PHYSICAL REVIEW A VOLUME 44, NUMBER 7 1 OCTOBER 1991

Effect of squeezed light on the photon-number probability distribution and sub-Poissonian distribution in the cascade three-level Jaynes-Cummings model

Lu-Bi Deng
Department of Physics, Southeast University, Nanjing, 210018, People's Republic of China

Son-Gen Sun
Center of Theoretical Physics, Chinese Center of Advanced Science and Technology (World Laboratory), Beijing, People's Republic of China
and Department of Physics and Chemistry, Nanjing Institute of Communication Engineering, Nanjing, People's Republic of China

(Received 28 November 1989; revised manuscript received 1 November 1990)

Quantum-statistical properties such as the photon-number probability distribution and sub-Poissonian distribution are studied by considering the resonant interaction of two radiation fields, one of which is in a squeezed state with a cascade-structure three-level atom. It is noteworthy to emphasize that the direction of squeezing, i.e., the angle θ, and the squeezing parameter r as well as the initial atomic state play important roles in the determination of the nonclassical behavior of the radiation field. At the same time we give a possible explanation for the effect of the ratio of the intensity of the two fields on the quantum-statistical properties.

PACS number(s): 42.50.Dv

I. INTRODUCTION

In the past few years, a great deal of work has been done on quantum-statistical properties of the multimode laser and the resonant interaction of multilevel atoms and multimode radiation.[1–4] In particular, the interaction of the cascade three-level atom with two fields has received extensive attention because it is closely associated with the two-mode two-photon laser and the two-mode two-photon coherent state (i.e., squeezed state). Li[5] proposed a cascade three-level Jaynes-Cummings (JC) model that greatly differs from the Λ structure coherent JC model in which there is no direct two-photon transition between the lowest and the uppermost level. Sun has studied the quantum-statistical properties for this case.[6] It is found that strange behavior can appear under certain conditions, but the quantum-statistical study of the interaction in which the fields are initially either both in squeezed states, or one field is in a coherent state and the other is in a squeezed state, is not considered.

In this paper, we investigate the photon-number probability $p(n_1)$ of the radiation field of mode 1 and the sub-Poissonian distribution of the radiation field in the interaction of the cascade three-level atom with two radiation fields (Fig. 1). The fields are either both in squeezed states, or one is in a squeezed state and the other is in a coherent state. The time evolution of these quantities that characterizes the quantum-statistical property of a stimulated emission field is calculated and plotted for different conditions. The results in this paper may be then compared with the results in Refs. 6 and 7.

As in Ref. 8, the atom-field interaction Hamiltonian of this cascade JC three-level system in the rotating-wave approximation can be written as

$$\hat{H}=\hbar\sum_{i=a,b,c}\omega_i\hat{S}_{ii}+\hbar\sum_{i=1}^{2}\Omega_i\hat{a}_i^\dagger\hat{a}_i +\hbar(\lambda_1\hat{a}_1\hat{S}_{ab}+\lambda_2\hat{a}_2\hat{S}_{bc}+\text{H.c.})\,, \tag{1}$$

where $\hbar\omega_i$ $(i=a,b,c)$ is the energy of atomic level $|i\rangle$, λ_i $(i=1,2)$ is the coupling coefficient, Ω_i is the frequency of field of mode i $(i=1,2)$, and $\hat{a}_i$ and $\hat{a}_i^\dagger$ $(i=1,2)$ are the destructive and creative operators of the radiation field. $\hat{S}_{ij}$ is the atomic project operator. The Hamiltonian can be rewritten as

$$\hat{H}=\hbar(\hat{H}_0+\hat{V})\,, \tag{2}$$

where the "free part" is

$$\hat{H}_0=\sum_{i=a,b,c}\omega_i\hat{S}_{ii}+\sum_{i=1}^{2}\Omega_i\hat{a}_i^\dagger\hat{a}_i\,, \tag{3}$$

and the "interaction part" is

$$\hat{V}=\lambda_1\hat{S}_{ab}\hat{a}_1+\lambda_2\hat{S}_{bc}\hat{a}_2+\text{H.c.} \tag{4}$$

