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Resonant periodic-gain surface-emitting semiconductor lasers and correlated emission in a ring cavity

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A semiclassical theory of a resonant periodic-gain (half-wave spatially periodic-gain segments) laser in the context of a semiconductor medium is presented using an oversimplified picture. Terms arise in the polarization of this periodic-gain medium that lead to enhanced light-matter interaction, doubling the gain coefficient, and enhancing mode-pulling effects. Discussion of the physical processes is extended to include a comparison with the ring-cavity correlated-emission laser, which also utilizes a periodic-gain medium and exhibits a vanishing phase fluctuation between the degenerate counterpropagating modes. A simple physical picture of radiations from a half-wave-periodic, radiating dipole array illustrates the common mechanism and important relationship between these lasers.

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I. INTRODUCTION

Recent reports of resonant periodic-gain (RPG) surface-emitting semiconductor lasers [1–4] have created a great deal of interest because of their potential application in optoelectronic integration [5], and two-dimensional arrays [6–8] for optical processing. These laser structures make use of half-wave-periodic, thin sections of gain medium (e.g., GaAs/$Al_{1-x}Ga_xAs$ quantum wells) similar to that proposed for a correlated emission in a ring cavity [9]. The concept of a correlated-emission laser (CEL) was first developed [10] some five years ago. In addition to its intrinsic interest to quantum optics, the CEL holds promise for applications in various areas of fundamental and applied physics, e.g., the laser gyroscope [11,12]. Several detailed investigations of various aspects of a CEL including linear [13–16] and nonlinear theories [17,18] have been reported. In these devices, two laser modes are coherently coupled either by preparing a three-level laser medium in a coherent superposition of upper states [10] or by using a spatially periodic gain medium in a ring cavity [9]. Figure 1(a) shows schematically a periodic gain medium CEL in a ring cavity. The periodic gain medium provides the correlation between the two degenerate counterpropagating waves in the ring cavity by constructive interference. When the light of a mode is partially reflected from a layer of the gain medium, constructive interference is achieved when the phase of counterpropagating mode matches that of the reflected wave. Much of this work, both theoretical [13–18] and experimental [19–24], has been directed towards three-level and two-photon systems [15].

In Fig. 1(b) the structure of a RPG surface-emitting laser with integrated epitaxial mirrors is shown [25–28] where only a few layers have been illustrated to simplify the picture. A standing-wave optical field is shown in registration with the quantum-well gain layers. This results

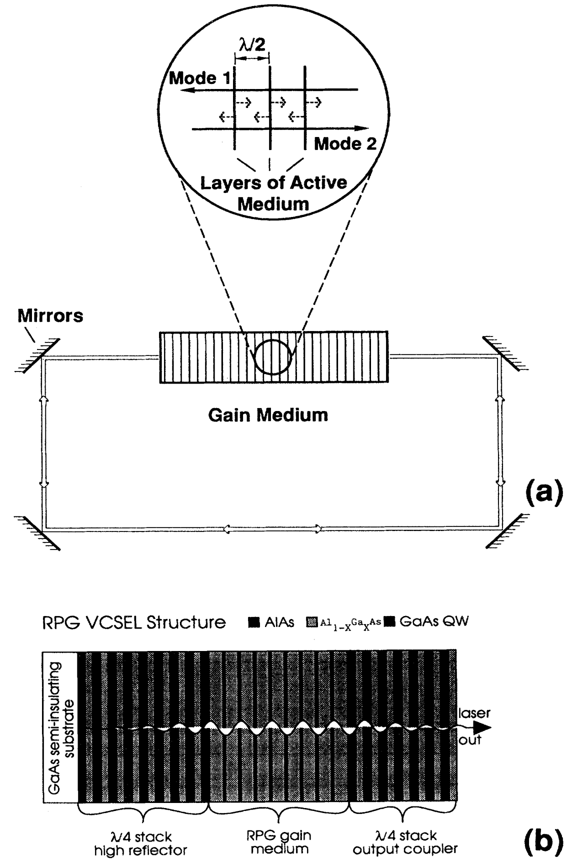

FIG. 1. Schematic representation of (a) a CEL in a ring cavity and (b) a RPG surface-emitting semiconductor laser. Both laser structures incorporate a $\lambda/2$ periodic-gain medium. For the CEL system this gain medium is placed in a ring cavity. For the RPG laser, high reflectors ($\lambda/4$ stacks of AlAs and $Al_{1-x}Ga_xAs$) are epitaxially grown along with the gain medium forming a high-Q Fabry-Pérot cavity.

in an enhanced light-matter interaction. These layers inherently operate in a single longitudinal mode because of the short cavity lengths and, hence, large free-spectral range compared to the gain bandwidth. Although a quantum theory of a CEL based on a spatially periodic gain medium in a ring cavity has been developed [29], the analysis was directed towards noise quenching and the enhanced light-matter interaction aspect (which is a key concept in RPG lasers) was not emphasized.

In this paper, we present an approximate semiclassical theory of RPG lasers (and compare with CEL), treating the valence and conduction bands of semiconductors as a homogeneously broadened two-level system [30] where the Fermi-Dirac distribution for the equilibrium carrier population is used instead of Maxwell-Boltzmann distribution. The RPG semiconductor medium is placed in a Fabry-Pérot cavity such that standing-wave optical-field interaction is enhanced by locating the thin sections of the gain medium (quantum wells) at the antinodes [1–4]. Enhancement in the gain coefficient and contributions to mode-pulling effects due to the RPG structure are evaluated. In RPG medium, amplified spontaneous emission in the directions transverse to the lasing axis is reduced because of the small overlap between optical field and gain sections [3]. It is straightforward to adapt the formalism for any two-level system, instead of two-band semiconductors which in reality are quite complex and require several approximations.

