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Quantum noise properties of the laser: Depleted pump regime

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We show that a laser may generate amplitude-squeezed or antibunched light in a regime well above threshold, where depletion of the ground state by incoherent pumping is significant. Simple analytical solutions are presented to illustrate which atomic-level schemes and pumping rates are optimal.

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I. INTRODUCTION

The search for novel sources of radiation, specifically antibunched and squeezed light, has received a great deal of theoretical and experimental attention in the past decade or more [1]. A number of passive nonlinear systems have been used to generate squeezed light, in continuous wave and pulsed fashion. Squeezing light with a large coherent amplitude is difficult, however, due to the sensitivity of the source nonlinear interferometers to noise [2]. Ideally, one would like to have access to a squeezed-state laser which would produce such high intensity squeezed output, for applications ranging from quantum limited optical communications and ultra-high-precision measurements to atom-field interactions [3].

Intensity squeezed light has been successfully generated using semiconductor lasers with sub-Poissonian pumping [4]. Control of the pumping statistics is crucial and is achieved by a large series resistor which regulates the pump current; its sub-Poissonian statistics are then transferred to the laser output. The sub-Poissonian pumping of other laser systems is not so simple, however, and their potential as squeezed-state sources is apparently diminished [5].

In this paper we discuss the operation of a laser well above threshold, where depletion of the ground-state population by the incoherent pumping process is taken into account. The model is originally due to Lax and Louisell (LL) [6], who derived quantum Langevin rate equations for the atomic populations and intracavity photon number by adiabatically eliminating the polarization of the lasing transition. This procedure is based on the assumption that the polarization damping rate is large by virtue of a large phase damping contribution, produced by, for example, elastic collisions. The model was recently used in a discussion of the quantum noise properties of lasers with sub-Poissonian pumping [7]. With the exception of Ref. [8], where squeezing in a three-level laser is discussed, in all the previous treatments the pumping rate is treated as a parameter, which relies on the ground state of the three atomic levels being essentially undepleted. This assumption is removed here, and depletion of the pump is properly included in our analysis. Depletion is important far enough above threshold where the steady-state photon number becomes a nonlinear function of the pump parameter w_{20}. We show that sub-Poissonian photon statistics of the intracavity field and squeezing of the output intensity may be achieved, and discuss the optimal parameter regimes. Our results are in agreement with a numerical treatment of a laser model without phase damping treated by Ralph and Savage [9], in which the polarization dynamics of the lasing transition is included. These authors emphasize that the regimes where interesting quantum noise effects occur are not beyond experimental bounds. Our analytic approach has the advantage of making the physics more transparent, and allows direct comparison with earlier work [7].

The remainder of this paper is organized as follows. In Sec. II, we outline the theoretical model of the laser, and discuss the steady-state operating conditions above threshold. The linearized quantum dynamics is treated in Sec. III, and simple analytic solutions for the intracavity photon number fluctuations and output intensity squeezing spectrum are derived. Section IV provides a discussion of our results, and in Sec. V we give some conclusions.

II. THEORETICAL MODEL

The atomic-level scheme for the laser is indicated in Fig. 1. We follow the notation of LL, where $\hat{N}_i$ ($i=0,1,2$) are the atomic population operators and $\hat{n}$ the laser mode photon number operator; w_{ij} represents the incoherent transition rate from level j to i, due to either spontaneous emission or pumping (w_{20} is the pump parameter) and $\Gamma_i=\sum_j w_{ji}$ is the total incoherent rate out of level i to all other levels. The stimulated emission coefficient $\Pi=2\mu^2/\Gamma$ with μ and Γ the electric dipole coupling and polarization decay rate for the lasing transition, respectively. The atomic polarization has been adiabatically eliminated, on the basis that $\Gamma=(\Gamma_1+\Gamma_2)/2+\Gamma_{\rm ph}$ is the largest decay rate in the problem, which requires $\Gamma_{\rm ph}$ the elastic collision damping rate to dominate the spontaneous decay rates. The quantum Langevin rate equations are given by

$$\begin{aligned}
\frac{d}{dt}\hat{N}_0 &= -w_{20}\hat{N}_0+\Gamma_1\hat{N}_1+w_{02}\hat{N}_2+\hat{G}_0 , \\
\frac{d}{dt}\hat{N}_1 &= -(\Gamma_1+\Pi\hat{n})\hat{N}_1+(w_{12}+\Pi\hat{n})\hat{N}_2+\hat{G}_1 , \\
\frac{d}{dt}\hat{N}_2 &= w_{20}\hat{N}_0+\Pi\hat{n}\hat{N}_1-(\Gamma_2+\Pi\hat{n})\hat{N}_2+\hat{G}_2 , \\
\frac{d}{dt}\hat{n} &= -\gamma\hat{n}+\Pi\hat{n}(\hat{N}_2-\hat{N}_1)+\hat{G}_p .
\end{aligned} \tag{2.1}$$