Thus in the interaction picture, the time evolution operator $\hat{U}(t,0)$ can be written as

$$\hat{U}(t,0)=e^{-i\hat{V}t}=\begin{bmatrix} 1+\lambda_1^2\hat{a}_1[(\cos\hat{\mu}t-1)/\hat{\mu}^2]\hat{a}_1^\dagger & -i\lambda_1\hat{a}_1(\sin\hat{\mu}t/\hat{\mu}) & \lambda_1\lambda_2\hat{a}_1[(\cos\hat{\mu}t-1)/\hat{\mu}^2]\hat{a}_2 \\ -i\lambda_1(\sin\hat{\mu}t/\hat{\mu})\hat{a}_1^\dagger & \cos\hat{\mu}t & -i\lambda_2(\sin\hat{\mu}t/\hat{\mu})\hat{a}_2 \\ \lambda_1\lambda_2\hat{a}_2^\dagger[(\cos\hat{\mu}t-1)/\hat{\mu}^2]\hat{a}_1^\dagger & -i\lambda_2\hat{a}_2^\dagger(\sin\hat{\mu}t)/\hat{\mu}) & 1+\lambda_1^2\hat{a}_2^\dagger[(\cos\hat{\mu}t-1)/\hat{\mu}^2)]\hat{a}_2 \end{bmatrix}, \tag{5}$$

where $\hat{\mu}=(\lambda_1^2\hat{a}_1^\dagger\hat{a}_1+\lambda_2^2\hat{a}_2\hat{a}_2^\dagger)^{1/2}$. Assume that the atom and the field are uncorrelated at the beginning. In terms of $\hat{U}(t,0)$, the density operator $\hat{\rho}(t)$ is related to its initial form $\hat{\rho}(0)$ by

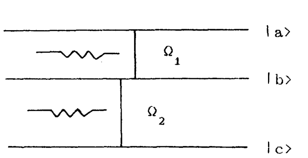

FIG. 1. Cascade three-level system. The atomic transition between $|a\rangle$ and $|b\rangle$ or between $|b\rangle$ and $|c\rangle$ is mediated by mode 1 or mode 2 with frequency Ω_1 (Ω_2).

$$\hat\rho(t)=\hat U(t,0)\rho(0)\hat U^{\dagger}(t,0)=\hat U(t,0)\rho_f(0)\otimes\rho_a(0)\hat U^{\dagger}(t,0)\ . \qquad (6)$$

Together with (5) representing matrix elements in the atomic basis, if $\hat L$ is an arbitrary operator, the average value of $\hat L$ is given by

$$\bar L=\langle\hat L\rangle=\mathrm{Tr}[\rho(t)\hat L]=\sum_{n_1,n_2}\langle n_1,n_2|\rho(t)\hat L|n_1,n_2\rangle\ . \qquad (7)$$

II. PHOTON-NUMBER PROBABILITY DISTRIBUTION

Recently Salynarayana *et al.*[7] analyzed the oscillatory photon-number distribution for a strongly squeezed coherent state. However, the effect of squeezed light on the photon-number probability distribution in the interaction of the cascade three-level atom with two fields was not considered in Ref. 7. In this cascade three-level atomic system, at the beginning, the fields are (a) both in squeezed states, (b) both in coherent states, or (c) one field is in a squeezed state but the other is in a coherent state when the initial atomic state is $|a\rangle$, $|b\rangle$, or $|c\rangle$, respectively. The corresponding expressions of photon-number probability distribution $p(n_1)$ of mode 1 are as follows:

$$p_x^a(n_1)=\sum_{n_2}\{[\lambda_2^2(n_2+1)+\lambda_1^2(n_1+1)\cos\bar\mu t]^2\bar\mu^{-4}+(\lambda_1^2 n_1\sin^2\bar\mu_3 t)(\bar n_1\bar\mu_3^2)^{-1}+[\lambda_1^2\lambda_2^2 n_1^2(n_2+1)(\cos\bar\mu_3 t-1)^2](\bar n_1\bar\mu_3^4)^{-1}\}\times W_x(\bar n_1,\bar n_2)\ , \qquad (8)$$

$$p_x^b(n_1)=\sum_{n_2}\{(\lambda_1^2 n_1\sin^2\bar\mu t)\bar\mu^{-2}+\cos^2\bar\mu_3 t+[\lambda_2^2(n_2+1)\sin^2\bar\mu_3 t]\bar\mu_3^{-2}\}W_x(\bar n_1,\bar n_2)\ , \qquad (9)$$