Relationships between a ring cavity CEL and a Fabry-Pérot cavity RPG surface-emitting laser are discussed. Although both lasers make use of spatially half-wave periodic-gain media, they differ in cavity feedback mechanism. In a CEL, the ring cavity does not influence the operating wavelength, rather counterpropagating running waves of the same frequency interact with periodic-gain medium and become correlated [29]. On the other hand, in RPG surface-emitting lasers [1–4], the Fabry-Pérot cavity mode strongly affects the operating frequency and influences [31] the interaction of the optical field with periodic-gain medium. A simple physical model, based on the radiation pattern of a periodic dipole array, demonstrates that, not surprisingly, common physical processes, e.g., quantum interference effects, govern the behavior of both CEL and RPG lasers.

The organization of this paper is as follows: In Sec. II we develop a semiclassical theory of RPG surface-emitting laser, where gain coefficient, mode pulling and pushing, and saturation terms are derived following the method of Ref. [32]. In Sec. III, the basic physical processes resulting from light-matter interactions in RPG surface-emitting lasers and CEL in a ring cavity are discussed. Common features and differences between RPG and CEL structure are identified. The relationships between these laser systems are further illuminated by considering the radiation pattern of a $\lambda/2$-spaced dipole array in Sec. IV. Finally, in Sec. V, concluding remarks summarize the present status of RPG lasers in the context of semiconductor-based CEL in a ring cavity.

II. SEMICLASSICAL THEORY OF RPG LASER

RPG surface-emitting semiconductor lasers were proposed [1] and demonstrated [1–4] recently. Progress has been rapid, and efficient cw operation under optical pumping has been achieved [26,27]. A schematic of the RPG structure is shown in Fig. 1(b); for details of various configurations Refs. [1–4, 25,26] should be consulted. In the following, an oversimplified semiclassical theory is developed using the density-matrix formalism.

The density-matrix formalism [32], developed originally for a two-level system, has been applied by several authors [30,33–37] to the analysis of the linear and nonlinear contributions to the optical gain in semiconductor lasers. The optical properties of a semiconductor are mainly determined by the conduction and the uppermost valence bands. In the case of quantum-well structures, the subband transitions with the $\Delta n=0$ selection rule dominate. Physically, the dominant optical transitions are those which involve an electron-hole pair whose wave functions have maximum spatial overlap, i.e., an electron in the conduction subband and holes in the valence subbands having the same quantum numbers. This simplified picture suggests an analogy with a two-level atomic system. A theoretical derivation, as well as a geometrical picture of an equivalent electronic dipole moment in a direct band-gap bulk semiconductor, has been given [35]. The Bloch functions for electrons in the conduction band and holes in the valence bands were used for calculation of dipole matrix elements. At band edges, the periodic parts of the electron and hole Bloch functions have *S*-like and *P*-like symmetries, respectively, and light- and heavy-hole wave functions are orthogonal to each other. For quantum wells, the band gap increases and the electron-hole interaction is modified as a result of spatial localization. The dipole strength increases and the degeneracy between light- and heavy-hole subbands at $K=0$ is lifted. For $GaAs/Al_{1-x}Ga_xAs$ quantum wells, the electron–heavy-hole band gap is smaller than that of the electron-light hole.

Here, density-matrix equations for a semiconductor, similar to those developed by Agrawal [30] and Kazarinov, Henry, and Logan [34], are used to determine the effects of the RPG spatial structure on the linear gain. The analysis begins with the relations between the medium polarization (driven by the electric fields) and the off-diagonal density-matrix elements and proceeds to calculate the polarization of the semiconductor medium inside a Fabry-Pérot cavity of total length L (along the z axis) following the procedure described in Ref. [32]. Then, the linear gain and frequency determining relations from the self-consistent laser theory are used to show the contribution of additional terms arising from the spatial periodicity of the medium.

As pointed out above, in the density-matrix approach for semiconductor lasers, the conduction-band state $|c\rangle$ and the corresponding valence-band state $|v\rangle$ participating in the band-to-band transitions are modeled as a "two-level system" analogous to that of Ref. [32]. The dipole moment between conduction and valence bands is denoted by d_{cv} and an explicit calculation is carried out following [35] except for the quantum confinement effects due to quantum wells. In a semiconductor medium, the carrier population follows Fermi-Dirac statistics (for both the electron and hole populations in their respective bands), as contrasted to the two-level atomic systems obeying Maxwell-Boltzmann statistics. Coulomb effects, carrier-carrier scattering, and phonon interaction all play an important role in establishing the Fermi-Dirac distribution on a subpicosecond time scale and medium behaves as a homogeneously broadened system [38].

The polarization $P(z,t)$ caused by the field $E(z,t)$ in a medium along the z axis can be obtained by taking the trace of the induced dipole moment d_{cv} with the density matrix and summing over all possible band-to-band transitions ω_t,

$$P(z,t)=\sum_{\omega_t}[d_{vc}\rho_{cv}(z,t)+d_{cv}\rho_{vc}(z,t)] \tag{1}$$

or

$$P(z,t)=\int d'D(\omega_t)[\rho_{cv}(z,t)+\rho_{vc}(z,t)]d\omega_t \tag{2}$$

where $d_{cv}=d_{vc}=d'$ is taken as real and $D(\omega_t)$ is the density of states per unit volume; ρ_{cv} and ρ_{vc} are the off-diagonal elements of the density matrix. The polarization in the medium in a Fabry-Pérot cavity can also be expanded in terms of complex amplitudes and the cavity eigenmodes [32],

$$P(z,t)=\sum_n \frac{P_n(t)}{2}\exp[-i(\nu_n t+\phi_n)]U_n(z)+\text{c.c.} , \tag{3}$$

where $P_n(t)$ is the complex amplitude, ν_n the frequency, ϕ_n the phase, and $U_n(z)$ the cavity mode profile for the nth-order mode of the empty resonator. The complex amplitude $P_n(t)$ is obtained from Eqs. (2) and (3) in the rotating-wave approximation [32,37],

$$P_n(t)=\frac{2}{\mathcal{L}}\exp[i(\nu_n t+\phi_n)] \times\int_0^L\left[\int D(\omega_t)d'\rho_{cv}(z,t)d\omega_t\right]U_n^*(z)dz , \tag{4}$$

where

$$\mathcal{L}\equiv\int_0^L|U_n(z)|^2dz$$

is a normalization factor.

