Structure of the electromagnetic field around the free electron in nonrelativistic QED

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We study, within the framework of nonrelativistic QED, the structure of the electromagnetic field in the neighborhood of a free spinless electron dressed by the interaction with the vacuum field. We introduce a suitable formalism that correlates electron position and field operators. The quantum average value obtained by applying correlated field operator to the dressed state gives the average value of the corresponding field quantity as a function of distance from the electron. The results obtained separately for the electric- and magnetic-field energy density around the particle display contributions that have quantum origin and that cancel in summing of the two, yielding the total energy density. In the total energy density only contributions that have a classical analog remain. These results are compared with those for the electromagnetic structure around a neutral atom, obtained in previous work [R. Passante and E. A. Power, Phys. Rev. A **35**, 188 (1987)].

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I. INTRODUCTION

The vacuum properties of a quantized field are modified by the presence of sources. The modifications can be described as due to a change in the field mode distribution [1], or as changes in the field fluctuations because of emission and absorption of virtual field quanta by the sources [2]. In the latter case, the sources are surrounded by a cloud of virtual quanta, contributing in general to the self-energy and modifying the primitive constants of the theory [3].

Recently, the local structure of the quantized electromagnetic field around neutral atoms and molecules in their ground state, in a range where nonrelativistic QED can be used have been studied [4,5]. In this case, where the electrons are localized in regions of the order of atomic dimensions, it has been shown that the structure of the electromagnetic field at large distance from the sources carries information about the source's level structure [5,6]. Moreover, it has proven meaningful to consider separately the electric and magnetic energy densities, these being connected respectively to the electric and magnetic van der Waals forces [7].

In this paper we study the structure of the electromagnetic field around a free electron and, in particular, the spatial structure of the electric and magnetic energy densities. This constitutes an aspect of the complicated structure of the photon cloud [8], which, as said previously, can be in principle connected to observable effects.

The process responsible for the cloud around the electron is the emission and absorption of virtual photons due to recoil events. This process is pictorially described by the electron-self-energy Feynman diagram [9], describing the matrix element connected to the total-energy shift.

In the context of hole theory, the structure of the virtual-electron-positron cloud was studied long ago [10]. This cloud extends up to the electron Compton wavelength $\lambda_C=\hbar/mc$, reducing within λ_C the bare-electron charge to the observed value, which there remains constant for larger distances. In this relativistic context, however, the transverse virtual-photon cloud around the electron has only been considered with regards to global self-energy effects.

We will treat this problem in the context of nonrelativistic QED, where it has been shown [5], at least in the case of neutral electromagnetic sources, that the electromagnetic energy density at a point a certain distance from the source is caused essentially by the virtual photons emitted by the source, which are allowed to reach that distance by the time-energy uncertainty principle. This qualitative statement is supported by both nonrelativistic and fully relativistic calculations [11] of the electromagnetic structure around a hydrogen atom, and seems to be general enough to be valid also in the case of charged sources. On the basis of this argument one expects that the field at distances $r>\lambda_C$ from the electron is a result mainly of low-frequency photons, such that one can use a nonrelativistic description. Here we shall limit our investigation of the structure of the electromagnetic cloud around a free electron to regions where nonrelativistic QED is expected to yield a correct answer.

Our procedure consists of evaluating the quantum average of suitable field operators on the dressed electron state. This is an eigenstate of the full Hamiltonian, and it will be calculated perturbatively, up to second order in the coupling constant, from the bare free-electron state with a given momentum. This dressed state, however, being a momentum eigenstate, is not localized in space and requires a description different from the atomic or

molecular cases treated previously [4–7]. In fact, if a direct calculation of the average of any local field quantity is naively performed on this state, it gives a constant independent of r. In order to be able to speak about the structure of the photon cloud around the electron, one must relate this cloud to the position of the electron, which seems to conflict with the assumption that the dressed electron is in a momentum eigenstate. We overcome this difficulty by introducing operators which correlate electron and field position in a way which is described in the next section in detail. In Sec. III we obtain the magnetic and the electric energy density around the free electron. The results are discussed in Sec. IV, where they are also compared to those for an atomic source obtained in previous work.

II. DRESSED FREE ELECTRON AND CORRELATED QUANTUM AVERAGES

We consider a spinless electron endowed, in view of our nonrelativistic approximation, with a physical charge $-e$ and rest bare mass m_0. The Hamiltonian of the electron interacting with the electromagnetic field, using the minimal coupling form and the nonrelativistic limit, is

$$\hat{H}=m_0c^2+\frac{[\hat{\mathbf{p}}+(e/c)\hat{\mathbf{A}}(\hat{\mathbf{x}})]^2}{2m_0}+\int d^3r\frac{|\hat{\mathbf{E}}_{\parallel}(\mathbf{r})|^2}{8\pi}+\int d^3r\frac{|\hat{\mathbf{E}}_{\perp}(\mathbf{r})|^2+|\hat{\mathbf{B}}(\mathbf{r})|^2}{8\pi}\ , \tag{2.1}$$

where $\hat{\mathbf{p}}$ is the canonical electron momentum conjugate to the electron position $\hat{\mathbf{x}}$.

In Eq. (2.1) the electromagnetic field is described within the Coulomb gauge, the vector potential $\hat{\mathbf{A}}$, satisfying $\nabla\cdot\hat{\mathbf{A}}=0$, and the electric-field operator $\hat{\mathbf{E}}$ is decomposed into longitudinal, $\hat{\mathbf{E}}_{\parallel}$, and transverse, $\hat{\mathbf{E}}_{\perp}$. The transverse-field operators can be analyzed in plane waves of wave vector $\mathbf{k}$ and polarization σ ($\sigma=1,2$) as follows:

$$\hat{\mathbf{A}}(\mathbf{r})=\sum_k \mathbf{A}_k[\hat{a}_k\exp(i\mathbf{k}\cdot\mathbf{r})+\text{H.c.}]\ , \tag{2.2a}$$

$$\hat{\mathbf{B}}(\mathbf{r})=\sum_k \mathbf{B}_k[\hat{a}_k\exp(i\mathbf{k}\cdot\mathbf{r})-\text{H.c.}]\ , \tag{2.2b}$$

$$\hat{\mathbf{E}}_{\perp}(\mathbf{r})=\sum_k \mathbf{E}_k^{\perp}[\hat{a}_k\exp(i\mathbf{k}\cdot\mathbf{r})-\text{H.c.}]\ , \tag{2.2c}$$

where k indicates the set of values $\{k_x,k_y,k_x,\sigma\}$.

In Eqs. (2.2), $\hat{a}_k^{\dagger}$ and $\hat{a}_k$ are creation and annihilation operators for photons with wave vector $\mathbf{k}$ and polarization σ, which satisfy the commutation rules $[\hat{a}_k,\hat{a}_{k'}^{\dagger}]=\delta_{k,k'}\equiv\delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma,\sigma'}$, and the coefficients appearing in the expansions are

$$\mathbf{A}_k=\mathbf{e}^{\sigma}(\boldsymbol{\kappa})\sqrt{2\pi\hbar c/kV}\ , \tag{2.3a}$$

$$\mathbf{B}_k=i[\boldsymbol{\kappa}\times\mathbf{e}^{\sigma}(\boldsymbol{\kappa})]\sqrt{2\pi\hbar ck/V}\ , \tag{2.3b}$$

$$\mathbf{E}_k^{\perp}=i\mathbf{e}^{\sigma}(\boldsymbol{\kappa})\sqrt{2\pi\hbar ck/V}\ , \tag{2.3c}$$

with $\boldsymbol{\kappa}=\mathbf{k}/k$, the unit vector along $\mathbf{k}$. The longitudinal electric field at $\mathbf{r}$ due to the electron located at $\mathbf{x}$ is $\hat{\mathbf{E}}_{\parallel}$ and can be written as the Fourier expansion [12]

$$\hat{\mathbf{E}}_{\parallel}(\mathbf{r})=\frac{e}{2\pi^2}\int d^3k\,\exp[i\mathbf{k}\cdot(\mathbf{r}-\hat{\mathbf{x}})]\left[\frac{i\mathbf{k}}{k^2}\right]\ . \tag{2.4}$$

