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Generation of tunable radiation using an underdense ionization front

W. B. Mori
Department of Physics, University of California at Los Angeles, Los Angeles, California 90024
and Department of Electrical Engineering, University of California at Los Angeles, Los Angeles, California 90024
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It is shown that when a light pulse is incident upon an underdense relativistic ionization front, the light pulse can be simultaneously upshifted in frequency and compressed in duration. The reflection and transmission coefficients are examined. Examples for both overdense and underdense fronts are given to show that a relativistic ionization front can generate tunable radiation from millimeter to ultraviolet wavelengths.

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In this paper we consider the use of relativistic ionization fronts to both frequency upshift and pulse compress already existing sources of radiation. This mechanism may provide tunable radiation from millimeter to ultraviolet wavelengths as well as subfemtosecond pulse lengths. In this technique a light pulse is sent towards a relativistic ionization front whose velocity is $v_0 \lesssim c$. Ahead of the front is unionized gas, while behind the front is stationary plasma. With the advent of intense short-pulse lasers the generation of ionization fronts through photon ionization is now possible [1]. The rapid variation in the refractive index (i.e., plasma density) at the front increases the frequency of the incident light pulse, and if the pulse is reflected, it is compressed since the number of oscillations remains constant.

Lamp, Ott, and Walker [2] considered the reflection of radiation when the front is overdense in the front's frame. They showed that in this case a fraction of the radiation is frequency upshifted by the factor $(1+v_0/c)/(1-v_0/c)\approx 4\gamma_0^2$, where $\gamma_0=(1-v_0^2/c^2)^{-1/2}$. They considered this as a possible method to generate millimeter and submillimeter radiation. Although an expression for the transmission coefficient was formally derived, Lampe, Ott, and Walker did not consider underdense fronts. Recently, Mori [3] and Sprangle, Esarey, and Ting [4] have reconsidered overdense fronts as a possible method to generate x rays and subfemtosecond pulses. The two major results of this paper are that even when the front is *underdense* substantial frequency upshifts may still occur and that the transmitted mode may actually be reflected. Furthermore, underdense fronts have distinct advantages over overdense fronts for generating radiation and short pulses in the infrared.

This paper is outlined as follows. First, we calculate the frequencies that are generated by relativistic ionization fronts. Second, we show that in the laboratory frame the direction of the transmitted wave can be opposite the incident wave in the underdense limit. Third, we present computer simulations that confirm the analysis. Last, we consider examples to determine the feasibility of this technique for generating various wavelengths.

We assume that a plane wave polarized in the $\hat{\mathbf{y}}$ direction is incident from the left with a frequency $\omega_i=k_ic$ upon an ionization front moving from right to left with velocity v_0. The analysis is simplest when done in the frame of the ionization front. We denote quantities in this frame by the subscript f. In its own frame the front is a stationary plane boundary that continually emits electrons and ions with a velocity v_0 away from the incoming radiation. In this frame the analysis becomes a boundary value problem with the conditions that **E** and **B** be continuous and that **J** vanish at the boundary. The boundary condition for **J** exists because the plasma at the boundary cannot have any transverse velocity since it has just been created. Since there are three boundary conditions, there must be three modes. These are a reflected, a transmitted, and a free streaming wave. In the front's frame each of these modes has the same frequency as the incident wave. The incident wave has been Doppler upshifted to $\omega_f=(1+v_0/c)\gamma_0\omega_i\approx 2\gamma_0\omega_i$ and $k_f=\omega_f/c$, while the plasma frequency is still ω_p, since it is a Lorentz invariant.