To obtain $P_n(t)$, the off-diagonal element of the density matrix $\rho_{cv}(z,t)$ is evaluated by solving the formal density equations for a two-level or a two-band, i.e., for a semiconductor laser [30,33–37] under steady-state conditions. We take a special case of a semiconductor medium from which RPG structure can be fabricated relatively easily. The density-matrix equations for a semiconductor laser are used from Refs. [30,34], with a simplified notation for the components of $\rho(z,t)$,

$$\dot\rho_{cc}=-\gamma_c(\rho_{cc}-\bar\rho_{cc})-\frac{i}{\hbar}(V_{cv}\rho_{vc}-V_{vc}\rho_{cv}) , \tag{5}$$

$$\dot\rho_{vv}=-\gamma_v(\rho_{vv}-\bar\rho_{vv})+\frac{i}{\hbar}(V_{cv}\rho_{vc}-V_{vc}\rho_{cv}) , \tag{6}$$

$$\dot\rho_{cv}=-(\gamma+i\omega_t)\rho_{cv}+\frac{i}{\hbar}V_{cv}(\rho_{cc}-\rho_{vv}) , \tag{7}$$

and

$$\dot\rho_{ve}=\dot\rho^*_{cv} \tag{8}$$

where the dot means the time derivative and where the light-matter interaction is contained in the term $V_{cv}=V^*_{vc}$, γ_c and γ_v are intraband energy relaxation rates for the conduction and valence band, respectively, and are connected to T_1 [32]. Here, γ is the polarization relaxation rate ($\gamma^{-1}=T_2$ where T_2 is the dipole dephasing time) and ω_t is the transition frequency. $\bar\rho_{cc}$ and $\bar\rho_{vv}$ are the occupation probabilities of electrons and holes in quasithermal equilibrium and are determined by quasi-Fermi levels of the conduction and valence bands, respectively. The quasi-Fermi levels result from the pump source, e.g., optical or electrical pumping. Spontaneous emission is not included in this simple model. The light-matter interaction term can be written explicitly as

$$V_{cv}=-\frac{d'}{2}\sum_n E_n(t)\exp[-i(\nu_n t+\phi_n)]U_n(z)+\text{c.c.}, \tag{9}$$

where the summation runs over all of the optical modes.

In order to calculate the first- and third-order terms of the induced polarization, Eqs. (5)–(8) are solved (see Appendix) using slowly varying amplitude approximation, leading to rate equation approximation. Contributions to the linear gain coefficient α, saturation parameter β, and frequency pulling and pushing terms are derived using the explicit expression for polarization in the self-consistency equations [32]. For a single longitudinal mode (RPG lasers inherently operate at a single longitudinal mode because of large mode spacing in the short cavity) including only up to third-order polarization terms, we find

$$\dot E=\frac{-\nu}{2Q}E+\alpha\left[1-\frac{\beta}{\alpha}E^2\right]E , \tag{10a}$$

$$\nu+\dot\phi=\Omega+\frac{\alpha(\omega_t-\nu)}{\gamma}\left[1-\frac{\beta}{\alpha}E^2\right] , \tag{10b}$$

where Ω and Q denote the passive mode and "Q" of the resonator. Here,

$$\alpha=\alpha_0\left[1-\frac{\sin(k_nL_z)}{k_nL_z}\cos\left[2\varphi_n+(m-1)\frac{\pi k_n}{k_r}\right]\times\frac{\sin(m\pi k_n/k_r)}{m\sin(\pi k_n/k_r)}\right] \tag{11a}$$

and, where α_0 depends on quasi-Fermi levels $\bar\rho_{cc}$ and $\bar\rho_{vv}$ and polarization relaxation rate γ [see Appendix, Eq. (A17b)],

$$\beta=\frac{3}{8}\frac{d'^2\gamma}{\hbar^2}\left[\frac{\gamma_c+\gamma_v}{\gamma_c\gamma_v}\right]\frac{\alpha_0}{(\omega_t-\nu_n)^2+\gamma^2}\left[1-\frac{4}{3}\frac{\sin(k_nL_z)}{k_nL_z}\cos\left[2\varphi_n+(m-1)\frac{\pi k_n}{k_r}\right]\frac{\sin(m\pi k_n/k_r)}{m\sin(\pi k_n/k_r)}\right.$$
$$\left.+\frac{1}{3}\frac{\sin(2k_nL_z)}{2k_nL_z}\cos\left[4\varphi_n+2\pi(m-1)\frac{k_n}{k_r}\right]\frac{\sin(2m\pi k_n/k_r)}{m\sin(\pi k_n/k_r)}\right] . \tag{11b}$$

β, the saturation parameter, depends both on polarization relaxation γ and intraband relaxation rates γ_c and γ_v and through α_0 it also depends on quasi-Fermi levels which take into account interband relaxations. Typically, large values of γ, 10^{13}/sec [34,36], allow the use of the rate-equation approximation. In the above expressions, $k_n = n\pi/L$ with n (integer) that the number of half wavelengths in the unpumped cavity mode and L the total cavity length, L_z ($\ll \lambda$) is the thickness of an individual quantum-well gain section, $k_r = 2\pi/\lambda_r$ where λ_r is the resonant wavelength set by the physical spacing of the quantum wells, m is the total number of quantum wells, and $\phi_n = k_n a_0$ where a_0 is the spacing of the first quantum well from the $z=0$ end of the resonator. Here, α_0 is the usual gain coefficient (scaled appropriately for the thin-gain sections) as given in the Appendix [cf. Eq. (A17b)]. These equations show a resonance behavior, the term $\{\sin(m\pi k_n/k_r)/[m \sin(\pi k_n/k_r)]\}$ is just $(-1)^{m+1}$ for $k_n = k_r$ and is of order $1/m$ for $k_n \neq k_r$, the width of the resonance scales inversely as the number of quantum wells. This behavior has been discussed previously for RPG lasers [3]. Additional insight can be gained into these equations by considering the resonance case $n=m$, $k_n = k_r$, and $\phi_n = \pi/2$. Under these conditions, $\cos[2\varphi_n + (m-1)\pi k_n/k_r] \rightarrow (-1)^m$ and Eqs. (11) simplify to