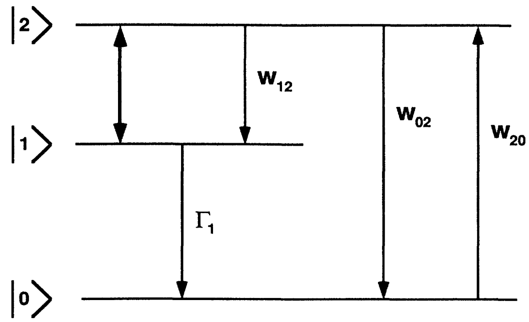

FIG. 1. Atomic-level scheme for the laser. Lasing occurs between levels 1 and 2.

The general form of the equations may be written down by inspection. The equation for $\hat{N}_0$ can be eliminated by the conservation of population condition

$$\hat{N}_0+\hat{N}_1+\hat{N}_2=\hat{1}_N \tag{2.2}$$

where N is the (constant) total number of lasing atoms and $\hat{1}_N$, the identity operator in the space of N atoms. As a result the noise operators $\hat{G}_i$ $(i=0,1,2)$ are linearly dependent:

$$\hat{G}_0+\hat{G}_1+\hat{G}_2=\hat{0}\ . \tag{2.3}$$

The diffusion matrix elements are given by the formula $\langle \hat{G}_i(t)\hat{G}_j(t')\rangle=2D_{ij}\delta(t-t')$, where the factor 2 is introduced for consistency with the notation of LL. The diffusion matrix for our laser may be calculated by using, for example, generalized Einstein relations [10], but in contrast to the model of LL, we include depletion of the ground state and its concomitant quantum fluctuations in the analysis. However, in every other respect the model is entirely conventional, in that the pumping statistics are Poissonian. The diffusion matrix elements are

$$\begin{aligned}
2D_{10}&=-\Gamma_1\hat{N}_1\ ,\\
2D_{11}&=(\Gamma_1+\Pi\hat{n})\hat{N}_1+(w_{12}+\Pi\hat{n})\hat{N}_2\ ,\\
2D_{12}&=-\Pi\hat{n}\hat{N}_1-(w_{12}+\Pi\hat{n})\hat{N}_2\ ,\\
2D_{1p}&=-2D_{2p}=\Pi\hat{n}\hat{N}_1+\Pi\hat{n}\hat{N}_2\ ,\\
2D_{22}&=w_{20}\hat{N}_0+\Pi\hat{n}\hat{N}_1+(\Gamma_2+\Pi\hat{n})\hat{N}_2\ ,\\
2D_{pp}&=\Pi\hat{n}\hat{N}_1+\Pi\hat{n}\hat{N}_2+\gamma\hat{n}
\end{aligned} \tag{2.4}$$

with $D_{ij}=D_{ji}$.

We proceed by perturbation theory, and analyze the quantum fluctuations of the populations and the photon number by linearizing about the steady-state mean value solutions of the laser in which the field is treated classically. The equations describing this semiclassical limit are just those of Eq. (2.1) in which the noise operators are dropped and the remaining operators are replaced by their mean values. These are just the usual Einstein rate equations. Mathematically this limit is justified by a scaling argument provided the number of laser atoms N is macroscopically large [11]. Two regimes of operation are delineated by the laser threshold. As the pump parameter w_{20} is increased a threshold value is reached, below this the mean laser intensity is zero, and above it is nonzero; lasing is said to occur. In this regime the inversion $D=N_2-N_1$ is fixed at a constant value independent of pumping. The steady-state solutions for the laser variables above threshold are