We determine the generated frequencies by calculating the wave numbers of the three waves in the front's frame from the respective dispersion relations and then Lorentz transforming the (ω,k) vectors back to the laboratory frame. The wave number of the reflected wave is $k_{rf}=-\omega_f/c$, so upon transforming back to the laboratory frame we obtain $\omega_r=\omega_i(1+v_0/c)/(1-v_0/c)\approx 4\gamma_0^2\omega_i$. Therefore the ionization front behaves as a moving mirror if the reflection coefficient does not vanish. The wave number of the transmitted wave is $k_{tf}=(\omega_f^2-\omega_f^2)^{1/2}$. The transmitted wave is evanescent if $\omega_f<\omega_p$. We therefore call the front overdense if $\omega_f<\omega_p$ and underdense if $\omega_f>\omega_p$. In the underdense limit the frequency and wave number of the transmitted wave in the laboratory frame are $\omega_t=\gamma_0(\omega_f-v_0k_{tf})\approx\omega_i(1+\omega_p^2/4\omega_1^2)$ and $k_t\approx(\omega_i/c)(1-\omega_p^2/4\omega_i^2)$. Interestingly, when $\frac{1}{4}(\omega_p^2/\omega_i^2)\gg 1$ the frequency upshift of the transmitted wave can be quite large and the wave is actually traveling *backwards* in the laboratory frame. That is, we will show that the wave is transmitted (i.e., to the right) in the front's frame with a group velocity less than v_0, so that in the laboratory frame it moves in the direction of the ionization front (i.e., to the left). To our knowledge these points have never before been recognized. It is worthwhile to note how the above upshifts compare to those obtained from flash ionization [5] and photon acceleration [6]. In flash ionization the frequency is upshifted by the factor $(1+\omega_p^2/\omega_i^2)^{1/2}$, while in photon acceleration it is upshifted by $2\sqrt{2\varepsilon}$ where ε is the amplitude of the "accelerating" plasma wave (normalized to $mc\omega_p/e$).

The dispersion relation of the free streaming mode is $k_{sf}=\omega_f/v_0$. When transformed back to the laboratory frame this mode has $\omega_0=\gamma_0(\omega_f-v_0k_{sf})=0$ and $k_s=[v_0/(c+v_0)]k_i$. Thus it is a static magnetic field, and it will be shown that when substantial upshifts occur (either the reflected or transmitted upshifts) most of the incident energy remains embedded in the plasma in this mode.

As when calculating the resulting frequencies, it is simplest to calculate the reflection and transmission coefficients in the front's frame and then transform the results back to the laboratory frame. The analysis in the front's frame is a boundary-value problem, with the three boundary conditions and the four modes alluded to previously. The incident and reflected fields are of the form

$$E_{yf}=I_f e^{i(\omega_f t-k_f x)}+R_f e^{i(\omega_f t+k_f x)}+\text{c.c.} ,$$

$$B_{zf}=I_f e^{i(\omega_f t-k_f x)}-R_f e^{i(\omega_f t+k_f x)}+\text{c.c.} ,$$

while the fields inside the streaming plasma are

$$E_{yf}=T_f e^{i(\omega_f t-k_{tf} x)}+N_f e^{i[\omega_f t-(\omega_f/v_0)x]}+\text{c.c.} ,$$

$$B_{zf}=T_f\frac{ck_{tf}}{\omega_f}e^{i(\omega_f t-k_{tf} x)}+N_f\frac{c}{v_0}e^{i[\omega_f t-(\omega_f/v_0)x]}+\text{c.c.} ,$$

$$\frac{4\pi}{c}J_{yf}=i\left[\frac{ck_{tf}^2}{\omega_f}-\frac{\omega_f}{c}\right]T_f e^{i(\omega_f t-k_{tf} x)}+i\left[\frac{c\omega_f}{v_0^2}-\frac{\omega_f}{c}\right]N_f e^{i[\omega_f t-(\omega_f/v_0)x]}+\text{c.c.} ,$$

where $k_{tf}^2=\omega_f^2-\omega_p^2$. After imposing the boundary conditions Lampe, Ott, and Walker obtained the reflection and transmission coefficients in the front's frame:

$$r_f\equiv\frac{R_f}{I_f}=\frac{1-\beta_0}{1+\beta_0}\frac{1-\sqrt{\varepsilon_f}}{1+\sqrt{\varepsilon_f}} ,$$

$$t_f\equiv\frac{T_f}{I_f}=\frac{1-\beta_0}{1-\beta_0\sqrt{\varepsilon_f}}\frac{2}{1+\sqrt{\varepsilon_f}} ,$$

$$n_f\equiv\frac{N_f}{I_f}=\frac{2\beta_0^2}{1+\beta_0}\frac{1-\sqrt{\varepsilon_f}}{1-\beta_0\sqrt{\varepsilon_f}} ,$$

where $\varepsilon_f=c^2k_{tf}^2/\omega_f^2$ and $\beta_0=v_0/c$. We note that for simplicity these results were derived for discontinuous fronts. Although a front can be approximately discontinuous in the laboratory frame it is in general continuous in its own frame. This is because the scale length of the front is γ_0 times longer and the incident wavelength is $2\gamma_0$ times shorter in the front's frame. The coefficients can be calculated analytically for continuous fronts. Lampe, Ott, and Walker [2] have shown that r_f is unchanged for the overdense case and it can be shown [7] that t_f becomes $\varepsilon_f^{-1/4}$ $[(1-\beta_0)/(1-\beta_0\sqrt{\varepsilon_f})]$. When $\omega_p^2/\omega_f^2\ll 1$ this reduces to the expression for the discontinuous front.