and

$$p_x^c(n_1)=\sum_{n_2}\{\lambda_1^2\lambda_2^2\bar n_1 n_2(\cos\bar\mu_1 t-1)^2\bar\mu_1^{-4}+(\lambda_2^2 n_2\sin\bar\mu_2 t)\bar\mu_2^{-2}+(\lambda_1^2 n_1+\lambda_2^2 n_2\cos\bar\mu_2 t)^2\bar\mu_2^{-4}\}W_x(\bar n_1,\bar n_2)\ , \qquad (10)$$

where the x index stands for c,c, s,c, and s,s, respectively.

Firstly from Fig. 2 we can see that for the initial atomic state $|a\rangle$, $p_{s,c}^a(n_1)$ is similar to Fig. 1 in Ref. 7. Otherwise for the initial atomic state $|b\rangle$, the interaction between the field of mode 2 and the transition $|b\rangle$ to $|c\rangle$ is much stronger than that corresponding to the initial atomic state $|a\rangle$ (since at $t=0^+$, the field of mode 2 interacts with the atom). The interaction makes the field of mode 1 approach a Poissonian distribution although the field of mode 1 for $\theta=0$ initially is expected to follow a super-Poissonian distribution. The difference between the statistical property of the field in this system and that of the field in the two-level atomic system is obvious; it shows that the initial atomic state plays an important role in the determination of photon-number probability distribution in this atomic system.

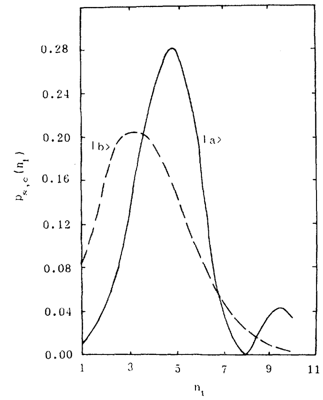

FIG. 2. $p_{s,c}(n_1)$ as a function of n_1 for $t=0.314$, $\lambda_1=\lambda_2=1$, $\bar n_2=10$, $r=0.8$, $\theta=0$. The field of mode 1 is in a squeezed state and the field of mode 2 in a coherent state.

By comparing Fig. 3(a) with Fig. 3(b), we can see that for different θ (the direction of squeezing[9]), the effect of the squeezed light of mode 2 on the field of mode 1 is greatly different; since $\theta=\pi/2$, i.e., the field of mode 2 follows a sub-Poissonian distribution, and the peak value of $p_{c,s}^a(n_1)$ for $\theta=\pi/2$ is approximately 0.5 that of $p_{c,s}^a(n_1)$ for $\theta=0$. This induces the field of mode 1 towards a sub-Poissonian distribution.

III. SUB-POISSONIAN DISTRIBUTION

As defined in Ref. 6, for a stimulated field, $F_{n_i}(t)=[\langle\hat n_i^2(t)\rangle-\langle\hat n_i(t)\rangle^2]/\langle\hat n_i(t)\rangle$. If $F_{n_i}(t)<1$, we have sub-Poissonian distribution, that is a nonclassical effect; in contrast, if $F_{n_i}(t)>1$, the stimulated field follows a super-Poissonian distribution; however, for a pure coherent state, i.e., Poissonian distribution, $F_{n_i}(t)=1$.

In this system for mode 1 we have

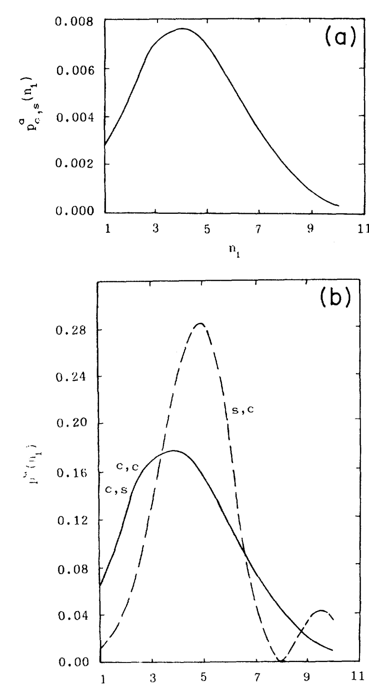

FIG. 3. $p^a(n_1)$ as a function of n_1 for $t=0.314$, $\lambda_1=\lambda_2=1$, the initial atomic state $|a\rangle$, and $n_2=10$, $r=0.8$. (a) Mode 1 in a coherent state, mode 2 in a squeezed state, $\theta=\pi/2$. (b) The three cases of the initial state of two fields, $\theta=0$.