$$\begin{aligned}
N_0&=\frac{N(w_{02}+\Gamma_1)+D(w_{02}-\Gamma_1)}{\Gamma_1+w_{02}+2w_{20}}\ ,\\
N_1&=\frac{Nw_{20}-D(w_{02}+w_{20})}{\Gamma_1+w_{02}+2w_{20}}\ ,\\
N_2&=\frac{Nw_{20}+D(\Gamma_1+w_{20})}{\Gamma_1+w_{02}+2w_{20}}\ ,\\
D&=\frac{\gamma}{\Pi}\ ,\\
n&=n_s\left[\frac{R}{R_{\rm th}}-1\right]
\end{aligned} \tag{2.5}$$

where $R=Nw_{20}$ is the net pumping rate, in the undepleted pump regime. The threshold pump "rate" and characteristic photon number are given by

$$\begin{aligned}
R_{\rm th}&=\frac{D\Gamma_1\Gamma_2}{\Gamma_1-w_{12}}\left[1+\frac{w_{20}(\Gamma_1+w_{12})}{\Gamma_1\Gamma_2}\right]\ ,\\
n_s&=\frac{D\Gamma_1\Gamma_2[1+w_{20}(\Gamma_1+w_{12})/\Gamma_1\Gamma_2]}{\gamma(\Gamma_1+\Gamma_2-w_{12})[1+2w_{20}/(\Gamma_1+\Gamma_2-w_{12})]}\ ,
\end{aligned} \tag{2.6}$$

respectively. For the pump parameter $w_{20}\ll\Gamma_1$, n is a linear function of R in agreement with the undepleted pump limit. However, for larger pump rates the depletion becomes important and n becomes nonlinear; this is illustrated in Fig. 2. Note that the laser threshold occurs at around $w_{20}=0.006\Gamma_1$. Depletion of the

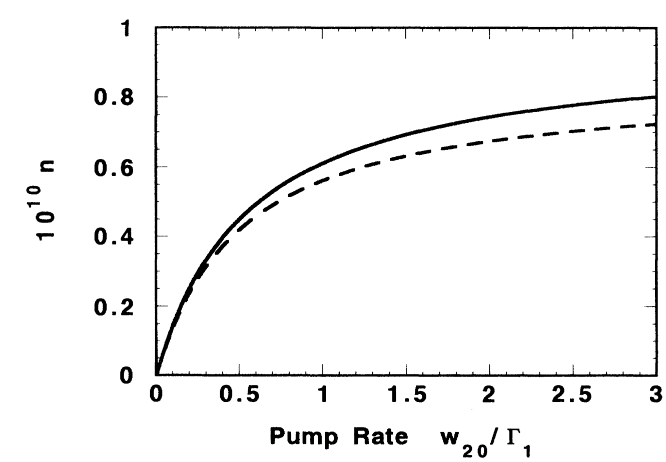

FIG. 2. Photon number as a function of pump rate above threshold. Parameters are $N=2\times10^9$, $\Pi=1\times10^{-10}$, $\gamma=0.01\Gamma_1$, and $w_{12}=0$, $w_{02}=0.1\Gamma_1$ (solid line); $w_{02}=0$, $w_{12}=0.1\Gamma_1$ (dashed line).

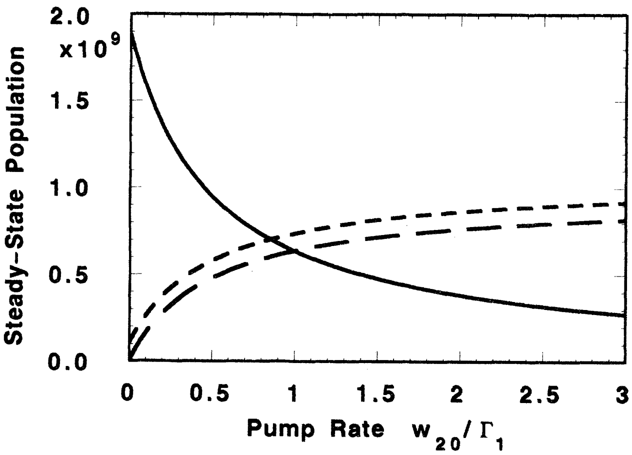

FIG. 3. Atomic populations for the same parameters as the dashed curve of Fig. 2. Solid line: N_0; long-dashed line: N_1; short-dashed line: N_2.

ground-state population is illustrated in Fig. 3.