The Coulomb self-energy in Hamiltonian (2.1) provides a contribution to the rest mass energy given by

$$\delta m_1c^2=\frac{1}{8\pi}\int d^3r|\hat{\mathbf{E}}_{\parallel}(\mathbf{r})|^2\ , \tag{2.5}$$

which diverges as the inverse of the electron radius [10] yielding, in view of (2.4), a divergence linear in the upper cutoff $k_{\max}=mc/\hbar$, as

$$\delta m_1c^2=\frac{\alpha\hbar c}{\pi}k_{\max}\ . \tag{2.6}$$

Including the Coulomb self-energy in the rest-mass term, the Hamiltonian (2.1) becomes

$$\hat{H}=\hat{H}_0+\hat{H}_1+\hat{H}_2+(m_0+\delta m_1)c^2\ , \tag{2.7}$$

where

$$\hat{H}_0=\sum_k\hbar\omega_k\hat{a}_k^{\dagger}\hat{a}_k+\frac{\hat{p}^2}{2m_0}\ , \tag{2.8a}$$

$$\hat{H}_1=\frac{e}{m_0c}\hat{\mathbf{A}}(\hat{\mathbf{x}})\cdot\hat{\mathbf{p}}\ , \tag{2.8b}$$

$$\hat{H}_2=\frac{e^2}{2m_0c^2}[\hat{A}(\hat{\mathbf{x}})]^2\ . \tag{2.8c}$$

We intend to evaluate the average value of appropriate field operators on the dressed electron state with momentum $\mathbf{p}$. This state is obtained perturbatively starting from the corresponding bare-electron state

$$|0\rangle=|\mathbf{p};\{0_N\}\rangle\ , \tag{2.9}$$

defined as

$$\hat{\mathbf{p}}|\mathbf{p};\{0_N\}\rangle=\mathbf{p}|\mathbf{p};\{0_N\}\rangle\ , \tag{2.10a}$$

$$\hat{a}_k|\mathbf{p};\{0_N\}\rangle=0\ . \tag{2.10b}$$

Up to terms of order e^2, the dressed state is

$$|\psi\rangle=\frac{1}{N}(|0\rangle+|\mathrm{I}\rangle+|\mathrm{II}\rangle_1+|\mathrm{II}\rangle_2)\ , \tag{2.11}$$

where $N=1+\langle\mathrm{I}|\mathrm{I}\rangle/2$ is the normalization factor. The corrections to the unperturbed state, in the context of a nonrelativistic approximation ($\mathbf{p}\ll mc$ and $\hbar\omega_k\ll 2mc^2$), are

$$|\mathrm{I}\rangle = -\frac{e}{m_0 c}\sum_k \frac{\mathbf{A}_k\cdot\mathbf{p}}{\hbar\omega_k}|\mathbf{p}-\hbar\mathbf{k};1_k\rangle\ , \tag{2.12}$$

$$|\mathrm{II}\rangle_1 = \frac{e^2}{m_0^2c^2}\sum_k\sum_{k'(\neq k)}\frac{[\mathbf{A}_k\cdot(\mathbf{p}-\hbar\mathbf{k}')]\mathbf{A}_{k'}\cdot\mathbf{p}}{\hbar^2\omega_k(\omega_k+\omega_{k'})}|\mathbf{p}-\hbar\mathbf{k}-\hbar\mathbf{k}';1_k,1_{k'}\rangle+\sqrt{2}\frac{e^2}{m_0^2c^2}\sum_k\frac{(\mathbf{A}_k\cdot\mathbf{p})^2}{2\hbar^2\omega_k^2}|\mathbf{p}-2\hbar\mathbf{k};2_k\rangle\ , \tag{2.13}$$

$$|\mathrm{II}\rangle_2 = -\frac{e^2}{2m_0c^2}\sum_k\sum_{k'(\neq k)}\frac{\mathbf{A}_k\cdot\mathbf{A}_{k'}}{\hbar(\omega_k+\omega_{k'})}|\mathbf{p}-\hbar\mathbf{k}-\hbar\mathbf{k}';1_k1_{k'}\rangle-\sqrt{2}\frac{e^2}{2m_0c^2}\sum_k\frac{A_k^2}{2\hbar\omega_k}|\mathbf{p}-2\hbar\mathbf{k};2_k\rangle\ . \tag{2.14}$$

Now we want to take the quantum average of the appropriate operators on $|\psi\rangle$ in order to investigate the electromagnetic field in the surroundings of the dressed electron. It is immediately seen that taking the expectation values of field operators alone is not really adequate. Indeed, since the total Hamiltonian is symmetric under space translations, the dressed electron is also completely delocalized in space. As a consequence, the expectation value of any field quantity that surrounds the particle will be constant over all space. For such a delocalized system, then, it is impossible to obtain in a naive way the structure of the photon cloud.

In order to overcome this difficulty we consider the simultaneous correlation between the electron-position operator and the value of a field operator at a given distance from the electron. The expectation value of such operators on the dressed electron state, after integration over all the possible electron positions, should give the average value of the field operator as a function of the distance from the electron.

This approach is similar, in the nonrelativistic context, to the procedure used by Weisskopf [10] in studying the charge distribution arising in relativistic QED from virtual-electron-positron-pair creation around a free electron. We shall adapt it to the investigation of the virtual-photon cloud, which is the main interest of our work.

We define the operator

$$\hat{g}_F(\mathbf{r}_1,\mathbf{r}_2)=\hat{F}(\mathbf{r}_1)\hat{P}(\mathbf{r}_2)\ , \tag{2.15}$$

where $\hat{P}(\mathbf{r}_2)$ is the projector in the electron-position eigenstate

$$\hat{P}(\mathbf{r}_2)=|\mathbf{r}_2\rangle\langle\mathbf{r}_2|\ , \tag{2.16}$$

and $\hat{F}(\mathbf{r}_1)$ is any local field operator built from the transverse and the longitudinal field at point $\mathbf{r}_1$, with $\hat{F}$ and $\hat{P}$ commuting.

It is possible to express (2.15) in terms of the distance $\mathbf{r}_1-\mathbf{r}_2$ and of the intermediate point $\mathbf{r}$:

$$\begin{aligned}&\boldsymbol{\rho}=\mathbf{r}_1-\mathbf{r}_2\ ,\\ &\mathbf{r}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)\ ,\end{aligned} \tag{2.17}$$

$$\hat{g}_F(\mathbf{r},\boldsymbol{\rho})=\hat{F}\left[\mathbf{r}+\frac{\boldsymbol{\rho}}{2}\right]\hat{P}\left[\mathbf{r}-\frac{\boldsymbol{\rho}}{2}\right]\ . \tag{2.18}$$

Equation (2.18) depends on both $\mathbf{r}$ and $\boldsymbol{\rho}$. We then define the correlation $\hat{G}_F(\boldsymbol{\rho})$, which is obtained from $\hat{g}_F(\boldsymbol{\rho},\mathbf{r})$ by integration over $\mathbf{r}$,

$$\hat{G}_F(\boldsymbol{\rho})=\int d^3r\,\hat{g}_F(\mathbf{r},\boldsymbol{\rho})\ . \tag{2.19}$$

$\hat{G}_F(\boldsymbol{\rho})$ depends only on the distance between the field at $\mathbf{r}_1$, and the electron, at $\mathbf{r}_2$. Naturally, $\hat{G}_F(\boldsymbol{\rho})$ can be connected to the observable described by $\hat{F}$. In fact, taking its expectation value on a state of the form $|\psi;f\rangle=|\psi\rangle\otimes|f\rangle$ where $|f\rangle$ is the state vector of the field and $|\psi\rangle$ describes the electron's state, one has

$$\begin{aligned}&\int d^3r\langle\psi;f|\hat{F}(\mathbf{r}_1)\hat{P}(\mathbf{r}_2)|\psi;f\rangle\\ &\quad=\int d^3r\langle f|\hat{F}(\mathbf{r}_1)|f\rangle\langle\psi|\mathbf{r}_2\rangle\langle\mathbf{r}_2|\psi\rangle\\ &\quad=\int d^3r\langle f|\hat{F}(\mathbf{r}+\boldsymbol{\rho})|f\rangle|\psi(\mathbf{r})|^2\ .\end{aligned} \tag{2.20}$$

The result coincides with the average value of the field quantity $\hat{F}$ at a given distance from the electron.

