$ plane intersects another curve (4.7) also on the I-$\tilde{\delta}$ plane at $\tilde{\delta}=\Gamma_2 G_{\max}/\gamma_c$, as obtained above.

In steady state, one has $\dot{\phi}=0$. The phase equation of motion (4.4) predicts mode pushing in steady state, since $\nu_c>\omega_c$ when $\tilde{\delta}>0$ (i.e., $\nu_c>\omega_L-\omega_{BA}$). In other words, the oscillation frequency ν_c is no longer bounded between the passive cavity frequency ω_c and the atomic transition frequency $\omega_L-\omega_{BA}$ in the DAP. These are also in contrast to the situation in the off-resonantly–pumped dressed-state laser as well as to that in an ordinary laser. It is of practical importance to know how to set the passive cavity frequency according to a required oscillation frequency and a detuning, etc. It follows from Eqs. (4.4) and (4.7) that we should set the passive cavity frequency to be

$$\omega_c = \nu_c + \tfrac{1}{2}\gamma_c(\tilde{\delta}/\Gamma_2) - G_{\max} . \tag{4.9}$$

From Eq. (4.9) we find that the oscillation frequency ν_c can still be expressed in the form of a weighted average,

$$\nu_c = \frac{\gamma_2(\omega_c+G_{\max}) + \frac{1}{2}\gamma_c(\omega_L-\omega_{BA})}{\Gamma_2+\frac{1}{2}\gamma_c} . \tag{4.10}$$

In comparison with Eq. (3.13), we see that the passive cavity frequency ω_c is replaced by $\omega_c+G_{\max}$. This is another noteworthy feature of the resonantly pumped dressed-state laser.

Finally, we point out one more difference between the resonantly pumped and the off-resonantly–pumped dressed-state lasers. In the resonant-pump case, lasing can also occur around the other (i.e., the upper) Rabi sideband with the linear gain peaks at $\delta=\chi-\Gamma_2$. In fact, the properties of the cavity field are symmetric about the detuning δ in the resonant-pump case. This result as well as the overall gain characteristics are not surprising in light of the probe absorption curve for a strong resonant-pump field [13]. This curve is symmetric about $\delta=0$, exhibits dispersionlike structures near $\delta=\pm\chi$, and arises solely from nonsecular terms in a dressed-atom picture [16].

V. DISCUSSION

Experimentally, the one-photon lasing in the off-resonant–pump case has been observed by Lezama, Zhu, and Mossberg [8]. The linear one-photon gain in the resonant–pump case is small compared with that in the off-resonant–pump case, since $G_{\max}/\alpha_0 \sim O(\gamma_2/\chi)$. However, it is of the same order of magnitude compared with the maximum two-photon gain in the off-resonant–pump case, which has recently been calculated by Lewenstein, Zhu, and Mossberg [9]. Experimentally, it would be easier to observe one-photon lasing from resonantly driven two-level atoms than to observe two-photon lasing from off-resonantly driven two-level atoms. The reason is simply that the *linear* two-photon gain (which, in fact, is far-detuned one-photon linear gain) is much smaller than maximum two-phonon gain and thus the realization of two-photon lasing needs triggering [21].

In summary, we have studied the operation of dressed-state lasers pumped by an off-resonant and a resonant external field separately. Lasing without inversion between bare atomic states can occur in both cases. In the off-resonant–pump case, lasing occurs at one of the Rabi sidebands, for which there exists population inversion between dressed-atom–field states; the gain is due to the population inversion between the dressed states [22]. In the resonant-pump case, lasing can occur near either of two Rabi sidebands, for which there is *no* population inversion between the dressed states. By going beyond the secular approximation, we showed that the gain comes from the atomic coherence between the dressed states. While the operation of the off-resonantly–pumped dressed-state laser is quite similar to that of an ordinary laser (provided appropriate quantities in the DAP are used), the resonantly pumped dressed-state laser exhibits some interesting features, such as (1) mode pushing and (2) larger laser intensity coming from smaller linear gain as the cavity-field detuning is changed.

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