The calculation of the output squeezing spectrum proceeds by using the linearized equations of motion to calculate the variance σ^2 in photon number, about the steady-state mean value n. The Mandel Q parameter can then be constructed from the mean and variance

$$Q=\frac{\sigma^2-n}{n} \tag{2.7}$$

where $Q>0$, $=0$, and <0 correspond to super-Poissonian, Poissonian, and sub-Poissonian photon statistics, respectively. The intensity squeezing spectrum of the output, normalized to unit shot noise is then given by, approximately [5,7],

$$V(\omega)=1+2Q\frac{1}{1+(\omega/\gamma)^2} \tag{2.8}$$

where ω is the spectral offset from the laser frequency. Sub-Poissonian photon statistics and concomitant intensity squeezing ($V<1$) in the output are signatures of the quantum-mechanical nature of the electromagnetic field. Equation (2.8) illustrates the close relationship between sub-Poissonian photon statistics inside the laser cavity and amplitude/intensity squeezing in the output. The factor of 2 which appears in the numerator is most important, and indicates that perfect amplitude noise reduction in the output is approached when the internal variance approaches one-half that for a Poissonian distribution, rather than zero, as might be expected. This is a consequence of the boundary condition at the output mirror [4,5,7].

III. LINEARIZED FLUCTUATION ANALYSIS

We now linearize the rate equations around the semiclassical steady-state values, by setting $\hat{D}=D+\Delta\hat{D}$, $\hat{N}_1=N_1+\Delta\hat{N}_1$, $\hat{N}_0=N_0+\Delta\hat{N}_0$, and $\hat{n}=n+\Delta\hat{n}$, in Eq. (2.1). The linearized equations are given by

$$\begin{aligned}
\frac{d}{dt}\Delta\hat{D}&=w_{20}\Delta\hat{N}_0+(\Gamma_1-\Gamma_2-w_{12})\Delta\hat{N}_1\\
&\quad-(2\Pi n+\Gamma_2+w_{12})\Delta\hat{D}-2\gamma\Delta\hat{n}+\hat{G}_2-\hat{G}_1\ ,\\
\frac{d}{dt}\Delta\hat{N}_1&=-(\Gamma_1-w_{12})\Delta\hat{N}_1+(w_{12}+\Pi n)\Delta\hat{D}+\gamma\Delta\hat{n}+\hat{G}_1\ ,\\
\frac{d}{dt}\Delta\hat{N}_0&=-w_{20}\Delta\hat{N}_0+(\Gamma_1+w_{02})\Delta\hat{N}_1+w_{02}\Delta\hat{D}+\hat{G}_0\ ,\\
\frac{d}{dt}\Delta n&=\Pi n\,\Delta\hat{D}+\hat{G}_p\ ,\\
\Delta\hat{N}&=0=\Delta\hat{N}_0+2\Delta\hat{N}_1+\Delta\hat{D}\ .
\end{aligned} \tag{3.1}$$

We have numerically calculated the photon number variance, and hence the Mandel Q parameter and intensity squeezing spectrum from the linearized Eq. (3.1) without further approximation. However, to gain some physical insight into the results we proceed analytically by adiabatically eliminating the atomic fluctuations. This is valid above threshold since the field changes over a relatively long time scale given by the inverse cavity bandwidth γ^{-1}. For simplicity we consider two alternative limits of the atomic-level scheme separately. As a notational convenience we drop the caret symbol on operator quantities from here on.

A. Spontaneous decay out of the lasing levels: $w_{12}=0$, $\Gamma_2=w_{02}$

The adiabatic atomic population fluctuation operators may be written

$$\begin{aligned}
\Delta N_0&=\frac{-(1-w_{02}/\Gamma_1)\Delta D+2G_0/\Gamma_1}{1+w_{02}/\Gamma_1+2w_{20}/\Gamma_1}\ ,\\
\Delta N_1&=\frac{1}{\Gamma_1}(\Pi n\,\Delta D+\Pi D\,\Delta n+G_1)\ ,\\
\Delta D&=\frac{-\gamma\Delta n+\dfrac{1}{1+w_{02}/\Gamma_1}\left[\left[\dfrac{2w_{20}/\Gamma_1}{1+w_{02}/\Gamma_1+2w_{20}/\Gamma_1}\right]G_0-\dfrac{w_{02}}{\Gamma_1}G_1+G_2\right]}{\left[\dfrac{(w_{02}+w_{20})}{1+w_{02}/\Gamma_1+2w_{20}/\Gamma_1}+\Pi n\right]}\ .
\end{aligned} \tag{3.2}$$