To see how the expectation value of $\hat{G}_F(\boldsymbol{\rho})$ is connected to integral observables, one integrates it over $\boldsymbol{\rho}$

$$\begin{aligned}\int d^3\rho\,\hat{G}_F(\boldsymbol{\rho})&=\int d^3r_1\int d^3r_2\hat{g}_F(\mathbf{r}_1,\mathbf{r}_2)\\ &=\int d^3r_1\hat{F}(\mathbf{r}_1)\int d^3r_2|\mathbf{r}_2\rangle\langle\mathbf{r}_2|\\ &=\int d^3r_1\hat{F}(\mathbf{r}_1)\ .\end{aligned} \tag{2.21}$$

So, when $\hat{F}$ is correlated to the electron position, its integral over all the possible distances and directions around the electron will simply coincide with its integral in the space. For example, if $\hat{F}$ describes the energy density of the field, $\hat{G}_F(\boldsymbol{\rho})$ will be the energy density at distance $\boldsymbol{\rho}$ from the electron and its integral over $\boldsymbol{\rho}$ will be the total field energy.

III. CORRELATED FIELDS

In order to obtain the electromagnetic-energy-density distribution around the electron we shall first calculate the magnetic energy density. Let us assume $\hat{F}=\hat{B}^2$ in Eq. (2.18), where $\hat{B}$ is the magnetic field, $\hat{F}$ is taken in a normal-ordered form so that the contributions from the vacuum can be separated. Equation (2.19) can then be written as

$$\hat{G}_{B^2}(\boldsymbol{\rho})=\int d^3r\,\hat{g}_{B^2}(\mathbf{r},\boldsymbol{\rho})+C_{\mathrm{ZP}}\ , \tag{3.1}$$

where

$$\hat{g}_{B^2}(\mathbf{r},\boldsymbol{\rho})=\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)=:[\hat{\mathbf{B}}(\mathbf{r}_1)]^2:\hat{P}(r_2) \tag{3.2}$$

is obviously normally ordered in the field operators and $\hat{\mathbf{B}}(\mathbf{r}_1)$, defined by Eq. (2.2b), is the magnetic field at point $\mathbf{r}_1$. The quantity C_{ZP} in (3.1) and in analogous expressions is the zero-point contribution to the correlation function stemming from the vacuum field fluctuations. In the present case

$$C_{ZP}=\frac{2\pi\hbar c}{V}\sum_k k \ . \tag{3.3}$$

According to the standard perturbative method, the expectation value of operator (3.2) on the dressed state (2.11) is, up to terms of order e^2,

$$\langle\psi|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle=\frac{1}{N^2}[\langle 0|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|0\rangle+\langle \mathrm{I}|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{I}\rangle$$
$$+2\,\mathrm{Re}\langle 0|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{II}\rangle_1+2\,\mathrm{Re}\langle 0|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{II}\rangle_2+2\,\mathrm{Re}\langle 0|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{I}\rangle] \ . \tag{3.4}$$

The first term on the right-hand side (RHS) of (3.4) vanishes because $\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)$ is normally ordered and the last term is zero because $\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)$ does not connect states which differ by one photon. The other terms in (3.4) can be written explicitly using the perturbative corrections to the state vector (2.12)–(2.14) and the explicit expression for the electron momentum eigenstate in the r representation:

$$\langle\mathbf{r}|\mathbf{p}\rangle=\frac{1}{\sqrt{V}}\exp\left[\frac{i}{\hbar}\mathbf{r}\cdot\mathbf{p}\right] . \tag{3.5}$$

Then the second term on the RHS of (3.4) can be expressed as

$$\langle \mathrm{I}|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{I}\rangle=-\frac{2}{V\hbar^2c^2}\frac{e^2}{m_0^2c^2}\sum_{k,k'}\left[(\mathbf{A}_k\cdot\mathbf{p})(\mathbf{A}_{k'}\cdot\mathbf{p})(\mathbf{B}_k\cdot\mathbf{B}_{k'})\frac{\exp[i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{r}_2-\mathbf{r}_1)]}{kk'}\right] , \tag{3.6}$$

where $\mathbf{B}_k$ and $\mathbf{A}_k$ are given by (2.3a) and (2.3b).

The third term in (3.4) can be expressed using the second-order correction for the state vector (2.13),

$$2\,\mathrm{Re}\langle 0|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{II}\rangle_1=\frac{2}{V\hbar^2c^2}\frac{e^2}{m_0^2c^2}\mathrm{Re}\sum_{k,k'}\left[(\mathbf{A}_k\cdot\mathbf{p})(\mathbf{A}_{k'}\cdot p')(\mathbf{B}_k\cdot\mathbf{B}_{k'})\frac{\exp[-i(\mathbf{k}+\mathbf{k}')\cdot(\mathbf{r}_2-\mathbf{r}_1)]}{kk'}\right] . \tag{3.7}$$

The remaining term in (3.4) is given by

$$2\,\mathrm{Re}\langle 0|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{II}\rangle_2=-\frac{2}{V}\frac{e^2}{m_0c^2}\mathrm{Re}\sum_{k,k'}\left[(\mathbf{A}_k\cdot\mathbf{A}_{k'})(\mathbf{B}_k\cdot\mathbf{B}_{k'})\frac{\exp[-i(\mathbf{k}+\mathbf{k}')\cdot(\mathbf{r}_2-\mathbf{r}_1)]}{\hbar(\omega_k+\omega_{k'})}\right] . \tag{3.8}$$

Now we evaluate the sums in terms (3.6)–(3.8). Since the method is essentially the same for all these terms, we will outline the calculations for the first one only.

Concentrating on (3.6), first we change sums into integrals as

$$\sum_k\rightarrow\frac{V}{8\pi^3}\int_0^\infty k^2dk\int d\Omega_k\sum_\sigma , \tag{3.9}$$

and we introduce the new variable:

$$\eta=k|\mathbf{r}_2-\mathbf{r}_1| \ . \tag{3.10}$$

In $\mathbf{k}$ space the z axis is chosen to be parallel to $\mathbf{r}_1-\mathbf{r}_2$; the angle between $\mathbf{k}$ and $\mathbf{r}_1-\mathbf{r}_2$ is indicated as θ_k. After these substitutions, we get for (3.6) the expression

$$\langle \mathrm{I}|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{I}\rangle=\frac{1}{|\mathbf{r}_2-\mathbf{r}_1|^4}\frac{e^2}{m_0^2c^2}\frac{1}{2^3\pi^4V}$$
$$\times\int_0^\infty\eta\,d\eta\int_0^\infty\eta'd\eta'\int d\Omega_k\int d\Omega_{k'}\sum_{\sigma,\sigma'}[\mathbf{e}^\sigma(\boldsymbol{\kappa})\cdot\mathbf{p}][\mathbf{e}^{\sigma'}(\boldsymbol{\kappa}')\cdot\mathbf{p}]$$
$$\times[\boldsymbol{\kappa}\times\mathbf{e}^\sigma(\boldsymbol{\kappa})]\cdot[\boldsymbol{\kappa}'\times\mathbf{e}^{\sigma'}(\boldsymbol{\kappa}')]\exp[-i(\eta\cos\theta_k-\eta'\cos\theta_{k'})] , \tag{3.11}$$

which depends on the fourth power of the distance $\mathbf{r}_1-\mathbf{r}_2$, on the angle between $\mathbf{r}_1-\mathbf{r}_2$, and on the electron momentum $\mathbf{p}$. The sum over polarizations is straightforward, by the use of orthonormality relation

$$\sum_{\sigma} e_1^{\sigma}(\boldsymbol{\kappa})e_j^{\sigma}(\boldsymbol{\kappa})=\delta_{i,j}-\kappa_i\kappa_j \ . \tag{3.12}$$

The integrals appearing in (3.11) are calculated by regularization

$$\int_0^{\infty} d\eta \equiv \lim_{\lambda\to 0}\int_0^{\infty} e^{-\lambda\eta}d\eta$$

in order to obtain convergent results when the integrand is an oscillating function. We get

$$\langle \mathrm{I}|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{I}\rangle=\frac{1}{2}\frac{\alpha\hbar c}{Vm_0^2c^2}\frac{p^2\sin^2\Theta}{|\mathbf{r}_2-\mathbf{r}_1|^4} \ , \tag{3.13}$$

where Θ is the angle between the electron momentum and the vector $\mathbf{r}_1-\mathbf{r}_2$ and $\alpha=e^2/\hbar c$ is the fine-structure constant.