$$\alpha = \alpha_0 \left[1 + \frac{\sin(k_n L_z)}{k_n L_z} \right] \tag{12a}$$

and

$$\beta = \frac{3}{8}\frac{d'^2\gamma}{\hbar^2}\left[\frac{\gamma_c+\gamma_v}{\gamma_c\gamma_v}\right]\frac{\alpha_0}{(\omega_t-\nu)^2+\gamma^2} \times \left[1+\frac{4}{3}\frac{\sin(k_nL_z)}{k_nL_z}+\frac{1}{3}\frac{\sin(2k_nL_z)}{2k_nL_z}\right] . \tag{12b}$$

For $k_n L_z \ll 1$, which is the case for quantum-well structures, the gain is doubled on resonance and the saturation parameter is increased by a factor of $\frac{8}{3}$.

For an insight into the modal frequency behavior, we neglect $\dot{\phi}$, i.e., ignore dispersion effects [32,36], and write the equation

$$\nu \simeq \Omega + \frac{(\omega_t - \nu)}{\gamma}(\alpha - \beta E^2) . \tag{13}$$

When the term βE^2 is small, we find that mode pulling is increased for RPG lasers (compared to the conventional uniform gain medium lasers) because of gain enhancement.

Equation (13) can be recast as

$$\nu = \frac{\Omega + S\omega_t}{1+S} , \tag{14}$$

where

$$S \equiv \frac{\alpha - \beta E^2}{\gamma} . \tag{15}$$

S is the stability factor. From Eq. (14) it is seen that the laser frequency ν is a weighted average of passive cavity mode frequency Ω and medium frequency ω_t with weighting factors unity and S, respectively. For $S \ll 1$ the laser frequency approaches Ω, and with $S \gg 1$ the operating frequency is "pulled" towards the medium frequency, especially in the case of a poor cavity Q, $\nu \approx \omega_t$.

In the steady-state case, Eq. (10a) gives

$$\alpha - \beta E^2 = \frac{\nu}{2Q} , \tag{16a}$$

which leads to a stability factor

$$S = \nu/2Q\gamma \tag{16b}$$

and

$$\nu = \frac{\gamma\Omega + (\nu/2Q)\omega_t}{\gamma + \nu/2Q} \tag{17}$$

which is the same as for the uniform gain medium lasers [32]. From this result, it is apparent that for low-Q cavities, i.e., $\nu/2Q \gg \gamma$, the operating frequency ν approaches ω_t, the medium frequency, and in case of high-Q cavities $\nu/2Q \ll \gamma$, the laser frequency ν approaches Ω, the cavity mode.

III. PHYSICAL PROCESSES IN RPG AND CEL LASERS

In this section, the fundamental processes resulting from enhanced light-matter interaction in RPG-based surface-emitting lasers [1–4,25–28] and a ring-cavity CEL [9,29] are discussed. The theory of correlated emission in the periodic-gain medium in a ring cavity was developed by Krause and Scully [29] using a fully quantum-mechanical treatment, where various coefficients for linear gain (α_{ij}) and nonlinear terms ($\beta_{ij;km}$) were derived. A complete quantum-mechanical formulation of the CEL problem was most appropriate to analyze the correlation of spontaneous emission which arises from the cross coupling of counterpropagating modes in the periodic-gain medium of a ring cavity. On the other hand, while considering a RPG medium in a Fabry-Pérot cavity, the electromagnetic fields can be described classically and a semiclassical laser theory described in Sec. II is sufficient to explain the results and predict the behavior.

Since both the ring-cavity CEL [9] and RPG surface-emitting [1–4] lasers (in a Fabry-Pérot resonator) utilize half-wave spatially periodic-gain medium, it is worthwhile to compare the fundamental principles involved in noise quenching via correlated spontaneous emission, and enhanced gain and saturation coefficients. The fundamental linewidth of laser radiation is due to spontaneous emission events in the lasing medium. In an atomic medium laser, this leads to the well-known Schawlow-Townes linewidth. In semiconductor devices, the strong coupling between the gain and the electronic contribution to the refractive index gives rise to substantial increases in this linewidth [39–41]. The linewidth in a RPG surface-emitting laser is as yet an open question. In the present discussion, we treat the problem as if it were a striated gain medium of independent oscillators and quote the results from the CEL calculations [29] for the diffusion coefficient of the relative phase angle $D(\theta)$

between the two degenerate counterpropagating modes in the ring cavity:

$$D(\theta)=(4\rho^2)^{-1}(\alpha_{11}+\alpha_{22}-2\alpha_{11}\cos\psi) - \tfrac{1}{8}(\beta_{11;11}+\beta_{22;22}+6\beta_{12;12}-8\beta_{11;22}\cos2\psi) , \quad (18)$$

where α_{ij} and $\beta_{ij;km}$ are gain and saturation coefficients, ρ denotes the average number of photons in each mode, and $\psi=\theta+(\nu_1-\nu_2)t$ where the subscripts refer to the two counterpropagating modes in the ring cavity.

In order to achieve noise quenching between the two modes, the diffusion coefficient $D(\theta)$ should vanish. This can be achieved when all gain and saturation coefficients become equal, i.e., $\alpha_{11}=\alpha_{22}=\alpha_{12}\equiv\alpha$ and $\beta_{11;11}=\beta_{22;11}=\beta_{12;12}=\beta_{11;22}\equiv\beta$. Then the diffusion coefficient will vanish provided $\psi=0$ or nonlinear saturation effects lead to $D(\theta)=0$. The equality of various coefficients α_{ij} and $\beta_{ij;km}$ can be achieved by interference of two counterpropagating modes in a ring cavity at the thin sections of a periodic-gain medium.