In order to determine the reflection and transmission coefficients in the laboratory frame the fields need to be Lorentz transformed. In the laboratory frame the electric field of the incident and transmitted waves (underdense limit) as functions of the fields in the front's frame are

$$E_i=\gamma_0(E_{fi}-\beta_0B_{fi})=\gamma_0(1-\beta_0)I_f ,$$

$$E_t=\gamma_0(E_{ft}-\beta_0B_{ft})=\gamma_0(1-\beta_0\sqrt{\varepsilon_f})T_f .$$

Therefore, the laboratory-frame transmission coefficient is

$$t\equiv\frac{E_t}{E_i}=\frac{1-\beta_0\sqrt{\varepsilon_f}}{1-\beta_0}t_f=\frac{2}{1+\sqrt{\varepsilon_f}} .$$

In an analogous fashion the laboratory-frame reflection and static magnetic field coefficients become

$$r\equiv\frac{E_r}{E_i}=\frac{1-\sqrt{\varepsilon_f}}{1+\sqrt{\varepsilon_f}},\quad n\equiv\frac{B_n}{E_i}=2\beta_0\frac{1-\sqrt{\varepsilon_f}}{1-\beta_0\sqrt{\varepsilon_f}} .$$

Lampe, Ott, and Walker [2] investigated r and n in the overdense limit $(2\gamma_0\omega_i<\omega_p)$. For this case, the reflection coefficient $|r|$ approaches unity. Therefore in the laboratory frame the electric field of the reflected wave is equal to that of the incident wave and the ionization front acts somewhat like a moving mirror. However, unlike the moving mirror there is not 100% photon reflection. The difference is that the moving mirror is a source of energy while the plasma produced in the ionization front case is not. Since the number of oscillations must remain constant, the reflected pulse is compressed by an amount equal to that of frequency upshift, so the energy (ε) and photon (η) reflectivities are

$$\varepsilon=\frac{1-\beta_0}{1+\beta_0}r\approx\frac{1}{4\gamma_0^2},\quad \eta=\frac{1-\beta_0}{1+\beta_0}\varepsilon\approx\left[\frac{1}{4\gamma_0^2}\right]^2 .$$

Since in the overdense limit the transmitted mode is evanescent, most of the energy must be in the static magnetic field. This can be verified by simple energy balance arguments. The amplitude of the static magnetic field is twice that of the incident electric field and the front moves towards the incident pulse, so the magnetic field exists in the plasma over a distance half that of the incident length. The kinetic energy of the static magnetic field can always be neglected.

In the underdense limit, $\omega_p^2/\omega_f^2\ll 1$, the transmission coefficient in the laboratory frame t approaches unity, while the magnetic field coefficient n and the transmitted frequency approach $n=2/(1+\omega_f^2/\gamma_0^2\omega_p^2)$ and $\omega_t=\omega_i(1+\gamma_0^2\omega_p^2/\omega_f^2)$. These provide two distinct limits. When $\omega_f^2/\gamma_0^2\omega_p^2\approx 4\omega_i^2/\omega_p^2\gg 1$, n becomes small, so most of the incident radiation is transmitted with a slight frequency upshift. On the other hand, when $\omega_f^2/\gamma_0^2\omega_p^2 \ll 1$ the frequency upshift can be very large, while n approaches 2. Since t is unity it would appear that energy conservation is violated. This can be resolved as follows. The ratio $\omega_f^2/\gamma_0^2\omega_p^2$ is the ratio of $\omega_f^2/\omega_p^2=\gamma_g^2=(1-v_g^2/c^2)^{-1}$ to γ_0^2, where $v_g=c\sqrt{\varepsilon_f}$ is the group velocity of the transmitted wave. Thus, when $4\omega_1^2/\omega_p^2=\gamma_g^2/\gamma_0^2<1$, the group velocity of the "transmitted" mode is less than v_0. Therefore, in the laboratory frame the energy in the "transmitted" mode travels backwards with the front and that is why quotation marks are used. This is also seen by noting that in this limit k_t is negative and is $-k_i[\gamma_0^2(\omega_p^2/\omega_f^2)]$. The energy in the "transmitted" mode is localized near the front and since the number of oscillations remains constant the pulse is compressed. Most of the energy is once again left in the magnetic field and energy is conserved.