Evaluation of (3.7) is straightforward because it differs from (3.6) just by an overall minus sign and by a change $\mathbf{k}\to-\mathbf{k}$ in the argument of the exponential. We obtain

$$2\,\mathrm{Re}\langle 0|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{II}\rangle_1=\frac{1}{2}\frac{\alpha\hbar c}{Vm_0^2c^2}\frac{p^2\sin^2\Theta}{|\mathbf{r}_2-\mathbf{r}_1|^4} \ . \tag{3.14}$$

Following the same procedure as above, (3.8) yields

$$2\,\mathrm{Re}\langle 0|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{II}\rangle_2=-\frac{5}{2\pi}\frac{\alpha\hbar^2}{Vm_0}\frac{1}{|\mathbf{r}_2-\mathbf{r}_1|^5} \ . \tag{3.15}$$

Substituting Eq. (3.13)–(3.15) into (3.4) and using $N=1$, since all the expressions are of second order in e, one has

$$\langle\psi|\hat{g}_{B^2}(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle = \frac{\alpha\hbar c}{Vm_0^2c^2}\frac{p^2\sin^2\Theta}{|\mathbf{r}_2-\mathbf{r}_1|^4}-\frac{5}{2\pi}\frac{\alpha\hbar^2}{Vm_0}\frac{1}{|\mathbf{r}_2-\mathbf{r}_1|^5} \ . \tag{3.16}$$

To obtain the average value of the square of the magnetic field at a distance ρ from the electron, we substitute Eq. (3.16) into Eq. (3.1), which gives

$$\langle\psi|\hat{G}_{B^2}(\rho)|\psi\rangle=\frac{\alpha\hbar c}{m_0^2c^2}\frac{p^2\sin^2\Theta}{\rho^4} -\frac{5}{2\pi}\frac{\alpha\hbar^2}{m_0}\frac{1}{\rho^5}+\frac{2\pi\hbar c}{V}\sum_k k \ . \tag{3.17}$$

We remark that all terms depending on the electron momentum and on ρ^{-4} originate from the perturbative effects of the first-order-correction Hamiltonian $\hat{H}_1$, whereas the ρ^{-5} dependence results from the effect of the second-order Hamiltonian $\hat{H}_2$, which is independent of the electron momentum.

Following the same procedure as for the magnetic energy density, one can obtain the electric energy density around the free electron, separating the part due to the transverse electric field, which includes radiative contributions, from that due to the longitudinal field. To this end, let us first put $\hat{F}=\hat{E}_{\perp}^2$ in (2.19) where we again separate the contribution of the normal-ordered form in order to obtain

$$\hat{G}_{E_{\perp}^2}(\boldsymbol{\rho})=\int d^3r\,\hat{g}_{E_{\perp}^2}(\mathbf{r},\boldsymbol{\rho})+C_{\mathrm{ZP}} \ , \tag{3.18}$$

where

$$\hat{g}_{E_{\perp}^2}(\mathbf{r},\boldsymbol{\rho})=\hat{g}_{E_{\perp}^2}(\mathbf{r}_1,\mathbf{r}_2)=:[\hat{\mathbf{E}}_{\perp}(\mathbf{r}_1)]^2:\hat{P}(r_2) \ . \tag{3.19}$$

Proceeding in the same fashion as in the previous section we obtain the analogous expression of Eq. (3.4), whose terms different from zero are

$$\langle \mathrm{I}|\hat{g}_{E_{\perp}^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{I}\rangle=\frac{8}{\pi^2V}\frac{\alpha\hbar c}{m_0^2c^2}\frac{p^2\cos^2\Theta}{|\mathbf{r}_2-\mathbf{r}_1|^4} \ , \tag{3.20}$$

$$2\,\mathrm{Re}\langle 0|\hat{g}_{E_{\perp}^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{II}\rangle_1=-\frac{8}{\pi^2V}\frac{\alpha\hbar c}{m_0^2c^2}\frac{p^2\cos^2\Theta}{|\mathbf{r}_2-\mathbf{r}_1|^4} \ , \tag{3.21}$$

$$2\,\mathrm{Re}\langle 0|\hat{g}_{E_{\perp}^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{II}\rangle_2=\frac{5}{2\pi}\frac{\alpha\hbar^2}{Vm_0}\frac{1}{|\mathbf{r}_2-\mathbf{r}_1|^5} \ . \tag{3.22}$$

We remark that the two contributions (3.20) and (3.21) have opposite sign. It follows that no perturbative contributions to the squared transverse field come from the first-order Hamiltonian $\hat{H}_1$.

Adding terms (3.20)–(3.22), we find

$$\langle\psi|\hat{g}_{E_{\perp}^2}(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle=\frac{5}{2\pi}\frac{\alpha\hbar^2}{Vm_0}\frac{1}{|\mathbf{r}_2-\mathbf{r}_1|^5}\equiv\frac{5}{2\pi}\frac{\alpha\hbar^2}{Vm_0}\frac{1}{\rho^5} \ . \tag{3.23}$$

According to Eq. (2.19), we have the following final expression for the square of the transverse field around the electron, analogous to Eq. (3.17):

$$\langle\psi|\hat{G}_{E_{\perp}^2}(\boldsymbol{\rho})|\psi\rangle=\frac{5}{2\pi}\frac{\alpha\hbar^2}{m_0}\frac{1}{\rho^5}+\frac{2\pi\hbar c}{V}\sum_k k \ . \tag{3.24}$$

We now define the correlation operator for the square of the total electric field, which has the form

$$\hat{G}_{E^2}(\boldsymbol{\rho})=\int d^3r:\left[\hat{\mathbf{E}}_{\perp}\left[\mathbf{r}+\frac{\boldsymbol{\rho}}{2}\right]+\hat{\mathbf{E}}_{\parallel}\left[\mathbf{r}+\frac{\boldsymbol{\rho}}{2}\right]\right]^2:\hat{P}\left[\mathbf{r}-\frac{\boldsymbol{\rho}}{2}\right]+C_{\mathrm{ZP}} \ . \tag{3.25}$$

In terms of $\hat{g}_{E_\perp^2}(\mathbf{r},\boldsymbol{\rho})$, defined in Eq. (3.19), and of the following two operators,

$$\hat{g}_{E_\parallel^2}(\mathbf{r},\boldsymbol{\rho})=\hat{g}_{E_\parallel^2}(\mathbf{r}_1,\mathbf{r}_2)=[\hat{E}_\parallel(\mathbf{r}_1)]^2\hat{P}(\mathbf{r}_2)\ , \tag{3.26}$$

$$\hat{g}_{\mathbf{E}_\perp\cdot\mathbf{E}_\parallel}(\mathbf{r},\boldsymbol{\rho})=\hat{g}_{\mathbf{E}_\perp\cdot\mathbf{E}_\parallel}(\mathbf{r}_1,\mathbf{r}_2)$$
$$=\hat{\mathbf{E}}_\perp(\mathbf{r}_1)\cdot\hat{\mathbf{E}}_\parallel(\mathbf{r}_1)\hat{P}(\mathbf{r}_2)\ , \tag{3.27}$$