$$[F^a_{n_i}(t)]_{i,j}=[\langle\Delta\hat n_1^a(t)\rangle/\langle\hat n_1^a(t)\rangle]_{i,j}=\{[\langle\hat n_1^a(t)^2\rangle-\langle\hat n_1^a(t)\rangle^2]/\langle\hat n_1^a(t)\rangle\}_{i,j}$$

$$=\Bigg[\sum_{n_1,n_2}\{n_1^2[\lambda_2^2(n_2+1)+\lambda_1^2(n_1+1)\cos\bar\mu t]^2\bar\mu^{-4}+[\lambda_1^2(n_1+1)^3\sin\bar\mu t]\bar\mu^{-2}$$

$$+[\lambda_1^2\lambda_2^2(n_1+1)^3(n_2+1)(\cos\bar\mu t-1)^2]\bar\mu^{-4}\}\overline{W}_{i,j}(\bar n_1,\bar n_2)$$

$$-\Bigg\{\bar n_1+\sum_{n_1,n_2}\{[\lambda_1^2(n_1+1)\sin^2\bar\mu t]\bar\mu^{-2}+\lambda_1^2\lambda_2^2(n_1+1)(n_2+1)(\cos\bar\mu t-1)^2\bar\mu^{-4}\}\overline{W}_{i,j}(\bar n_1,\bar n_2)\Bigg\}^2\Bigg]$$

$$\times\Bigg\{\bar n_1+\sum_{n_1,n_2}\{[\lambda_1^2(n_1+1)\sin^2\bar\mu t]\bar\mu^{-2}+\lambda_1^2\lambda_2^2(n_1+1)(n_2+1)(\cos\bar\mu t-1)^2\bar\mu^{-4}\}\overline{W}_{i,j}(\bar n_1,\bar n_2)\Bigg\}^{-1}, \qquad (11)$$

$$[F^b_{n_1}(t)]_{i,j}=\Bigg[\sum_{n_1,n_2}[n_1^2-\lambda_1^2n_1(2n_1-1)(\sin^2\bar\mu_1t)\bar\mu_1^{-2}]\overline{W}_{i,j}(\bar n_1,\bar n_2)-\sum_{n_1,n_2}[\lambda_1^2n_1(\sin^2\bar\mu_1t)\bar\mu_1^{-2}W_{i,j}(\bar n_1,\bar n_2]^2\Bigg]$$

$$\times\Bigg[\bar n_1-\sum_{n_1,n_2}\lambda_1^2n_1(\sin^2\bar\mu_1t)\bar\mu_1^{-2}\overline{W}_{i,j}(\bar n_1,\bar n_2)\Bigg]^{-1}, \qquad (12)$$

$$[F^c_{n_1}(t)]_{i,j}=\Bigg[\sum_{n_1,n_2}[n_1^2-\lambda_1^2\lambda_2^2n_1n_2(2n_1-1)(\cos\bar\mu_2t-1)^2\bar\mu_2^{-4}]\overline{W}_{i,j}(\bar n_1,\bar n_2)$$

$$-\Bigg[\bar n_1-\sum_{n_1,n_2}\lambda_1^2\lambda_2^2n_1n_2(\cos\bar\mu_2t-1)^2\bar\mu_2^{-4}\overline{W}_{i,j}(\bar n_1,\bar n_2)\Bigg]^2\Bigg]\Bigg/\Bigg[\bar n_1-\sum_{n_1,n_2}\lambda_1^2\lambda_2^2n_1n_2(\cos\bar\mu_2t-1)^2$$