We next present computer simulations that confirm the above analysis. In Fig. 1 simulation results of reflection from an overdense front are shown. In the simulations the front moves from right to left towards an incident electromagnetic wave. The front is essentially discontinuous since it rises over one cell. The velocity of the front is $v_0/c=\sqrt{3}/2$, so $\gamma_0=2$ and the frequency of the incident wave is $\omega_i/\omega_p=0.28$. The power spectrum of the reflected wave is shown in Fig. 1(a). The broad peak is centered at $[(1+\beta_0)/(1-\beta_0)]\omega_i\approx 4\omega_i$. The narrow peak is at ω_i and is the radiation reflected from the front after it has stopped moving. In Fig. 1(b) the absolute value of the reflected radiation is plotted versus time. It clearly shows that the electric field amplitude of the upshifted radiation is equal to that of the incident radiation.

In Fig. 2 we present results for an underdense front. Parameters were chosen so that $\omega_f/\gamma_0^2\omega_p^2 \ll 1$, since in this case substantial upshifts are possible. The simulation parameters are $\gamma_0=100$ and $\omega_i/\omega_p=0.125$. In Fig. 2(a) we show the power spectrum of the radiation leaving the left-hand (reflected boundary). The large peak is centered at the theoretically predicted (dashed line) $\omega=\omega_i[1+\frac{1}{4}(\omega_p^2/\omega_i^2)]\approx 2.125\omega_p$ (an upshift by a factor of 17). Therefore, as predicted, the radiation in this so-called "transmitted" mode actually leaves the left-hand boundary. We have done simulations where $\frac{1}{4}(\omega_p^2/\omega_i^2)<1$ and have seen the transmitted radiation exit

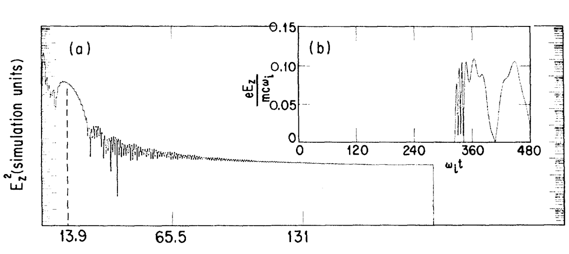

FIG. 1. Reflection from an overdense front. (a) The power spectrum of the reflected radiation. (b) The time history of the reflected radiation.

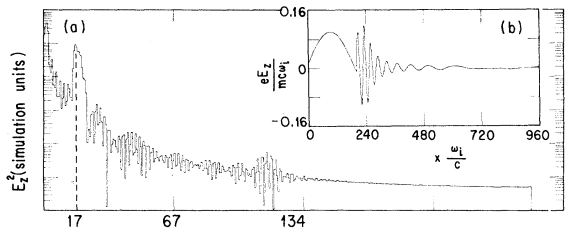

FIG. 2. "Reflection" from an underdense front. (a) The power spectrum of the radiation leaving the left-hand boundary. (b) A plot of the electric field vs x.

the right-hand boundary with the predicted upshift. In Fig. 2(b) the electric field versus position is plotted at $\omega_p t=175$. The ionization front is at $x=35c/\omega_p$ at this time. The upshifted radiation is clearly seen to pile up at the head of the front. The maximum amplitude is seen to be equal to the incident amplitude $eE_i/mc\omega_p=0.0125$, which is in agreement with $t=1$. As the front moves from right to left the amplitude of the upshifted radiation does not change, but the number of upshifted oscillations increases since it is equal to the number that has been incident on the front. Last, we note that as predicted a static magnetic field was left in the plasma in both cases and that in simulations with continuous fronts the upshifted radiation was relatively unchanged.