Eq. (3.25) becomes

$$\hat{G}_{E^2}(\boldsymbol{\rho})=\int d^3r[\hat{g}_{E_\perp^2}(\mathbf{r},\boldsymbol{\rho})+2\hat{g}_{\mathbf{E}_\perp\cdot\mathbf{E}_\parallel}(\mathbf{r},\boldsymbol{\rho})$$
$$+\hat{g}_{E_\parallel^2}(\mathbf{r},\boldsymbol{\rho})]+C_{\mathrm{ZP}}\ . \tag{3.28}$$

The expectation value of operator (3.26) is easily evaluated. The Coulomb field depends only on the position operator, so when it is multiplied by projector $\hat{P}(\mathbf{r}_2)$ it reduces just to the classical Coulomb field of a point electron located at $\mathbf{r}_2$. Physically, this is a consequence of the fact that the longitudinal field is instantaneous and at a given time it depends on the simultaneous position of the charge, so one expects that the quantum fluctuations of the charge do not play a role in Eq. (3.25). In fact, direct evaluation of the correlation operator for the square of the longitudinal electric field around the dressed electron gives

$$\langle\psi|\hat{g}_{E_\parallel^2}(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle$$
$$=\frac{1}{N^2}[\langle 0|\hat{g}_{E_\parallel^2}(\mathbf{r}_1,\mathbf{r}_2)|0\rangle+\langle \mathrm{I}|\hat{g}_{E_\parallel^2}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{I}\rangle]$$
$$=\langle 0|\hat{g}_{E_\parallel^2}(\mathbf{r}_1,\mathbf{r}_2)|0\rangle=\frac{e^2}{V|\mathbf{r}_1-\mathbf{r}_2|^4}\ , \tag{3.29}$$

which coincides with the classical expression.

The fact that the instantaneous longitudinal field at fixed distance from the source is not affected by quantum fluctuations leads one to expect that it interferes incoherently with any fluctuating quantity, and that the average value of interference term (3.27) on the dressed state vanishes. This is confirmed by a direct calculation of (3.27):

$$\langle\psi|\hat{g}_{\mathbf{E}_\perp\cdot\mathbf{E}_\parallel}(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle=2\,\mathrm{Re}\langle 0|\hat{g}_{\mathbf{E}_\perp\cdot\mathbf{E}_\parallel}(\mathbf{r}_1,\mathbf{r}_2)|\mathrm{I}\rangle=0\ . \tag{3.30}$$

Adding Eqs. (3.23), (3.29), and (3.30), we have

$$\langle\psi|\hat{g}_{E^2}(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle=\frac{e^2}{V|\mathbf{r}_1-\mathbf{r}_2|^4}+\frac{5}{2\pi}\frac{\alpha\hbar^2}{Vm_0}\frac{1}{|\mathbf{r}_1-\mathbf{r}_2|^5}\ . \tag{3.31}$$

Finally we integrate (3.31) over $\mathbf{r}$ and add the zero-point term to it, obtaining the square of the total electric field at distance $\boldsymbol{\rho}$ from the dressed electron

$$\langle\psi|\hat{G}_{E^2}(\boldsymbol{\rho})|\psi\rangle=\frac{e^2}{\rho^4}+\frac{5}{2\pi}\frac{\alpha\hbar^2}{m_0}\frac{1}{\rho^5}+\frac{2\pi\hbar c}{V}\sum_k k\ . \tag{3.32}$$

We remark that the only radiative correction to the classical electric energy density depends on ρ^{-5} and that this is perturbative effect of the second-order term, $\hat{H}_2$, of the interaction Hamiltonian.

IV. DISCUSSION OF THE RESULTS

The magnetic-field energy density at distance ρ from the electron can be obtained from Eq. (3.17) as

$$\mathcal{H}_{\mathrm{mag}}(\boldsymbol{\rho})$$
$$=\frac{\langle\psi|\hat{G}_{B^2}(\boldsymbol{\rho})|\psi\rangle}{8\pi}$$
$$=\frac{\alpha\hbar c}{m_0^2c^2}\frac{p^2\sin^2\Theta}{8\pi\rho^4}-\frac{5}{16\pi^2}\frac{\alpha\hbar^2}{m_0}\frac{1}{\rho^5}+\frac{\hbar c}{4V}\sum_k k\ , \tag{4.1a}$$

while the electric-field energy is analogously derived from Eq. (3.32):

$$\mathcal{H}_{\mathrm{el}}(\boldsymbol{\rho})=\frac{\langle\psi|\hat{G}_{E^2}(\boldsymbol{\rho})|\psi\rangle}{8\pi}$$
$$=\frac{e^2}{8\pi\rho^4}+\frac{5}{16\pi^2}\frac{\alpha\hbar^2}{m_0}\frac{1}{\rho^5}+\frac{\hbar c}{4V}\sum_k k\ . \tag{4.1b}$$

The total energy density surrounding the charge at distance ρ is the sum of the previous quantities, that is

$$\mathcal{H}_f(\boldsymbol{\rho})=\frac{e^2}{8\pi\rho^4}+\frac{\alpha\hbar c}{m_0^2c^2}\frac{p^2\sin^2\Theta}{8\pi\rho^4}+\frac{\hbar c}{2V}\sum_k k\ . \tag{4.2}$$

We see that contributions to the magnetic and to the electric energy density depending on ρ^{-5} have opposite sign and cancel.

Of the surviving terms in (4.2) two are proportional to ρ^{-4}, the first coming from the electric-field energy density and the second from the magnetic one; the last surviving contribution is the zero-point energy density. The first term coincides with the classical electric energy density surrounding a nonrelativistic moving charge, while the second has the same structure as the classical magnetic energy density around a free moving charge [13]. So, apart from the zero-point-energy contributions, the space distribution of the total energy density around the quantum-mechanical electron coincides with that around a classical nonrelativistically moving charge.

In order to see explicitly the origin of the contributions to the electric and magnetic energy densities having a classical analog, we shall obtain, within our QED treatment, those contributions to the magnetic and to the electric field having an expectation value of the dressed state different from zero and whose square is expected to give origin to the classical energy density terms above.

According to our previous treatment of quantities quadratic in the field, now we shall calculate the expectation values on our dressed state of the correlation operators for the magnetic and the electric field which, following the definition (2.19), are

$$\hat{\mathbf{G}}_{\mathbf{B}}(\boldsymbol{\rho})=\int d^3r\,\hat{\mathbf{g}}_{\mathbf{B}}(\mathbf{r},\boldsymbol{\rho})\ , \tag{4.3a}$$

$$\hat{\mathbf{G}}_{\mathbf{E}}(\boldsymbol{\rho})=\int d^3r\,\hat{\mathbf{g}}_{\mathbf{E}}(\mathbf{r},\boldsymbol{\rho})\ , \tag{4.3b}$$

where, using (2.15), we have defined

$$\hat{\mathbf{g}}_B(\mathbf{r},\rho)=\hat{\mathbf{g}}_B(\mathbf{r}_1,\mathbf{r}_2)=\hat{\mathbf{B}}(\mathbf{r}_1)\hat{P}(\mathbf{r}_2)\ . \tag{4.4}$$

The operator related to the electric field can be separated in a transverse and a longitudinal contribution

$$\hat{\mathbf{g}}_E(\mathbf{r},\rho)=\hat{\mathbf{g}}_E(\mathbf{r}_1,\mathbf{r}_2)=\hat{\mathbf{g}}_{E_\perp}(\mathbf{r}_1,\mathbf{r}_2)+\hat{\mathbf{g}}_{E_\parallel}(\mathbf{r}_1,\mathbf{r}_2)\ , \tag{4.5}$$

which, in terms of fields, can be written explicitly as

$$\hat{\mathbf{g}}_E(\mathbf{r}_1,\mathbf{r}_2)=\hat{\mathbf{E}}_\perp(\mathbf{r}_1)\hat{P}(\mathbf{r}_2)+\hat{\mathbf{E}}_\parallel(\mathbf{r}_1)\hat{P}(\mathbf{r}_2)\ . \tag{4.6}$$

