Acceleration of solitary ion-acoustic surface waves

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We consider the interaction between long-wavelength ion-acoustic and electron-plasma surface waves on a semi-infinite plasma. It then turns out that an ion-acoustic solitary wave can be accelerated when the amplitude of the electron-plasma surface wave varies in time.

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Surface waves appear in various kinds of plasmas [1], and their amplitudes are often so large that nonlinear effects are important [2,3]. In order to simplify the analysis of these waves, it is advantageous to adopt the semi-infinite plasma model, where the unperturbed electron and ion density is constant ($=N_0$) in the region $z>0$, whereas the region $z\leq 0$ is filled with a dielectric medium. Considering, for simplicity, an unmagnetized plasma, it is then well known [2,4] that two kinds of electrostatic waves can exist, namely, high-frequency electron-plasma surface waves with frequency $\omega_h \approx \omega_{pe}/\sqrt{2}$, and low-frequency ion-acoustic surface waves with frequency $\omega_l \approx k_\parallel^{(l)} v_s$, where $k_\parallel^{(l)}=|\mathbf{k}_\parallel^{(l)}|$, and $\mathbf{k}_\parallel^{(l)}$ is the wave vector parallel to the boundary. Here we have denoted the electron-plasma frequency by $\omega_{pe}=(N_0 q_e^2/\epsilon_0 m_e)^{1/2}$ where q_e is the electron charge and m_e the electron mass, and the ion sound velocity by $v_s=[(T_e+T_i)/m_i]^{1/2}$ where T_e and T_i are the electron and ion temperatures and m_i is the ion mass.

The purpose of this Brief Report is to investigate how the presence of the high-frequency electron-plasma surface wave influences the propagation of the low-frequency ion-acoustic surface wave. The electric field of the high-frequency wave, which is supposed to have appeared due to some external excitation mechanism, is written in the form

$$\mathbf{E}_h=\mathbf{E}_0\exp[-i(\mathrm{Re}\,\omega_h)t+\gamma t+i\mathbf{k}_\parallel^{(h)}\cdot\mathbf{r}-k_\parallel^{(h)}|z|]$$

where $\gamma=\mathrm{Im}\,\omega_h$ represents the increment, or decrement, of the high-frequency wave amplitude, and $k_\parallel^{(h)}$ is the magnitude of the wave vector $\mathbf{k}_\parallel^{(h)}$ parallel to the boundary.

In order to study the low-frequency motion, we start from the ion continuity equation

$$\partial_t n_1+\nabla\cdot(N_0\mathbf{v}+n_1\mathbf{v})=0 \tag{1}$$

and the ion and electron momentum equations

$$\partial_t\mathbf{v}+\mathbf{v}\cdot\nabla\mathbf{v}=-(q_i/m_i)\nabla\phi_l-(T_i/m_i)\nabla n_1/N_0 \tag{2}$$

and

$$0\approx-(q_e/m_e)\nabla\phi_l-(T_e/m_e)\nabla\ln(N_0+n_{1e})-(q_e^2/m_e^2\omega_{pe}^2)\nabla|\mathbf{E}_h|^2 \tag{3}$$

where n_1 is the ion density perturbation, $\mathbf{v}$ the ion fluid velocity, ϕ_l the low-frequency part of the potential, and $q_i(=-q_e)$ the ion charge. In Eq. (3) we have neglected the electron slow motion, and included the ponderomotive force term $\nabla|\mathbf{E}_h|^2$ due to the electron-plasma surface wave. We regard the plasma as quasineutral [5], i.e., the electron density perturbation $n_{1e}\approx n_1+\lambda_D^2\nabla^2 n_1$ where $\lambda_D^2=T_e/m_e\omega_{pe}^2$.

We now introduce the surface density $S(\mathbf{r}_\parallel,t)=\int_{-0}^{\xi+0}dz\, n_1$, where $\mathbf{r}_\parallel=x\hat{\mathbf{x}}+y\hat{\mathbf{y}}$, and where $\xi(\mathbf{r}_\parallel,t)$ is the surface displacement from the equilibrium position $z=0$. Integrating Eq. (1) across the plasma boundary, and derivating in time, we then rewrite (1) as

$$\partial_t^2 S+N_0\partial_t v_z|_{\xi+0}+\int_{-0}^{\xi+0}dz\, N_0\partial_t\nabla_\parallel\cdot\mathbf{v}+\partial_t\nabla_\parallel\cdot(S\mathbf{u})=0 \tag{4}$$

where $v_z|_{\xi+0}=\partial_t\xi+\mathbf{u}\cdot\nabla\xi$ and $\mathbf{u}(\mathbf{r}_\parallel,t)=\mathbf{v}_\parallel|_{\xi+0}$.