To show the role of periodic-gain medium, we reproduce the expressions for α_{ij} and $\beta_{ij;km}$ from Ref. [29] in original notation:

$$\alpha_{ij}\equiv\alpha_0\int_{-l/2}^{l/2} n(z)u_i(z)u_j^*(z)dz \quad (19)$$

and

$$\beta_{ij;km}\equiv\beta_0\int_{-l/2}^{l/2} n(z)u_i(z)u_j(z)u_k^*(z)u_m^*(z)dz , \quad (20)$$

where $u_l(z)$ are normal mode functions and $n(z)$ is linear density of gain medium. For traveling waves in a ring cavity the normal mode functions can be expressed as

$$u_1(z)\approx\exp(ikz), \quad u_2(z)\approx\exp(-ikz) . \quad (21)$$

It is easy to show that with spatially periodic $n(z)$ with $z=j\pi/k$, i.e., $(j\lambda/2)$ periodicity with j an integer, the diffusion coefficient vanishes, whereas for a uniform gain medium with $n(z)=n_0$ (constant) it does not. It is important to note that the ring cavity does not influence the lasing frequency; rather the periodic-gain medium provides a constructive interference between the propagating modes.

On the other hand, in RPG surface-emitting lasers the Fabry-Pérot cavity plays an important role [31]. The Fabry-Pérot cavity formed by the integrated multilayer high reflectors around a 4–5-μm-thick RPG semiconductor provides a standing-wave optical field. The antinodes of the standing-wave optical field must be in registration with the thin sections of gain medium (i.e., quantum wells) for an optimal interaction between the light and active material. Also, the short cavity length and consequent large longitudinal mode spacing leads to single longitudinal mode operation of such microlasers. As seen from Eqs. (12a) and (12b) in Sec. II, additional terms arise in both the linear gain and nonlinear saturation coefficients as a result of the $\lambda/2$ periodic medium. The filling factor (L_z/λ_r) in Eq. (A17b) in the Appendix simply indicates that the gain is proportional to the cumulative thickness of the active medium. Compared to uniform gain medium of the same total length, RPG medium can provide twice the gain but it also saturates at lower intensities. It is interesting to note that the linear gain and saturation coefficients, i.e., Eqs. (12a) and (12b) can also be derived from Eqs. (19) and (20) simply using $u_j(z)$ as Fabry-Pérot mode functions for $i=j$, etc.

For the frequency behavior of RPG lasers, Eq. (13) of Sec. II predicts strong mode-pulling effects which should lead to a stable frequency operation. The nonlinear saturation term, however, counteracts at high intensities and reduces the stability factor given in Eq. (15). It is interesting to note that the steady-state modal frequency behavior of the RPG laser under saturation conditions approaches that of a uniform medium laser. However, under pulsed and modulated conditions, RPG would exhibit highly stable frequency operation.

IV. RADIATION PATTERN OF A PERIODIC DIPOLE ARRAY

Insight into the common physics underlying the behavior of both RPG and CEL lasers can be gained by considering the radiation patterns associated with a three-dimensional periodic radiating dipole array. Assume, as in Fig. 2, an array of dipole oscillators with equal dipole moments, $p=pe_3$, aligned in the z direction and located by the position vectors

$$\mathbf{r}=\sum_j m_j a_j e_j , \quad (22)$$

where the a_j are the unit-cell distances in the e_j direc-

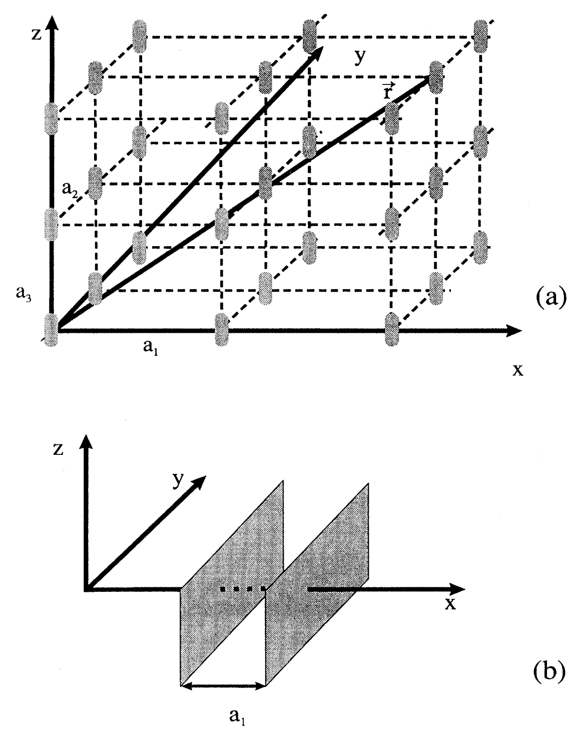

FIG. 2. A periodic dipole array. All of the dipoles are oriented in the z direction and spaced by a_1, a_2, and a_3 along the coordinate axes, respectively.

tions and m_j are integers. The radiated power from this array is simply given by [42]

$$S = S_0(n_1n_2n_3)^2\cos^2\theta\left[\prod_{j=1}^{3}\left[\frac{\sin(n_j\gamma_j/2)}{n_j\sin(\gamma_j/2)}\right]^2\right], \quad (23)$$

where

$$\gamma_1 = ka_1\sin(\theta)\cos(\varphi)+\beta_1 ,$$

$$\gamma_2 = ka_2\sin(\theta)\sin(\varphi)+\beta_2 ,$$

$$\gamma_3 = ka_3\cos(\theta)+\beta_3 ,$$

and $k=\omega/c$, n_j is the number of oscillators in the jth direction, β_j is the phase shift between adjacent oscillators in the jth direction, and $S_0=(p/2\pi\epsilon_0)^2$ is the radiation intensity of a single oscillator. Note the similarity between the structure of this equation and the equation for the gain in the RPG structure derived earlier [cf. Eq. (11a)].