Assuming operation well above threshold where $n \gg n_s$ we find, using (3.2) and (2.5),

$$\frac{d}{dt}\Delta n = -\gamma \Delta n + G(t) \quad (3.3)$$

where

$$G(t) = \frac{1}{1+w_{02}/\Gamma_1+2w_{20}/\Gamma_1} G_2 - \frac{w_{02}/\Gamma_1+2w_{20}/\Gamma_1}{1+w_{02}/\Gamma_1+2_{w20}/\Gamma_1} G_1 + G_p \ . \quad (3.4)$$

Note that Eq. (3.4) reduces to Eq. (2.21) of Ref. [7], in the limit of an undepleted pump $w_{20}/\Gamma_1 \ll 1$. More generally pump depletion renormalizes the coefficients of the atomic noise operators in (3.4), and the effect of this will be discussed in Sec. IV. The steady-state variance in photon number is given by the equation

$$\sigma^2 \delta(t-t') = \frac{\langle G(t)G(t')\rangle}{2\gamma} \quad (3.5)$$

which using Eqs. (2.4) and (3.4) leads to

$$\sigma^2 = \tfrac{1}{2}n + \frac{\tfrac{1}{2}n}{(1+w_{02}/\Gamma_1+2w_{20}/\Gamma_1)^2} \times \left[1 + \frac{2w_{02}}{\Gamma_1} + \left[\frac{2w_{20}}{\Gamma_1} + \frac{w_{02}}{\Gamma_1} \right]^2 \right] + \frac{w_{02}D}{\gamma(1+w_{02}/\Gamma_1+2w_{20}/\Gamma_1)^2} \ . \quad (3.6)$$

The first term on the right is due to vacuum fluctuations transmitted by the output coupler. The second term is associated with pumping and spontaneous decay processes. The last term is directly proportional to n_s and is small when compared to n for operating conditions well above threshold. We then find

$$\sigma^2 = n - \frac{2w_{20}/\Gamma_1}{(1+w_{02}/\Gamma_1+2w_{20}/\Gamma_1)^2} n + O(n_s) \quad (3.7)$$

so that the photon statistics are sub-Poissonian. Thus from Eqs. (2.7) and (2.8) we predict intensity squeezing ($V<1$) in the output. Figure 4 shows the intensity squeezing at the laser frequency ($\omega=0$, in the rotating frame of reference), as a function of pump rate, for three different values of the stimulated emission coefficient Π.

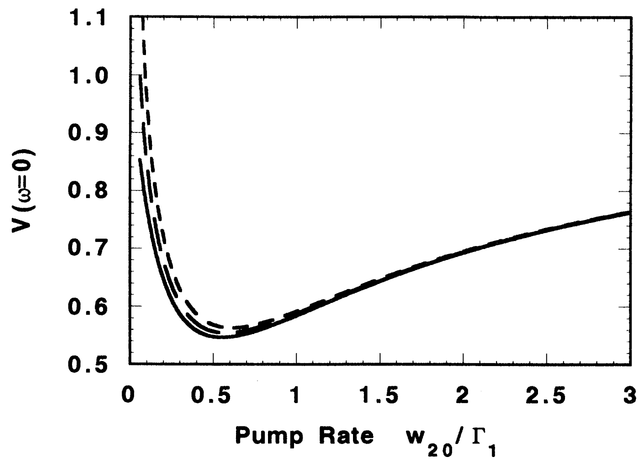

FIG. 4. Amplitude squeezing at the laser frequency as a function of the pump rate for parameters $N=2\times10^9$, $\gamma=0.01\Gamma_1$, $w_{12}=0$, $w_{02}=0.1\Gamma_1$ and $\Pi=10^{-9}$ (solid line), 10^{-10} (long-dashed line), and 5×10^{-11} (short-dashed line). Shot-noise level corresponds to $V=1$.

The magnitude of Π depends not only on the atomic species, through the transition oscillator strength, but also on the polarization damping rate of the lasing transition. With increase in Π, the degree of squeezing saturates at around 45% below shot-noise level. The physics of this result will be discussed in Sec. IV.