The integral in (4) has to be deduced from Eqs. (2) and (3). Thus, multiplying (3) by m_e/m_i, and adding (2) and (3) we eliminate $\nabla\phi_l$. The $\nabla\ln(N_0+n_{1e})$ term in the resulting equation is approximated by $\nabla(n_{1e}/N_0-n_1^2/2N_0^2)$. Integrating each term across the surface one consequently obtains

$$\int_{-0}^{\xi+0}dz\, N_0\partial_t\nabla_\parallel\cdot\mathbf{v}=-v_s^2\nabla_\parallel^2\left[S+\lambda_D^2\nabla_\parallel^2 S-\int_{-0}^{\xi+0}dz\,\frac{n_1^2}{2N_0}\right]-N_0\nabla\cdot(\mathbf{u}v_z)_{\xi+0}\;. \tag{5}$$

Here we have for simplicity assumed that $T_e\gg T_i$. The $\int dz\, n_1^2$ term in (5) has been estimated previously [5]. Thus, following Ref. [5], we obtain $\int dz\, n_1^2\approx-2q_e N_0 S\phi_0/T_e$ where $\phi_0\,[=\phi_l(z=0)]$ is the electrostatic potential at the boundary. Furthermore, we shall only consider velocities $\mathbf{u}$ and densities S which are functions of the coordinate $\eta\equiv x-\int^t V(t')dt'$, where

$V(t)$ is a time-dependent parameter. As n_1 is zero inside the plasma we can then, by eliminating ϕ_l from (2) and (3), approximate $\mathbf{u}$ by $(Q|\mathbf{E}_0|^2\hat{\mathbf{x}}/V)\exp(-2kz+2\gamma t)$, where $Q\equiv q_e^2/m_i m_e \omega_{pe}^2$, and where, for notational simplicity, we write k instead of $k_\parallel^{(h)}$. Inserting (5) into Eq. (4), and neglecting higher-order nonlinear terms, we thus derive the equation

$$\partial_t^2 S + N_0 \partial_t^2 \xi - v_s^2 \nabla_\parallel^2 (S + \lambda_D^2 \nabla_\parallel^2 S + q_e S \phi_0 / T_e) - Q|\mathbf{E}_0|^2 \nabla_\parallel^2 [(1-2k\xi)S]\exp(2\gamma t) = 0 \ . \quad (6)$$

Equation (6) agrees with Eq. (9) in Ref. [5] for the particular case where the high-frequency field is absent ($\mathbf{E}_0=0$) and the boundary is fixed ($\xi=0$). We also note that Eq. (6) reminds us of, but differs significantly, unless $\gamma=0$ and ion nonlinearities are neglected, from the case [6] where the plasma is confined by an external electromagnetic field. Similarly to Ref. [6], we regard ξ as a function of S and $\nabla_\parallel^2 S$, i.e., $\xi = a_0 S + a_1 S^2 + a_2 \nabla_\parallel^2 S$ where $a_{0,1,2}$ are constants which are defined from the confinement properties of the plasma [6]. Furthermore, in order to avoid a very lengthy analysis, we now consider the limit $\gamma t \gg 1$, assuming that all variables depend on $\eta = x - \int^t V(t')dt'$ where $V(t)=V(0)\exp(\gamma t)$. In this limit, which differs completely from that of our previous paper [5], we can in (6) neglect the $v_s^2\nabla_\parallel^2(\ldots)$ term, and approximate $2k\xi$ by $2a_0 kS$. If $1+N_0 a_0 - Q|\mathbf{E}_0|^2/V^2(0) = -4N_0 a_2/L^2$, it then turns out that Eq. (6) has a solitary wave solution

$$S = \frac{S_0}{\cosh^2(\eta/L)} \ . \quad (7)$$

Here we have denoted the soliton width by L, and the soliton amplitude by S_0, where $S_0 = 6a_2/[a_1 + 2ka_0 Q|\mathbf{E}_0|^2/N_0 V^2(0)]L^2$. Estimating [6] the coefficients $a_{0,1,2}$ we suggest the typical value $S_0 \approx 3N_0 T_e m_i V^2(0)/q_e^2 |\mathbf{E}_0|^2 k L^2$.

To conclude, we have in this Brief Report shown that a finite amplitude surface charge solitary solution (7) can appear due to the interaction between the high-frequency electron-plasma surface wave, and the low-frequency ion-acoustic wave. For large times, the soliton velocity $V(t)$ $[=V(0)\exp(\gamma t)]$ increases (exponentially) with time, until the growth of the high-frequency field ceases (i.e., $\gamma=0$). The accelerated surface charge soliton radiates electromagnetic energy, which should be looked for in experiments [1].

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