For stimulated emission, the phase relationships between these oscillators are simply set by the distances a_j and the propagation direction of the initial plane wave. For an incident wave propagating in the xy plane at an angle of ψ to the x axis and polarized in the z direction, the phase shifts are $\beta_1=ka_1\cos(\psi)$, $\beta_2=ka_2\sin(\psi)$, and $\beta_3=0$. Thus, in the equatorial xy plane the expression for the radiated power simplifies to

$$S=(n_1n_2n_3)^2S_0\frac{\sin^2(n_1\gamma_1/2)}{n_1^2\sin^2(\gamma_1/2)}\,\frac{\sin^2(n_2\gamma_2/2)}{n_2^2\sin^2(\gamma_2/2)} \quad (24)$$

with

$$\gamma_1=ka_1[\cos(\varphi)-\cos(\psi)]$$

and

$$\gamma_2=ka_2[\sin(\varphi)-\sin(\psi)] .$$

Figure 3 shows the angular distribution of the radiation from an ensemble of $32(x)\times100(y)\times100(z)$ oscillators with phases determined by an incident wave traveling along the x axis from the left ($\psi=0$) with $a_2=a_3=1/(40k)$; the values for ka_1 are shown. These angular distributions have been normalized by $[(n_1n_2n_3)^2S_0]^{-1}$, i.e., the radiation in the forward direction as a result of the coherent addition of the fields from all the dipoles, is $(n_1n_2n_3)^2$ more intense than the radiation from a single dipole. The important point to note is that the radiation intensity in the backward direction is equal to that in the forward direction for a half-wave periodic structure (top). Deviations from this periodicity lead to a suppression of the backward radiation. Of course, the sensitivity of the backward radiation to the periodicity, or equivalently the wavelength, scales inversely as n_1. For increased n_2 the angular spread of the lobes is reduced, but the intensities in the forward and backward directions are unchanged. The radiation pattern in the equatorial plane is independent of n_3.

This backward radiation is a manifestation of the factor-of-2 enhancement of the gain in the RPG structure. The radiated fields add coherently to both forward and backward waves, i.e., they couple optimally to a standing-wave pattern, even in the absence of a Fabry-Pérot cavity. This also results in the elimination of spontaneous emission fluctuations in the phase between the two counterpropagating modes in a CCEL ring cavity. A photon spontaneously emitted into, say, the forward direction gives rise to amplified spontaneous emission and fluctuations in both the amplitude and phase of the radiation in the forward and backward propagating modes become correlated. On the $\lambda/2$ resonance, precisely the same fluctuation occurs for the counterpropagating

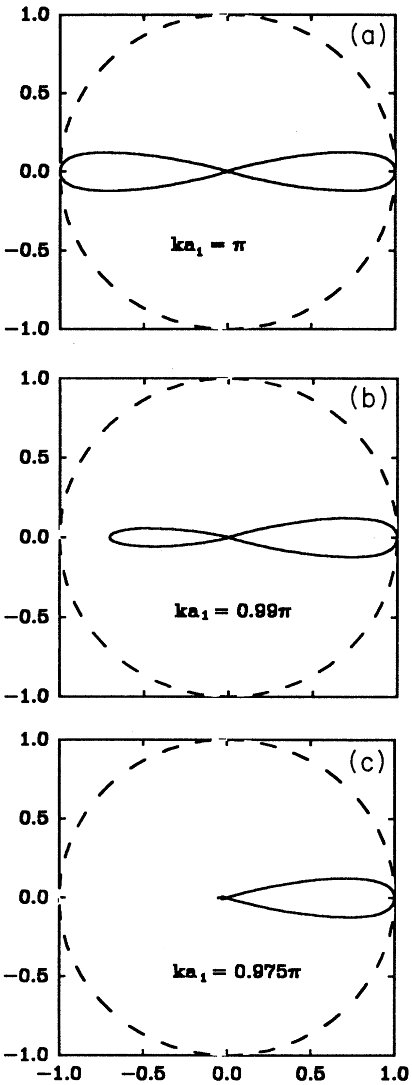

FIG. 3. Angular radiation pattern in the equitorial xy plane for a periodic array of $32(x)\times100(y)\times100(z)$ dipoles. The dipoles are closely spaced ($\lambda/40$) in the y and z directions. The spacing in the x direction is given in each segment. The relative phases have been adjusted to correspond to excitation by a plane wave incident from the x direction. Note that for a $\lambda/2$ spacing, (a) the radiation pattern is twofold symmetric with equal intensities in both the forward and backward directions. Away from the $\lambda/2$ condition, (b) and (c), the backward radiation decreases dramatically. This symmetry is responsible both for the suppression of noise fluctuations in a CEL and the gain enhancement in RPG lasers.

waves as a result of the bidirectional radiation (and gain) pattern. Off-resonance, and in particular for a homogeneous gain medium, this relationship is not preserved and the amplified spontaneous emission phase fluctuations for counterpropagating waves are uncorrelated.