B. Spontaneous decay between lasing levels: $w_{02}=0$, $\Gamma_2=w_{12}$

Following the same procedure as in III A we find the equation for Δn well above threshold ($n \gg n_s$),

$$\frac{d}{dt}\Delta n = -\gamma \Delta n + G(t) \quad (3.8)$$

where

$$G(t) = \left[\frac{1-w_{12}/\Gamma_1}{1+2w_{20}/\Gamma_1} \right] G_2 - \left[\frac{2w_{20}/\Gamma_1+w_{12}/\Gamma_1}{1+2w_{20}/\Gamma_1} \right] G_1 + G_p \quad (3.9)$$

and the steady-state variance in photon number is given by

$$\sigma^2 = \tfrac{1}{2}n + \tfrac{1}{2}\frac{w_{12}}{\Pi} + \tfrac{1}{2}\left[\frac{n+w_{12}/\Pi}{1-w_{12}/\Gamma_1} \right] \left[\frac{w_{12}}{\Gamma_1} + \frac{(1-w_{12}/\Gamma_1)^2+(2w_{20}/\Gamma_1+w_{12}/\Gamma_1)^2}{(1+2w_{20}/\Gamma_1)^2} \right] . \quad (3.10)$$

The first term is due to vacuum fluctuations transmitted by the output coupling mirror. The second term is proportional to the constant atomic inversion. Rewriting the equation for σ^2 we find

$$\sigma^2 = \tfrac{1}{2}\left[\frac{n+w_{12}/\Pi}{1-w_{12}/\Gamma_1} \right] \left[1 + \frac{(1-w_{12}/\Gamma_1)^2+(2w_{20}/\Gamma_1+w_{12}/\Gamma_1)^2}{(1+2w_{20}/\Gamma_1)^2} \right] \quad (3.11)$$

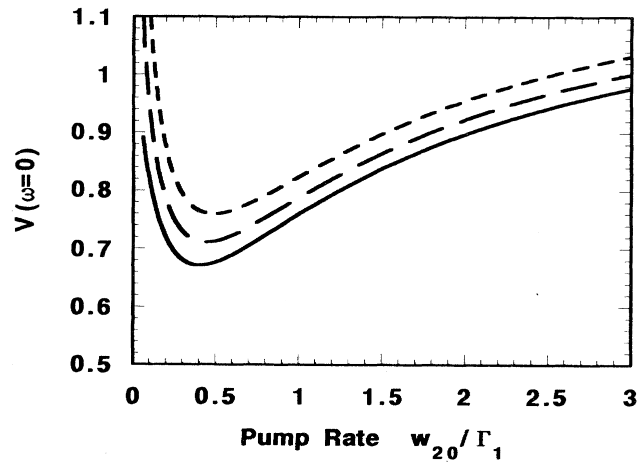

FIG. 5. Amplitude squeezing at the laser frequency as a function of the pump rate for parameters $N=2\times10^9$, $\gamma=0.01\Gamma_1$, $w_{02}=0$, $w_{12}=0.1\Gamma_1$, and $\Pi=10^{-9}$ (solid line), 10^{-10} (long-dashed line), and 5×10^{-11} (short-dashed line). Shot-noise level corresponds to $V=1$.

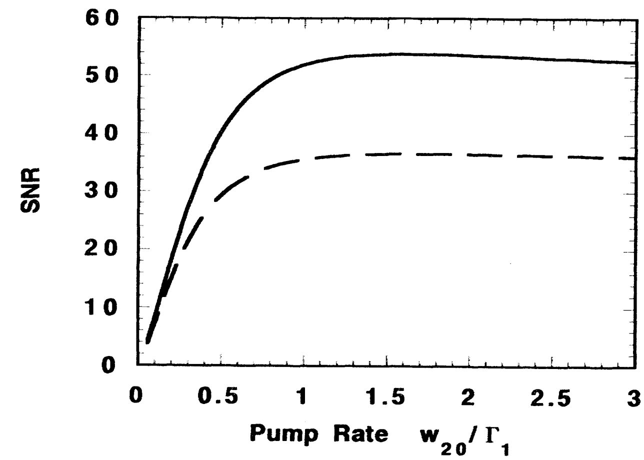

FIG. 6. Signal-to-noise ratio for the laser with the parameters of Fig. 2: $w_{02}=0.1$, $w_{12}=0$ (solid line); $w_{12}=0.1$, $w_{02}=0$ (dashed line).

which displays sub-Poissonian photon statistics of the intracavity field, and thus amplitude squeezing of the output, provided the pumping rate is an appreciable fraction of the spontaneous decay rate of the lower lasing level, and the upper level spontaneous decay rate is small. For the parameters of Fig. 5, increase in the stimulated emission coefficient Π, the squeezing saturates at around 33% below shot noise. Qualitatively Fig. 5 is similar to Fig. 4, except that the degree of squeezing is reduced.