Let us evaluate the expectation value of these operators on the dressed state (2.11). The term related to the magnetic field gives, always up to terms of order e^2,

$$\begin{aligned}&\langle\psi|\hat{\mathbf{g}}_B(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle\\&\quad\equiv 2\,\mathrm{Re}\langle 0|\hat{\mathbf{g}}_B(\mathbf{r}_1,\mathbf{r}_2)|0\rangle\\&\quad=-\frac{2e}{Vm_0c}\mathrm{Re}\sum_k \mathbf{B}_k(\mathbf{A}_k\cdot\mathbf{p})\frac{\exp[-i\mathbf{k}\cdot(\mathbf{r}_2-\mathbf{r}_1)]}{\hbar\omega_k}\ ,\end{aligned} \tag{4.7}$$

and evaluating the sum, we have

$$\begin{aligned}\langle\psi|\hat{\mathbf{g}}_B(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle&=-\frac{e}{Vc}\left[\frac{\mathbf{p}}{m_0}\times\frac{\mathbf{r}_1-\mathbf{r}_2}{|\mathbf{r}_1-\mathbf{r}_2|^3}\right]\\&\equiv-\frac{e}{Vc}\left[\frac{\mathbf{p}}{m_0}\times\frac{\boldsymbol{\rho}}{\rho^3}\right]\ .\end{aligned} \tag{4.8}$$

According to Eq. (4.3a), we obtain the magnetic field around the charge by integrating this quantity over $\mathbf{r}$

$$\mathbf{B}(\boldsymbol{\rho})\equiv\langle\psi|\hat{\mathbf{G}}_B(\boldsymbol{\rho})|\psi\rangle=-\frac{e}{c}\left[\frac{p}{m_0}\times\frac{\boldsymbol{\rho}}{\rho^3}\right]\ . \tag{4.9}$$

Evidently, (4.9) coincides with the classical expression for the magnetic field of a moving charge [13] at distance ρ and its square coincides with the first term in Eq. (3.17).

As for operator (4.5), the first term, $\hat{g}_{E_\perp}$, is related to the transverse electric field and describes a fluctuating quantum effect. Consequently, its average on the dressed stationary state (2.11) must be zero. In fact,

$$\begin{aligned}&\langle\psi|\hat{\mathbf{g}}_{E_\perp}(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle\\&\quad=-\frac{2e}{Vm_0c}\mathrm{Re}\sum_k \mathbf{E}_\perp(\mathbf{A}_k\cdot\mathbf{p})\frac{\exp[-i\mathbf{k}\cdot(\mathbf{r}_2-\mathbf{r}_1)]}{\hbar\omega_k}=0\ .\end{aligned} \tag{4.10}$$

The second term $\hat{g}_{E_\parallel}$ gives

$$\langle\psi|\hat{\mathbf{g}}_{E_\parallel}(\mathbf{r}_1,\mathbf{r}_2)|\psi\rangle=-\frac{e}{V}\frac{\mathbf{r}_1-\mathbf{r}_2}{|\mathbf{r}_1-\mathbf{r}_2|^3}\equiv-\frac{e}{V}\frac{\boldsymbol{\rho}}{\rho^3}\ , \tag{4.11}$$

so that the electric field surrounding the dressed electron coincides with the classical field surrounding a nonrelativistic moving charge

$$\mathbf{E}(\rho)\equiv\langle\psi|\hat{\mathbf{G}}_E(\boldsymbol{\rho})|\psi\rangle=-e\frac{\boldsymbol{\rho}}{\rho^3} \tag{4.12}$$

identical to the Coulomb field around a static classical charge. This is due to the fact that, within our nonrelativistic approximation, we neglected corrections at the order of v^2/c^2.

As expected, one obtains some of the terms, which depend on ρ, of the total energy density from the squares of the average values of the magnetic and electric fields given by Eqs. (4.9) and (4.12). However, in the magnetic and electric energy density, Eqs. (4.1a) and (4.1b), terms depending on ρ^{-5} also appear that cannot be derived by squaring the classical fields. These terms arise from corrections to the zero-order fields, representing fluctuations whose average value is zero and that change the zero-point average value of $\hat{B}^2$ and $\hat{E}^2$ around the electron, permanently, and independent of the electron motion. In fact, these contributions persist also for an electron at rest ($p=0$), and we can then think of them as due to the virtual-photon cloud surrounding the electron in the sense of being a result of the interference between the virtual photons emitted and reabsorbed by the electron and the zero-point-field fluctuations.

We may rewrite Eqs. (4.1a) and (4.1b) for the electron at rest as

$$\mathcal{H}_{\mathrm{mag}}(\rho)=-\frac{5}{16\pi^2}\frac{e^2}{\rho^4}\frac{\lambda_C}{\rho}+\frac{\hbar c}{4V}\sum_k k\ , \tag{4.13}$$

$$\mathcal{H}_{\mathrm{el}}(\rho)=\frac{e^2}{8\pi\rho^4}+\frac{5}{16\pi^2}\frac{e^2}{\rho^4}\frac{\lambda_C}{\rho}+\frac{\hbar c}{4V}\sum_k k\ . \tag{4.14}$$

Equation (4.13) shows that the magnetic energy density around the electron is lower than that of the free field vacuum. Equation (4.14) shows the modification of the electric energy density which is represented as a power-law correction to the Coulomb energy density. Thus the correction is large near the electron, although the validity of (4.14) is limited by our approximation of distances larger than λ_C. We must stress that this result is nonrelativistic and due to low-energy photons. Fully relativistic calculations at the same order in e, although presumably significantly different at distances smaller than λ_C, can be expected to give only small corrections in the range of distances for which our approach is valid. A correction to the Coulomb energy density around a charged particle due to the high-frequency photons is known to exist which, for a particle at rest, behaves as a power law at distances larger than λ_C [14]. This correction, however, is due to the vacuum polarization induced by the electric field of the charge and is of the sixth order in the coupling constant.

We now turn to the total energy of the dressed system, and we integrate Eq. (4.2). The first term, which comes from the Coulomb energy density, has already been evaluated [see Eq. (2.5)]; it provided a rest-mass energy $\delta m_1 c^2$. The second term comes from the classical magnetic energy density surrounding a moving electron. We can write its integral in a compact form using the fact that $\alpha\hbar c/\rho^4$ is the square of the Coulomb field and introducing definition (2.5) of δm_1:

$$\frac{\alpha\hbar c}{m_0^2c^2}\frac{p^2}{8\pi}\int d^3\rho\frac{\sin^2\Theta}{\rho^4}=\frac{\alpha\hbar c}{m_0^2c^2}\frac{p^2}{8\pi}\frac{2}{3}\int d^3\rho\frac{1}{\rho^4}$$
$$=\left[\frac{4}{3}\frac{\delta m_1}{m_0}\right]\frac{p^2}{2m_0}$$
$$=\left[\frac{\delta m}{m_0}\right]\frac{p^2}{2m_0}\ , \tag{4.15}$$

where

$$\delta m=\tfrac{4}{3}\delta m_1\ . \tag{4.16}$$

A contribution to the field energy of this form appears in any bound system [15], it is ascribed to the cloud of virtual photons surrounding the atom. It is the energy density of a transverse field, but it is not an effect of the radiation field, rather an effect of the so-called "velocity field," which arises from the Lorentz transformation of the static Coulomb field of a steady charge. This is due to the well-known fact that separation of longitudinal and transverse parts of a field is not relativistically invariant. A different, covariant, separation into velocity field, describing "charge photons," and radiation field, describing "radiation photons," was proposed and it was suggested that only "radiation photons" might be considered as physical photons. Under this point of view, term (4.15) is due to emission and reabsorption of only "charge photons."