V. CONCLUDING REMARKS

Before concluding, we summarize the present experimental status of RPG lasers and initial trial experiments using RPG medium in a ring cavity. Since the first demonstration of optically pumped RPG lasers [1–4], a great deal of progress has been made and cw operation of these laser structures has been achieved. Recently, high-efficiency (>45%), narrow-linewidth (~0.025 nm) cw lasing at room temperature has been demonstrated both in the $GaAs/Al_{1-x}Ga_xAs$- [26,27] and $In_{1-y}Ga_yAs/Al_{1-x}Ga_xAs$-based RPG structures. The $In_{1-y}Ga_yAs/Al_{1-x}Ga_xAs$ material system is very promising for the ring-cavity CEL, because the GaAs substrate is transparent at the lasing wavelengths. RPG structures with 20-period 8-nm-thick $In_{0.2}Ga_{0.8}As$ quantum wells and $Al_{0.2}Ga_{0.8}As$ half-wave spacers sandwiched between $Al_{0.25}Ga_{0.75}As/AlAs$ integrated multilayer high-reflectors, all fabricated in a single metal-organic chemical-vapor deposition growth cycle, have delivered ~40-mW cw power at ~930–940 nm at room temperature. Single-ended power efficiencies >43% for optical pumping at 740 nm have been demonstrated. Based on these results, we have grown RPG structures with 40 and 60 periods of 8-nm-thick $In_{0.2}Ga_{0.8}As$ quantum wells and $Al_{0.2}Ga_{0.8}As$ half-wave spacers on GaAs substrates with both sides polished. Initial photoluminescence studies show intense radiation centered around ~930 nm with an anisotropic distribution in a narrow angle rather than a uniform photoluminescence (PL) from a Lambertian source. This directionality of amplified spontaneous emission is consistent with the calculations in Sec. IV. However, to use these $In_{1-y}Ga_yAs$ structures in CEL ring cavity, an antireflection coating (reflectivity ~0.1%) is required because of high Fresnel reflectivity of $Al_{1-x}Ga_xAs$ and GaAs surfaces (~30%) and it is very difficult to use these high-index materials ($n=3.40-3.64$) at very large Brewster's angles.

In conclusion, we have developed a semiclassical theory for resonant periodic gain lasers using the particular case of the surface-emitting semiconductor lasers. The theory based on a simple two-band model analogous to the two-level atomic system predicts the gain enhancement and frequency selectivity in the resonant periodic-gain laser. The common physics of the resonant periodic-gain medium and correlated emission ring cavity laser is also discussed.

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APPENDIX

The solution of the density-matrix equations is carried out using a perturbative procedure [32], starting from a formal integration of Eq. (7) over the time interval from $-\infty$ to t (which includes all the contributions to the polarization up to the time t). The analytic evaluation is carried out in the rate equation approximation [32] which makes the assumptions that the population difference $(\rho_{cc}-\rho_{vv})$ and other quantities ϕ_n, d', and $E_n(t)$ do not vary appreciably in a time period γ^{-1} (dipole dephasing time T_2). For simplification, only a single-mode interaction is considered such that only the nth mode terms are used from Eq. (9) of Sec. II. The following expression is obtained for ρ_{cv}:

$$\rho_{cv}=\frac{-id'E_n(t)}{2\hbar}(\rho_{cc}-\rho_{vv})\times\frac{\exp[-i(\nu_n t+\phi_n)]}{\gamma+i(\omega_t-\nu_n)}U_n(z)\ . \tag{A1}$$

Substitution of Eq. (A1) for ρ_{cv}, and using $\rho_{vc}=\rho^*_{cv}$ and $V_{cv}=V^*_{vc}$ in Eqs. (5) and (6) yields

$$\dot\rho_{cc}=-\gamma_c(\rho_{cc}-\bar\rho_{cc})-R(\rho_{cc}-\rho_{vv})\ , \tag{A2}$$

$$\dot\rho_{vv}=-\gamma_v(\rho_{vv}-\bar\rho_{vv})+R(\rho_{cc}-\rho_{vv}), \tag{A3}$$

where

$$R=\frac{d'^2}{2\hbar^2}E_n^2|U_n(z)|^2\frac{\gamma}{\gamma^2+(\omega_t-\nu_n)^2}\ . \tag{A4}$$

In a steady-state case the rate equations for carriers, i.e., ρ_{cc} and ρ_{vv} for electrons and holes, respectively, Eqs. (A2) and (A3) give

$$\rho_{cc}-\rho_{vv}=\frac{\bar\rho_{cc}-\bar\rho_{vv}}{1+R/R_s}\ , \tag{A5}$$

where

$$R_s=\gamma_c\gamma_v/(\gamma_c+\gamma_v)\ .$$

Substituting this in Eq. (A1) and then the resulting expression for ρ_{cv} into Eq. (4) leads to

$$P_n(t)=(-i/\mathcal{L}\hbar)\int_0^L\int\frac{D(\omega_t)d'^2E_n(t)}{\gamma+i(\omega_t-\nu_n)}\times\frac{(\bar\rho_{cc}-\bar\rho_{vv}}{1+R/R_s}d\omega_t|U_n(z)|^2dz\ . \tag{A6}$$

An exact evaluation of the integral over ω_t is difficult because R also involves terms containing $(\omega_t-\nu_n)$. However a simplification is possible from the consideration that only those transition frequencies are effective in the polarization which are nearly resonant with the optical mode ν_n in the cavity. This amounts to including a modified density of states in the interval $\Delta\omega$ of interest; so $D(\omega_t)\Delta\omega$ is replaced with $D(\omega)$ (number of states in the spectral interval $\Delta\omega$) resulting in

$$P_n(t)=-\frac{(\omega_t-\nu_n)+i\gamma}{(\omega_t-\nu_n)^2+\gamma^2}\frac{D(\omega)}{\mathcal{L}\hbar}\times\int_0^L d'^2E_n(t)\frac{\bar\rho_{cc}-\bar\rho_{vv}}{1+R/R_s}|U_n(z)|^2dz\ . \tag{A7}$$

This relation is the same as that found for gas lasers [32]. To evaluate the integral in (A7) the usual procedure is to expand $(1+R/R_s)^{-1}$ under the assumption that $E_n(t)$, the field amplitude, is small, and hence R is much smaller than the saturation parameter R_s. Therefore, we give here only up to third-order polarization terms explicitly,

$$P_n(t)=-\frac{1}{(\omega_t-\nu_n)-i\gamma}\frac{D(\omega)}{\mathcal{L}\hbar}\left[\int_0^L d'^2E_n(t)(\bar\rho_{cc}-\bar\rho_{vv})|U_n(z)|^2dz-\int_0^L d'^2E_n(t)\frac{(\bar\rho_{cc}-\bar\rho_{vv})R}{R_s}|U_n(z)|^2dz\right]+\cdots\ ,$$

where the ellipsis represents higher-order terms, or ignoring higher-order terms,

$$P_n(t)\approx-\frac{1}{(\omega_t-\nu_n)-i\gamma}\frac{D(\omega)}{\mathcal{L}\hbar}(I_1+I_2)\ . \tag{A8}$$