IV. DISCUSSION

It is clear from the results presented that when the pumping rate w_{20} is comparable with the lower lasing level spontaneous decay rate Γ_1, sub-Poissonian photon statistics and output squeezing result. Neither are expected from conventional laser theories in which the pumping rate is treated as a parameter, independent of the laser operating conditions. In this limit the ground state 0 acts as a reservoir of population, and the pump can transfer population to the excited state 2, independent of the laser dynamics. However, the ability of the pump to populate the upper lasing level is conditional on whether the ground state is being replenished sufficiently fast to support the pumping, and this depends on the net stimulated and spontaneous emission rates from the lasing levels.

An analysis of the derivation of Eq. (3.7) indicates that the predicted sub-Poissonian photon statistics and squeezing are due to a reduction in the role of pump noise and spontaneous emission from the upper atomic level, when w_{20} is increased from the undepleted pump regime (where the photon statistics are Poissonian and the output is shot noise limited), towards Γ_1. For larger pump rates these noise terms continue to decrease, and one might expect the degree of squeezing to increase. The lower-level decay has little effect on squeezing in the undepleted pump regime provided only that $\Gamma_1 \gg \Gamma_2$. However, as the pump rate approaches Γ_1 this becomes increasingly important, since the random spontaneous emission events are then resolved, causing a reduction in the degree of squeezing. The two opposing tendencies may be seen by inspection of the pump rate dependence of the coefficients of the noise terms in Eq. (3.4). Physically this is due to mutual dependence of the pumping and lower-level decay which is manifest as w_{20} is increased. The quantum noise features are illustrated in Figs. 4 and 5.

An important measure of the significance of the squeezing reported here is the signal-to-noise ratio ($R_{\rm SN}$) of the output, illustrated in Fig. 6. The $R_{\rm SN}$ is defined as the ratio

$$R_{\rm SN}=\frac{n}{NV(\omega=0)} \tag{4.1}$$

where for convenience we have scaled the result to the number of lasing atoms N. To compare with a laser in the undepleted pump regime, which produces an almost coherent output, observe the linear portion of the curves, for $w_{20} \ll \Gamma_1$. The $R_{\rm SN}$ initially increases as the pump is depleted, and then effectively saturates or more correctly peaks and falls slowly, as spontaneous emission noise from the lower lasing level reduces the degree of squeezing.

V. CONCLUSION

In the undepleted pump regime above threshold, and with Poissonian pumping statistics, it is well known that the laser photon statistics are Poissonian, and the output is coherent or shot-noise limited. We have considered modifications to these quantum noise properties of a laser with a fixed number of atoms as the pump parameter is increased from above threshold, where the ground state is not significantly depleted, to well above threshold where the ground-state population is largely depleted. We have shown by analysis of two examples, that sub-Poissonian photon statistics and amplitude/intensity squeezing of the output may be found in an intermediate regime where $w_{20} \lesssim \Gamma_1$, provided that $\Gamma_1 \gg \Gamma_2$ (the usual design criterion for a laser). In the model considered, the most favorable regime is when the spontaneous emission between the lasing levels is minimized $w_{12}=0$. At higher pump rates, of less relevance to experimental investigation, the squeezing is reduced by spontaneous emission noise from the lower lasing level which although not a significant noise source for low pumping, acts as a bottleneck, and becomes increasingly important as w_{20} in increased.

Finally we note our analytical theory based on quantum Langevin rate equations complements the numerical analysis of Ralph and Savage who found output squeezed amplitude fluctuations, in a model in which the laser polarization dynamics is included [8,9]. Our analysis shows that the polarization dynamics is not important for this observation. The latter authors also emphasize that the interesting quantum noise properties are amenable to experimental investigation, and we hope our analysis will assist in such investigations. Important questions to address concern the feasibility of pumping far (≈ 100 times) above threshold, and the role of atomic number fluctuations in the laser medium.

Note added in proof. We have recently received copies of interesting work by H. Ritsch, P. Zoller, C. W. Gardiner, and D. F. Walls [12], and by T. C. Ralph and C. M. Savage. These concern the production of sub-Poissonian light from four (or more) level lasers, and the role of coherent pumping in such systems.

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[1] See the special feature editions: J. Mod. Opt. **34** , (6) (1987); **34**, (7) (1987); J. Opt. Soc. Am. B **4**, (10) (1987).

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