Let us write now the total energy as the sum of the field and the electron energy, dropping the rest-mass terms (which included the Coulomb self-energy) and the zero-point-field energy:

$$\langle\psi|\hat{H}|\psi\rangle=\left\langle\psi\left|\frac{[\hat{\mathbf{p}}+(e/c)\hat{\mathbf{A}}(\hat{\mathbf{x}})]^2}{2m_0}\right|\psi\right\rangle+\left[\frac{\delta m}{m_0}\right]\frac{p^2}{2m_0}\ . \tag{4.17}$$

The first term on the RHS is the expectation value of the electron's kinetic energy on the dressed state, $\mathcal{H}_k$; the second is field energy, $\mathcal{H}_F$.

The kinetic energy can be written also in terms of the velocity operator $\hat{\mathbf{v}}=\hat{\mathbf{p}}+(e/c)\hat{\mathbf{A}}(\mathbf{x})$ as

$$\mathcal{H}_k\equiv\tfrac{1}{2}m_0\langle\psi|\hat{\mathbf{v}}^2|\psi\rangle\ . \tag{4.18}$$

The explicit expression for the average kinetic energy $\mathcal{H}_k$ can be evaluated. It can be written as a sum of three terms:

$$\mathcal{H}_k=\left\langle\psi\left|\frac{\hat{p}^2}{2m_0}\right|\psi\right\rangle+\frac{e}{m_0c}\langle\psi|\hat{\mathbf{A}}(\hat{\mathbf{x}})\cdot\hat{\mathbf{p}}|\psi\rangle$$
$$+\frac{e^2}{2m_0c^2}\langle\psi|[\hat{A}(\hat{\mathbf{x}})]^2|\psi\rangle\ ,$$

where the first term is the bare kinetic energy

$$\left\langle\psi\left|\frac{\hat{p}^2}{2m_0}\right|\psi\right\rangle\equiv\left\langle 0\left|\frac{\hat{p}^2}{2m_0}\right|0\right\rangle=\frac{p^2}{2m_0} \tag{4.19}$$

and the second term is an effect due to the interaction and can be calculated explicitly, using the expression for the dressed state and definition (4.16),

$$\frac{e}{m_0c}\langle\psi|\hat{\mathbf{A}}(\hat{\mathbf{x}})\cdot\hat{\mathbf{p}}|\psi\rangle=-\left[\frac{\delta m}{m_0}\right]\frac{p^2}{m_0}\ . \tag{4.20}$$

The third term, up to order e^2, can be written in terms of the average value on the unperturbed vacuum state of the field

$$\frac{e^2}{2m_0c^2}\langle\psi|[\hat{\mathbf{A}}(\hat{\mathbf{x}})]^2|\psi\rangle\equiv\frac{e^2}{2m_0c^2}\langle\{0\}|[\hat{A}(\hat{\mathbf{x}})]^2|\{0\}\rangle\ . \tag{4.21}$$

This contribution is an effect of the interaction too, and it has been interpreted as the kinetic energy of the electron due to its motion forced by the vacuum fluctuating field [10]. Adding Eqs. (4.19)–(4.21), we obtain the kinetic energy $\mathcal{H}_k$ on the dressed state:

$$\mathcal{H}_k=\left[\frac{p^2}{2m_0}-\left[\frac{2\delta m}{m_0}\right]\frac{p^2}{2m_0}\right]$$
$$+\frac{e^2}{2m_0c^2}\langle\{0\}|[\hat{A}(\hat{\mathbf{x}})]^2|\{0\}\rangle$$
$$\cong\frac{p^2}{2(m_0+2\delta m)}+\frac{e^2}{2m_0c^2}\langle\{0\}|[\hat{A}(\hat{\mathbf{x}})]^2|\{0\}\rangle\ . \tag{4.22}$$

Equation (4.22) can be written in terms of the average value of the velocity operator

$$\langle\hat{\mathbf{v}}\rangle\equiv\langle\psi|\hat{\mathbf{v}}|\psi\rangle=\left[1-\frac{\delta m}{m_0}\right]\frac{\mathbf{p}}{m_0}\cong\frac{\mathbf{p}}{m_0\delta m}\ , \tag{4.23}$$

so that it becomes

$$\mathcal{H}_k=\tfrac{1}{2}m_0\langle\hat{\mathbf{v}}\rangle^2+\frac{e^2}{2m_0c^2}\langle\{0\}|[\hat{A}(\hat{\mathbf{x}})]^2|\{0\}\rangle\ . \tag{4.24}$$

The first contribution to the kinetic energy has a classical analog. The second one is the contribution to the kinetic energy of the electron's random motion due to the vacuum fluctuations of the field. It is a nonclassical effect, but it contributes to the total energy with a rest-mass term which is negligible in the nonrelativistic approach [16]. In the following, we will drop this term too.

Using expression (4.23), also the second term on the RHS of Eq. (4.17), which is the p-dependent contribution of the field energy, can be written in terms of the average velocity of the electron in the form

$$\mathcal{H}_F=\left[\frac{\delta m}{m_0}\right]\frac{1}{2}m_0\langle\hat{\mathbf{v}}\rangle^2\ . \tag{4.25}$$

Adding $\mathcal{H}_k$ and $\mathcal{H}_F$ we have the total energy of the system

$$\langle\psi|\hat{H}|\psi\rangle=\mathcal{H}_k+\mathcal{H}_F=\tfrac{1}{2}(m_0+\delta m)\langle\hat{\mathbf{v}}\rangle^2\ . \tag{4.26}$$

To accelerate a particle having mass m_0 up to the average velocity $\langle\hat{\mathbf{v}}\rangle$ requires an energy $m_0\langle\hat{\mathbf{v}}\rangle^2/2$; but also

means a certain amount of energy must be added to modify the field corresponding to the field energy. The result is that the effective mass of the coupled system is the dressed mass $m = m_0 + \delta m$.

We remark that in our approach the role of the field energy in the total energy of the system becomes particularly clear and that the contributions kept into account in Eq. (4.26) are of classical nature.

The self-energy (4.26) can also be written in terms of the momentum $\mathbf{p}$ of the bare electron, adding the corresponding expressions for $\mathcal{H}_k$ and $\mathcal{H}_F$, (4.22) and (4.15), obtaining the well-known result:

$$\mathcal{H}_k + \mathcal{H}_F = \frac{p^2}{2(m_0+2\delta m)} + \frac{\delta m}{m_0}\frac{p^2}{2m_0} \equiv \frac{p^2}{2(m_0+\delta m)} . \tag{4.27}$$

This result can also be derived using the well-known standard perturbative formula for the correction to the total energy up to order e^2:

$$\left[\frac{e}{m_0 c}\right]^2 \left\langle 0 \left| \hat{\mathbf{A}}(\hat{\mathbf{x}})\cdot\hat{\mathbf{p}} \frac{1}{E_0 - \hat{H}_0} \hat{\mathbf{A}}(\hat{\mathbf{x}})\cdot\hat{\mathbf{p}} \right| 0 \right\rangle = -\frac{p^2}{2m_0}\frac{\delta m}{m_0} . \tag{4.28}$$

The usual interpretation of this formula in terms of emission and reabsorption of virtual photons is a useful tool to calculate the matrix element, but it does not allow us to understand the role of the field energy.

Let us compare these results with those obtained for the virtual-photon cloud surrounding the hydrogen atom [7] in the so-called "near zone," in which the electromagnetic cloud results mainly from field frequencies higher than the characteristic frequencies of the bare atom. We expect that with these high frequencies, the electron behaves as if it were free, because they are able to induce transitions in the continuum spectrum of the hydrogen atom.

Before proceeding with the comparison, it is appropriate to stress the conceptual differences between the definitions used in this paper and those adopted in the study of the dressed hydrogen atom [7]. As explained before, (3.17) and (3.32) do not describe fields calculated at a fixed point in space; indeed, fields generated by a nonlocalized charge have to be constant throughout space. In fact, (3.17) and (3.32) are expectation values of correlated electron position and field operators, and in this sense we can interpret these quantities as fields calculated at position $\boldsymbol{\rho}$ relative to the electron.