Consider the first integral I_1 in Eq. (A8). In the RPG medium, thin-gain sections (quantum wells) are spaced at half-wave intervals. The Fabry-Pérot cavity modes are represented by $U_n(z)=\sin k_nz$ where $k_n=n\pi/L$. Since only the quantum-well region provides the gain, i.e., $(\bar\rho_{cc}-\bar\rho_{vv})\neq0$ only in these regions, the integral over the cavity length can be divided into n integrals each extending over the quantum-well thickness in each half-wave section with appropriate phase correlation between integrals. Thus,

$$I_1=d'^2E_n(t)\int_0^L \sin^2k_nz\left\{\sum_1^m\left[u_s\left[a_0+\frac{\pi(m-1)}{k_r}-\frac{L_z}{2}\right]-u_s\left[a_0+\frac{\pi(m-1)}{k_r}+\frac{L_z}{2}\right]\right]\right\}dz\ . \tag{A9}$$

Here u_s are unit step functions, $k_r=2\pi/\lambda_r$ where λ_r is the half-wave resonance set by the structure, a_0 is the distance from the edge of the cavity ($z=0$) to the center of the first quantum well, and m is the number of quantum wells of thickness L_z.

The transition dipole d' is approximately constant over the quantum-well dimension and has been pulled out of the integral. After some algebra, an analytic expression is derived for I_1, viz.,

$$I_1=d'^2E_n^2(t)(\bar\rho_{cc}-\bar\rho_{vv})\frac{mL_z}{2}\left[1-\frac{\sin k_nL_z}{k_nL_z}\cos\left[2\varphi_n+(m-1)\frac{\pi k_n}{k_r}\right]\frac{\sin(m\pi k_n/k_r)}{m\sin(\pi k_n/k_r)}\right]\ , \tag{A10}$$

where $\varphi_n=k_na_0$.

The second integral I_2 is similarly evaluated using the value of R from Eq. (A4),

$$I_2=\frac{md'^4E_n(t)}{2\hbar^2R_s}(\bar\rho_{cc}-\bar\rho_{vv})\frac{\gamma}{(\omega_t-\nu_n)^2+\gamma^2}\frac{3L_z}{8}\left[1-\frac{4}{3}\frac{\sin(k_nL_z)}{k_nL_z}\cos\left[2\varphi_n+(m-1)\frac{\pi k_n}{k_r}\right]\frac{\sin(m\pi k_n/k_r)}{m\sin(\pi k_n/k_r)}+\frac{1}{3}\frac{\sin(2k_nL_z)}{2k_nL_z}\cos\left[4\varphi_n+2\pi(m-1)\frac{k_n}{k_r}\right]\frac{\sin(2m\pi k_n/k_r)}{m\sin(2\pi k_n/k_r)}\right]\ . \tag{A11}$$

Substituting Eqs. (A10) and (A11) for integral terms in Eq. (A8), the complex polarization $P_n(t)$ can be written up to third order as

$$P_n(t)=P_n^{(1)}(t)+P_n^{(3)}(t)\ , \tag{A12}$$

where the superscripts refer to the dependence of $E_n(t)$. Gain and lasing frequency are determined using the self-consistency equations from the semiclassical laser theory [32], namely,

$$\dot E_n+\frac{\nu}{2Q_n}E_n=\frac{-\nu}{2\epsilon_0}\mathrm{Im}[P_n(t)]\ , \tag{A13}$$

$$\nu_n+\dot\phi_n=\Omega_n-\frac{\nu}{2\epsilon_0E_n}\mathrm{Re}[P_n(t)]\ , \tag{A14}$$

where Q_n is the cavity Q and Ω_n is the nth mode of the cold cavity. Using the explicit expression for $P_n(t)$ in Eqs. (A13) and (A14) we find for the single-mode case,

$$\dot E=\frac{-\nu}{2Q}E+(\alpha-\beta E^2)E\ , \tag{A15}$$

$$\nu+\dot\phi=\Omega+\frac{\omega_t-\nu}{\gamma}(\alpha-\beta E^2)\ , \tag{A16}$$

where

$$\alpha=\alpha_0\left[1-\frac{\sin(k_nL_z)}{k_nL_z}\cos\left[2\varphi_n+(m-1)\frac{\pi k_n}{k_R}\right]\times\frac{\sin(m\pi k_n/k_r)}{m\sin(\pi k_n/k_r)}\right] \tag{A17a}$$

with

$$\alpha_0=\frac{\nu d'^2}{2\epsilon_0\hbar}\left[\frac{\gamma}{(\omega_t-\nu)^2+\gamma^2}\right]D(\omega)(\bar{\rho}_{cc}-\bar{\rho}_{vv})\left[\frac{L_z}{\lambda_r}\right] \tag{A17b}$$

and

$$\beta=\frac{3}{8}\left[\frac{d'^2\gamma}{2\hbar}\right]\left[\frac{\gamma_c+\gamma_v}{\gamma_c\gamma_v}\right]\frac{\alpha_0}{(\omega_t-\nu)^2+\gamma^2}\left[1-\frac{4}{3}\frac{\sin(k_nL_z)}{k_nL_z}\cos\left[2\varphi_n+(m-1)\frac{\pi k_n}{k_r}\right]\frac{\sin(m\pi k_n/k_r)}{m\sin(\pi k_n/k_r)}+\frac{1}{3}\frac{\sin(2k_nL_z)}{2k_nL_z}\cos\left[4\varphi_n+2(m-1)\frac{\pi k_n}{k_r}\right]\frac{\sin(2m\pi k_n/k_r)}{m\sin(2\pi k_n/k_r)}\right]. \tag{A17c}$$

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