$$\times\bar\mu_2^{-4}\overline{W}_{i,j}(\bar n_1,\bar n_2)\Bigg], \qquad (13)$$

where i or j stands for s (squeezed state) or c (coherent state). By numerical summation, we can sufficiently analyze the behavior of the radiation field in this structure regardless of the initial state of the two fields. Figure 4 shows that three curves of $[F_{n_1}(t)]_{c,s}$ change with time. When the atom starts in $|c\rangle$ (because $\theta=0$) the mode-2 field follows a super-Poissonian distribution naturally, and the strongest interaction of the mode-2 field with the transition from $|c\rangle$ to $|b\rangle$ greatly destroys the coherence of the mode-1 field, which would induce a change in its sub-Poissonian distribution. However, because of a small ratio of the intensity $\bar n_2/\bar n_1$, mode-1 field still appears as a sub-Poissonian distribution. The reason is that the cascade three-level system is different from the Λ structure three-level atomic system. There is no direct two-photon transition between $|a\rangle$ and $|c\rangle$ or two-photon absorption between $|c\rangle$ and $|a\rangle$,[10] so when the atom starts in $|a\rangle$, this effect is obviously the weakest.

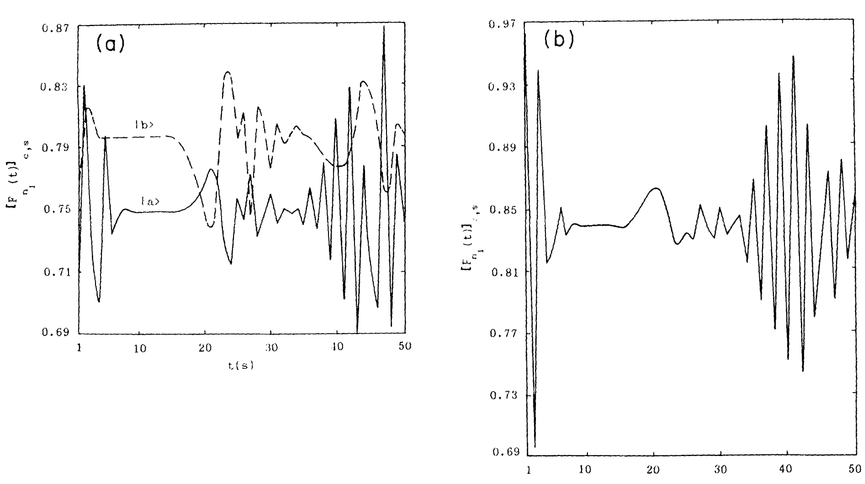

FIG. 4. $[F_{n_1}(t)]_{c,s}$ vs λt for $\lambda_1=\lambda_2=\lambda=1$, $\bar n_1=10$, $\bar n_2=4$, and $\theta=0$, the initial atomic states $|a\rangle,|b\rangle,|c\rangle$; s, squeezed state; c, coherent state.

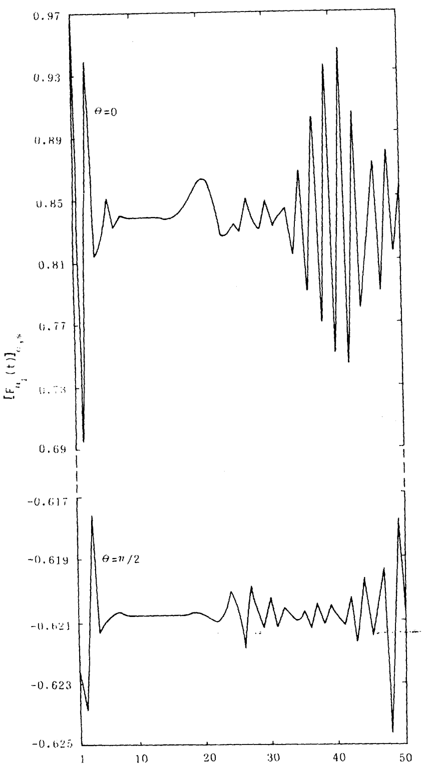

FIG. 5. $[F^c_{n_1}(t)]_{c,s}$ vs λt for $\lambda_1=\lambda_2=\lambda=1$, $r=0.8$, $\bar{n}_1=10$, $\bar{n}_2=4$, and the initial atomic state $|c\rangle$; s, squeezed state; c, coherent state.