We now compare our quantities with the field observables surrounding the hydrogen atom which we assume to be localized in space due to the large proton mass. In this case physical quantities related to the electromagnetic fields can be evaluated at the observation point $\mathbf{R}$, where $\mathbf{R}=\mathbf{0}$ is the position of the nucleus. In the near zone the average value of the square of the magnetic field on the dressed ground state $|\phi\rangle$ is [7]

$$\langle\phi|[\hat{\mathbf{B}}(\mathbf{R})]^2|\phi\rangle = B_{\mathrm{ZP}}^2 - \frac{5}{2\pi}\frac{\alpha\hbar^2}{m_0}\frac{1}{R^5} + 2\left[\frac{6}{\pi\hbar c}\zeta(\beta_1^l)^2\frac{1}{R^4}\right] , \tag{4.29}$$

where B_{ZP} is the zero-point value. β_1^l is defined in terms of an integral of a spherical Bessel function $j_1(x)$

$$\beta_1^l = \lim_{\lambda\to 0}\int_0^\infty dx\, e^{-\lambda x} x j_1(x) \tag{4.30}$$

and ζ is a constant.

It is possible to evaluate both the parameter β_1^l and the quantity ζ, using appropriate definitions [7], obtaining

$$\zeta = \frac{2e^2\hbar}{9\pi m_0^2 c}\frac{\hbar^2}{a_0^2}, \quad \beta_1^l = \frac{\pi}{2} . \tag{4.31}$$

We introduce into ζ the average momentum of the electron in the hydrogen atom, which is related to the Bohr radius a_0, according to the minimum uncertainty relation

$$a_0^2 = \frac{\hbar^2}{p_0^2} ; \tag{4.32}$$

consequently, Eq. (4.29) then can be written as

$$\langle\phi|[\hat{B}(\mathbf{R})]^2|\phi\rangle = -\frac{5}{2\pi}\frac{\alpha\hbar^2}{m_0}\frac{1}{R^5} + \frac{\alpha\hbar c}{m_0^2 c^2}\frac{2p_0^2}{3R^4} + B_{\mathrm{ZP}}^2 . \tag{4.33}$$

It is clear now that the magnetic field surrounding the hydrogen atom in the "near zone" coincides with the field "surrounding" a free electron [Eq. (3.17)], except for the fact that in the atomic case the charge is localized around a fixed point and its momentum is averaged over the solid angle. The physical interpretation we have given to the terms of Eq. (3.17) now can be applied to the magnetic field around the hydrogen atom. It is worth stressing that the calculation for the free electron is valid at distances from the charge larger than the Compton wavelength, while Eq. (4.33), which has been derived in the context of dipole approximation, is valid in a "near zone" which does not overlap with the charge distribution of the atom, being well outside the Bohr radius.

The square of the transverse electric field in the "near zone" around the atom is [7]

$$\langle\phi|[\hat{E}_\perp(\mathbf{R})]^2|\phi\rangle = \frac{4}{3\pi^2}\frac{e^2\hbar}{m_0 c}(\gamma_2^2 + 2\gamma_0^2)\frac{1}{R^5} + E_{\mathrm{ZP}}^2 , \tag{4.34}$$

where the parameters γ can be evaluated according to the definition [7]

$$\gamma_N^M = \lim_{\lambda\to 0}\int_0^\infty dx \int_0^\infty dy \frac{x^M y^M}{x+y} j_N(x) j_N(y) e^{-\lambda(x+y)} , \tag{4.35}$$

which gives

$$\gamma_2^2 = \frac{13}{8}\pi, \quad \gamma_0^2 = \frac{\pi}{8} , \tag{4.36}$$

consequently

$$\langle\phi|[\hat{\mathbf{E}}(\mathbf{R})]^2|\phi\rangle=\frac{5}{2\pi}\frac{\alpha\hbar^2}{m_0}\frac{1}{R^5}+E_{\mathrm{ZP}}^2\ , \tag{4.37}$$

which coincides with the expectation value of the square of the transverse electric field around the free electron, given in Eq. (3.24) with E_{ZP} the zero-point electric-field value.

The parallelism between field observables around the free charge, defined in terms of correlations, and field observables around neutral and localized sources does not extend to the case in which the longitudinal field is analyzed. The longitudinal field of the hydrogen atom is given by [7]

$$\hat{\mathbf{E}}_{\parallel}(\mathbf{R})=-\nabla_R\left[\frac{e}{R}-\frac{e}{|\mathbf{R}-\hat{\mathbf{x}}|}\right], \tag{4.38}$$

where $\mathbf{x}$ is the electron position. $\mathbf{R}=\mathbf{0}$ is the position of the nucleus. Forced fluctuations of the electron cause an instantaneous electric dipole moment; the longitudinal field surrounding this system is then very different from the simple Coulomb field, since it fluctuates and also its interference with the transverse field is different from zero.

V. CONCLUSIONS

In QED a free point particle, such as the idealized electron we have considered in our paper, acquires a complicated structure as a result of the interaction with the electromagnetic field. The most widely explored second-order effect of this structure is relativistic, which yields a substructure due to the polarization of the vacuum. Another part of this complicated structure consists of a virtual-photon cloud created by the interaction around the particle and related to a modification of the zero-point fluctuations of the electromagnetic field. The latter structure can be investigated by nonrelativistic QED.

We have investigated one aspect of the spatial form of this substructure by considering the average values of the magnetic and electric energy density around the dressed free particle.

A free electron dressed by its interaction with the electromagnetic field vacuum is, however, delocalized, so that the naive quantum average of any field operator would be constant in space. We have therefore introduced suitable correlation operators connecting the field value at a given point with the probability of finding the particle at another point at the same time. The average values of these operators working on the dressed state are appropriate to describe the field observables at a given fixed distance from the particle. Using this procedure we have found that the energy density of both the magnetic field and the electric field in the vicinity of the source appear to be modified from their unperturbed values.

The changes in the magnetic-field energy density, with respect to the bare vacuum value, have been shown to arise from two different contributions. The first is dependent on the electron momentum and coincides with the contribution expected classically and is proportional to the squared magnetic field of a moving charge in the nonrelativistic limit. Like its classical counterpart, this contribution goes as the inverse of the fourth power of the distance from the particle, and is responsible for the mass shift of the electron as in classical electrodynamics. The second contribution to the energy density has no classical analog, is independent of the motion of the electron, and is a permanent effect around the charge: it has been shown to decay in proportion with the inverse of the fifth power of the distance from the electron. The expectation value of the magnetic field around the particle coincides instead with the analogous classical distribution, because the contribution to the magnetic field due to the emission and absorption of virtual transverse photons is fluctuating and averages to zero.

The electric-field-energy-density distribution can also be interpreted as composed of two terms, the first of which coincides with the square of the classical Coulomb field around the charge. In the context of nonrelativistic QED its structure is independent of the particle's motion, provided of course that its momentum is constant in time. The second contribution arises from the transverse field and it is the electric counterpart of the nonclassical term in the magnetic-field energy density, having the same spatial structure and opposite sign.

These nonclassical contributions to the magnetic and electric energy densities, as discussed, do not contribute to the total energy density of the system. The consequence is that the cloud of virtual transverse photons does not contribute to the mass shift of the charged particle. This is at variance with the case of the scalar field around a neutral source [2,17] or with that of the virtual-meson cloud around the nucleon in the bag model [18], where the virtual cloud contributes to the total energy shift. This seems to explain also why in the relativistic treatment of the electron self-energy it is only the electron-positron virtual cloud that modifies the Coulomb longitudinal field, thereby contributing to the mass shift [10]. However, the presence of the virtual cloud in QED has an effect on the field surrounding the charge by changing the average values of the squares of the magnetic and electric fields. These averages can influence separately electrical neutral polarizable bodies and magnetizable ones [7], so that the structure of the cloud around a free charged particle is observable, at least in principle.

Finally, we have shown that the transverse electromagnetic structure around a nonrelativistic electron is the same as for hydrogen atom in a zone which is close to the atom, but larger than the Bohr radius.

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