We next consider the potential of this mechanism for generating radiation in various wavelength ranges. We assume that photon ionization creates the front. Hence, to lowest order, the front's velocity is the group velocity of the ionizing laser in plasma. This gives $\gamma_0=\omega_{\text{ion}}/\omega_p$, where ω_{ion} is the frequency of the ionizing laser. The reflected wave will therefore have a frequency of $4(\omega_{\text{ion}}^2/\omega_p^2)\omega_i$. In order for the front to be overdense, $2\gamma_0\omega_i<\omega_p$ or $2\omega_{\text{ion}}\omega_i<\omega_p^2$. Hence, in the overdense limit, the reflected frequency is bounded by $2\omega_{\text{ion}}$. Therefore, in order to generate x rays, higher-order effects need to be used to either increase γ_0 beyond $\omega_{\text{ion}}/\omega_p$ or create wakes with large density spikes behind the front. This is beyond the scope of this paper.

If instead the front is underdense, $\omega_p/\omega_f \ll 1$, the transmitted mode can be both upshifted and pulse compressed by the factor $\gamma_0^2(\omega_p^2/\omega_f^2)$. If $\gamma_0=\omega_{\text{ion}}/\omega_p$, the frequency of this mode is bounded by ω_{ion} rather than $2\omega_{\text{ion}}$. As a result both underdense and overdense fronts can be used straightforwardly to generate radiation from the millimeter to the optical wavelengths. However, the use of an underdense front can have two distinct advantages. In the overdense case the frequency is upshifted by the factor $4\gamma_0^2$, while for the underdense case the factor is $\frac{1}{4}(\omega_p^2/\omega_i^2)$. As the ionizing laser moves through the gas the pulse shape is altered causing v_0 to change. Slight changes in v_0 will cause significant changes in γ_0, and this in turn could have deleterious consequences to the upshifted spectrum. (This could be used to chirp pulses.) This is not a problem in the underdense case because the upshift is independent of γ_0. In addition, for the same incident and upshifted frequencies, the plasma density needs to be higher for the overdense case. Therefore, more energy is needed in the ionizing laser pulse. For example, to upshift 33-GHz radiation to 3 THz ($\lambda = 100$ μm) would require that $4\gamma_0^2$ or $\frac{1}{4}(\omega_p^2/\omega_i)$ be 100. We assume a 0.5-μm ionizing laser wavelength. If an overdense front is used then $4\gamma_0^2 = 100$, so $\gamma_0 = 5$ or $n_0 = 1.5 \times 10^{20}$ cm^{-3}. On the other hand, if an underdense front is used the density need only be $n_0 = 4 \times 10^{15}$ cm^{-3}.

The ionization-front scenario described here can be used to produce pulses of unprecedented duration. Since the output pulse will contain the number of oscillations that are incident on the front, the pulse is compressed not only whenever the frequency is upshifted but also whenever the ionization front moves a distance of only a fraction of the incident pulse length. For example, if a 1×10^{19} cm^{-3} front is generated by passing a 0.5-μm ionizing laser through 0.2 mm of gas then a single cycle of 790-GHz radiation can be upshifted to 0.25 μm. The generated single-cycle 0.25-μm pulse will be only 0.8 fs long.

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[1] M. D. Perry, O. L. Landon, A. Szoke, and E. M. Campbell, Phys. Rev. A **37**, 747 (1988).
[2] M. Lampe, E. Ott, and J. H. Walker, Phys. Fluids **21**, 42 (1978); V. I. Semenova, Sov. Radiophys. **10**, 599 (1967).
[3] W. B. Mori, Bull. Am. Phys. Soc. **34**, 2069 (1989).
[4] P. Sprange, E. Esarey, and A. Ting, Phys. Rev. A **41**, 4463 (1990).
[5] C. L. Jiang, IEEE Trans. Antennas Propag. **AP-23**, 83 (1975); S. C. Wilks, J. M. Dawson, and W. B. Mori, Phys. Rev. Lett. **61**, 337 (1988) .
[6] S. C. Wilks, J. M. Dawson, W. B. Mori, T. Katsouleas, and M. E. Jones, Phys. Rev. Lett. **62**, 2600 (1989).
[7] W. B. Mori, Bull. Am. Phys. Soc. **35**, 2020 (1990).
[8] H. C. Kapteyn and M. M. Murnane, J. Opt. Soc. Am. B **8**, 1657 (1991).