θ, i.e., the direction of squeezing of the mode-2 field, in Fig. 5, we have plotted curves of $[F_{n_1}(t)]_{c,s}$ for $\theta=0$ and $\pi/2$, respectively. Because of the different statistical distributions of the mode-2 field itself, we have $[F_{n_1}(t)]_{c,s}<0$ for $\theta=\pi/2$ and $[F_{n_1}(t)]_{c,s}>0$ for $\theta=0$; in addition, the amplitude of oscillation for $\theta=\pi/2$ becomes smaller than that for $\theta=0$. We guess that for $t>0$, there exist a large number of "interference" terms, depending on the phase of the squeezed light field of mode 2, which affect the dipole moment of the transition between the level $|a\rangle$ and $|b\rangle$ or between $|b\rangle$ and $|c\rangle$.

Finally, in Fig. 6, we have plotted $[F_{n_1}(t)]_{c,s}$ for ratios of $\bar{n}_1/\bar{n}_2$, and the initial atomic state $|b\rangle$; the greater the

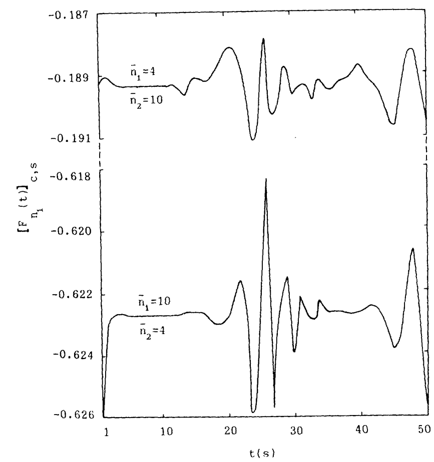

FIG. 6. $[F_{n_1}(t)]_{c,s}$ vs λt for $\lambda_1=\lambda_2=\lambda=1$, $r=0.8$, $\theta=\pi/2$, and the initial atomic state $|b\rangle$; mode 1, coherent state; mode 2, squeezed state.

$\bar{n}_1/\bar{n}_2$ is, the smaller the $[F_{n_1}(t)]_{c,s}$ is. The possible explanation for this is that at $t>0$, because of the initial atomic state $|b\rangle$, two transitions $|b\rangle\rightarrow|a\rangle$ and $|b\rangle\rightarrow|c\rangle$ exist, and the competition of the two transitions affects the quantum-statistical properties.

IV. CONCLUSION

In the cascade three-level system, by numerical computation, we can see that the initial atomic state and the different ratios of $\bar{n}_1/\bar{n}_2$ as well as the initial phase of the squeezed light strongly affect the interaction of two fields with the cascade three-level atom. Although at $t=0$ $[F_{n_1}(t)]_{c,s}=0$, after a while, the effect of the different initial conditions is different in the determination of $p(n_1)$ and $[F_{n_1}(t)]_{c,s}$, and different from the interaction of a single-mode field with a two-level atom. Here the interaction of the squeezed light with the transition between two levels will affect the statistical property of another light field that initially is in a pure coherent state through the time evolution of the atomic occupation and the dipole moment of the two levels itself. Its effect is stronger than that in the "Λ"-type three-level atomic system.

[1]S. Singh and M. S. Zubairy, Phys. Rev. **21**, 281 (1980).
[2]S. Zhu and D. Su, Phys. Rev. A **25**, 3169 (1982).
[3]N. N. Bogolubov, Jr. and F. L. Klein, Phys. Lett. **101A**, 201 (1984).
[4]S. Sun and X. Li, Phys. Rev. A **36**, 5844 (1987).
[5]X. Li, Acta Phys. Sin. **34**, 833 (1985).
[6]S. Sun, J. Phys. B: At. Mol. Opt. Phys. **23**, 2379 (1990).
[7]M. Venkata Satyanarayana, P. Rice, Reeta Vyas, and H. J. Carmichael, J. Opt. Soc. Am. B **6**, 228 (1989).
[8]D. Grischowsky *et al.*, Phys. Rev. A **12**, 2514 (1975).
[9]A. M. Abdel-Hafez *et al.*, Phys. Rev. A **35**, 1634 (1987).
[10]P. Alsing and M. S. Zubairy, J. Opt. Soc. Am. B **4**, 177 (1987).